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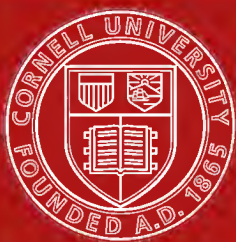
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TRANSFORMERS.

*A TREATISE ON
THE THEORY, CONSTRUCTION, DESIGN, AND USES OF
TRANSFORMERS, AUTO-TRANSFORMERS,
AND CHOKING COILS.*

BY

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PROFESSOR OF ELECTRICAL ENGINEERING AT THE MERCHANT VENTURERS'
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With 18 Plates and 332 Figures in the Text.



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√

PREFACE.

THIS book has been written with the object of giving a thorough exposition of the scientific principles upon which the action of a transformer is based, of the application of these principles to the design of such apparatus, and of their construction and uses. Much material has, of course, been collated from various sources, but much of the matter is original, and is here published for the first time. In particular, an entirely new method of designing by the second author is given in Chapter X., which eliminates the series of trial designs which have hitherto been customary, and leads to a good design straight away. This method is so simple and direct that even an inexperienced designer can easily get at once a preliminary design to meet any given conditions with full confidence that he is making the best use of the material employed.

The method also leads to very definite statements as to the relative value of different types of transformers, or different materials, and as to the effects of the several variables on the cost. This chapter has caused a delay of over eighteen months in the publication of the book, as all the rest was in type before the clue was obtained which enabled the method here given to be evolved.

The authors have aimed at uniformity and simplicity in the symbols used throughout the book, and with this end have adopted the principle of always using the same symbol for any quantity, and of using that one for no others. When necessary, distinction is drawn between different quantities of the same kind by using suffices; at first sight this sometimes appears cumbersome in places, but when the method becomes familiar this feeling disappears, and the saving by its use is enormous. The suffices are practically self-explanatory, and the whole of the symbols can be mastered with a minimum of mental

effort. The second author has devised a mode of distinguishing between the various values of an alternating function without the use of suffices, which would clash with the suffices used for distinguishing between different quantities of the same kind, and without the use of different types, which is both difficult to remember, difficult to write, and interferes with the principle of confining one style of type to one group of quantities (geometrical and mechanical, electric, magnetic, thermal, etc.). The same symbol letter is used for all values, and by itself stands for effective values whether alternating or not: an *I* worked into the symbol indicates the instantaneous value, an *M* the maximum value, and the usual algebraic convention of a bar over it denotes the mean value. But for the expense involved, an *E* or *R* might with advantage be used for the R.M.S. value of an alternating quantity. These symbols have required new types, and the authors wish to express their thanks to the publishers for their willingness to incur the necessary expense, and for their patience under the delay occasioned by the tenth chapter. Arrangements are being made whereby these special types will be available for general use at a reasonable cost.

Units are a vexed question, on which the book is cosmopolitan. Theory is quite independent of units, and should be discussed without any reference to them. No assumptions whatever as to units are involved in any of the formulæ given in this book, and all converting numbers, which so often needlessly encumber analytical investigations of electrical matters, are strictly tabooed from the formulæ. Units are not required until the magnitudes of particular quantities or constants have to be expressed, and there, alas! so many people omit them. To make everything as ready as possible for reference, the units are always explicitly stated whenever any magnitudes are given, whether in the text, in tables, or in graphs, and all constants are expressed in all the units that are likely to be of service. Engineers have already thrown over the C.G.S. unit of magnetic field in favour of the ampere-turn per millimetre or per inch; we would like to see them emancipated in the same way from the Line and its derivatives, practically the only C.G.S. units that they do employ. The Volt-second and its thousandal submultiples are in reality more convenient, and belong to the same system as the other electrical units in universal use.

The authors believe in designing in millimetres for metric shops, but if inches have to be worked to they prefer getting out the design straight away in terms of that unit rather than first in millimetres and then converting.

It saves labour, and the only difference is that the various constants have to be read on a different scale when they are taken from the curves. For the convenience of designers and students, the publishers are also issuing blank schedules for design for the most important types arranged on the lines of the examples worked out at the end of Chapter X.

The authors are very much indebted to the various persons and firms mentioned in the text who have so kindly furnished particulars of their apparatus or experiments, supplied drawings, lent electros, or in any other way assisted in the production of the book. Without such co-operation a book of this description would lose much of its value.

H. B.

D. R.

BRISTOL, *November* 1910.

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ERRATA.

Page 32, title to fig. 3·07, *for* “Kapito,” *read* “Capito.”
 „ 34, „ 3·12, do. do.
 „ 154, „ 9·25, *for* “Société Anonyme, Charleroi,”
read “Ateliers de Constructions
 Electriques de Charleroi.”

TRANSFORMERS.

CHAPTER I.

GENERAL PRINCIPLES.

Principle of Action.—When two circuits are so placed that a current in one of them sets up a magnetic flux through the other, we say that we have “mutual induction” between them. For example, if we have two coils, A and B, one of which, A, may be joined to a source D, while the other, B, is connected to a galvanometer G; then by closing the circuit of A, we produce a flux which is interlinked with B, giving rise to an induced electro-motive force, and, since the circuit of B is closed, to a current through the galvanometer. This induced current is only momentary, and lasts until the current in A has attained a steady value. In order to obtain an uninterrupted current through G, we require a continually varying flux, which can be set up by an oscillating current only. Such a current is most suitably procured from an alternator.

The field which is set up by an alternating current changes incessantly in magnitude and direction, so that the induced E.M.F. in coil B is obliged to do likewise. Consequently, not only the original current in A, but also the induced one in B, are both alternating.

The above figure shows that the magnetic flux through coil A is greater than the flux interlinked with B. Even within A, the flux per unit area varies, and if the coil consists of a number of convolutions (see fig. 1·02), those in the middle of the coil enclose more flux than those near the ends, and the flux through B will again be considerably less. The flux which is

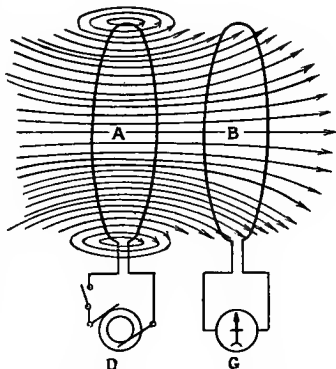


FIG. 1·01.—Principle of an Induction Coil.

interlinked with A only, constitutes the "leakage flux," and its amount is greater the more we separate B from A. It may, however, be considerably reduced by arranging the two coils concentrically.

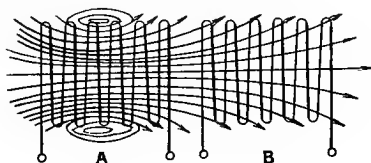


FIG. 1'02.—Magnetic Leakage.

The combination of coils A and B is called an "induction coil" or "transformer." Its inductive action may be greatly intensified by placing both coils on a soft iron ring. This reduces at once the reluctance of the magnetic path and the percentage leakage flux, while it increases the total flux con-

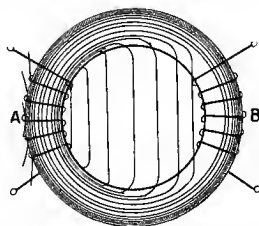


FIG. 1'03.—Field of an Iron Ring Transformer.

siderably. The magnetic field of an iron ring induction coil is shown in the next figure, from which it is evident that the leakage flux may be further reduced by moving coil B towards A. Its value will only be a small fraction of the total flux when A is placed so as to enclose B. If this arrangement

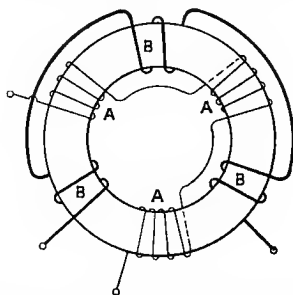


FIG. 1'04.—Transformer with Subdivided Windings.

is not suitable, it is advisable to subdivide both coils and to arrange A and B in alternate sections, as shown in fig. 1'04.

The alternating flux, which gives rise to an induced E.M.F. in B, also sets up an E.M.F. in A on account of self-induction. Let N_1 and N_2 be the

number of turns in coils A and B respectively, and \mathcal{P} the flux interlinked with both windings; then the instantaneous E.M.F.'s in the two coils are very nearly

$$\left. \begin{aligned} E_1 &= -N_1 \frac{d\mathcal{P}}{dT} \\ E_2 &= -N_2 \frac{d\mathcal{P}}{dT} \end{aligned} \right\} \quad \quad \quad 1.01.$$

Hence,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \text{and} \quad q = \frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \quad \quad 1.02,$$

where E_1 and E_2 are the R.M.S. values of the E.M.F.'s, and q is called the "ratio of transformation."

The inducing coil A, which is fed from an external source, is usually called the primary winding, or simply the "primary," and the coil B carrying the induced current, the "secondary." If the secondary E.M.F. is the greater, we speak of the apparatus as a "step-up" transformer; if it is the smaller, we have a "step-down" one.

Equation 1.02 is correct only for an ideal induction coil having no magnetic leakage. In practice, such an apparatus cannot be made; but by sufficiently subdividing the two windings, or by using a suitable concentric arrangement, it may be very nearly approached.

When the secondary circuit is completed through a bank of lamps, or other load, its winding will carry a current which more or less opposes the magnetic effect of the primary current. In order to maintain the flux necessary for inducing the E.M.F.'s, the primary current now increases until the resultant excitation due to both windings is just that required to produce this flux. If, as is usual at full load, this resultant excitation required for magnetising is very small compared with the excitation of either coil separately, the primary and secondary currents will be in almost opposite phases.

Magnetic Circuit.—The path of the magnetic flux of a transformer may be totally within the iron, as in the ring transformer illustrated previously, or its course may lie partly in the iron and partly in air. The most noteworthy of the latter type was Swinburne's hedgehog transformer, shown in fig. 1.05. Its core

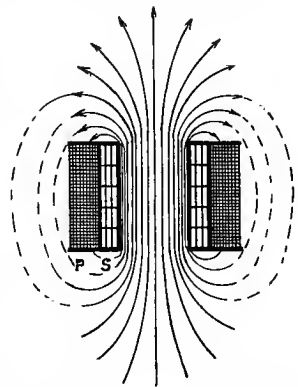


FIG. 1.05.—Swinburne's Hedgehog Transformer.

consisted of a bundle of soft iron wires with their ends spread out, resembling

the back of a hedgehog. It was claimed for this design that, as the hysteresis losses are small on account of the small amount of iron, the annual waste of energy of an occasionally loaded transformer (such as is used for lighting purposes only) would be lower than the waste in one the magnetic path of which lies totally within the iron. For it must be understood that the primary of a lighting transformer runs for 24×365 hours annually, while the secondary is partly or fully loaded for a few hours only each day. This advantage of the hedgehog type over the other one is, however, more imaginary than real, while its disadvantages are numerous.

In all modern designs the magnetic circuit lies totally within the iron, the hedgehog type having been discarded.

Core and Shell Transformers.—The iron frame of a modern transformer consists of cores and yokes. If the cores are surrounded by the two

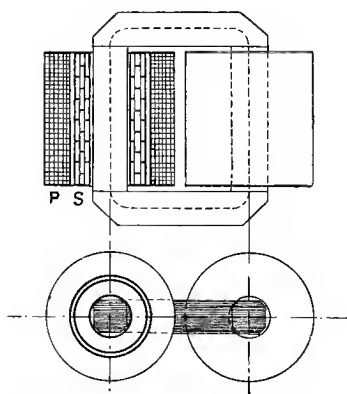


FIG. 1'06.—Single-Phase Core Transformer.

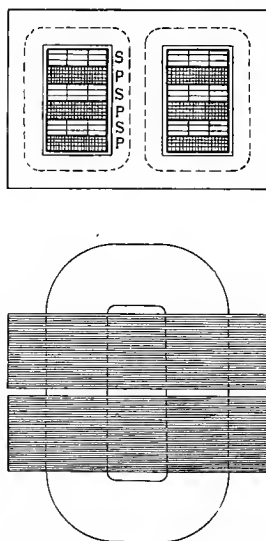


FIG. 1'07.—Single-Phase Shell Transformer.

windings, as shown in fig. 1'06, the transformer is of the “core type”; but if the windings are embedded in the iron, besides surrounding one or more cores, as in fig. 1'07, the design is called a “shell transformer.”

Two-Phase Transformer.—Core and shell transformers may be built for more than one phase. For two-phase currents we generally employ two single-phase designs, but a certain amount of iron may be saved by employing a common magnetic return, as illustrated in fig. 1'08. Let \mathcal{P}_I and \mathcal{P}_{II} be the maximum fluxes of cores I and II respectively, and \mathcal{P}_r the maximum

flux carried by the common return : then, since ϕ_I and ϕ_{II} differ in phase by nearly 90° , the resultant flux is found from

$$\phi_r^2 = \phi_I^2 + \phi_{II}^2 = 2\phi^2,$$

so that

$$\phi_r = \sqrt{2}\phi \quad . \quad . \quad . \quad 1.03.$$

If we assume that the flux density in the common return is the same as

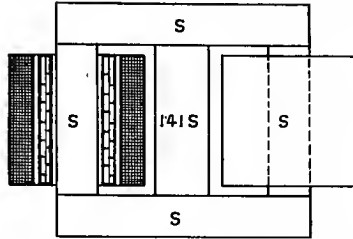


FIG. 1.08.—Two-Phase Core Transformer.

that in the cores, the cross-sectional area of the common magnetic path need only be $\sqrt{2}$ times that of either limb, instead of twice as much. The saving in iron is consequently about 10 or 15 per cent.

Three-Phase Transformers.—Three-phase transformers may also be built with a common return, or the different phases may be magnetically

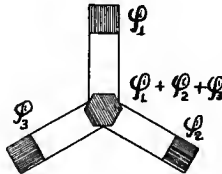
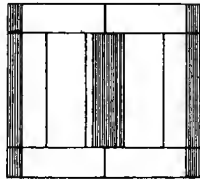


FIG. 1.09.—Three-Phase Core Transformer with Common Magnetic Return.

interlinked, as in figs. 1.10 and 1.11, of which the former represents a symmetrical and the latter an unsymmetrical design having the magnetic reluctances of the three phases unequal.

Consider the three-phase transformer with the common return. The latter will carry the sum of the fluxes from cores I, II, and III. But as this sum is zero at any instant—assuming that the fluxes follow sine laws, and

and, since we assume symmetry and sine waves,

$$\phi_1 + \phi_2 + \phi_3 = 0. \quad 1.07,$$

at any instant.

By combining these four equations we obtain

$$\left. \begin{aligned} \phi_1 &= \frac{\phi_I - \phi_{II}}{3} \\ \phi_2 &= \frac{\phi_{II} - \phi_{III}}{3} \\ \phi_3 &= \frac{\phi_{III} - \phi_I}{3} \end{aligned} \right\} \quad 1.08.$$

It may now be assumed that the magnetomotive forces and magnetic reluctances of the three phases are equal, producing equal amplitudes of fluxes. Then, if OA represents the maximum flux in core I (fig. 1.13), the

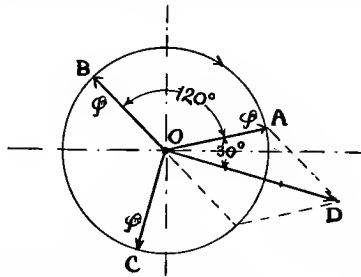


FIG. 1.13.—Vector Diagram for Fluxes in Symmetrical Three-Phase Transformer.

amplitude of the flux in yoke 1 will be equal to the amplitude of $\frac{\phi_I - \phi_{II}}{3}$, *i.e.* of $\frac{OD}{3}$. But $OD = \sqrt{3} OA$, and consequently $\frac{OD}{3} = \frac{\phi}{\sqrt{3}}$.

For constant flux densities we may therefore make the area of the yoke equal to the area of a core $\div \sqrt{3}$, or,

$$S_y = \frac{1}{\sqrt{3}} S_c \quad 1.09.$$

If we consider the unsymmetrical transformer shown in fig. 1.11, in which the reluctances of the three phases differ, it will be evident that the flux in the yoke is equal to that in a core. Nevertheless, we save material even here as against three single-phase transformers, as we require only three cores instead of six, which would be the number for three single-phase designs.

Polyphase shell transformers are but rarely made. A design manufactured by the Schuckert Company is illustrated in fig. 1.14.

In nearly all single-phase transformers of the core type the primary and

secondary windings are distributed between two cores, unless the output is very small, when both coils are made to surround the same limb.

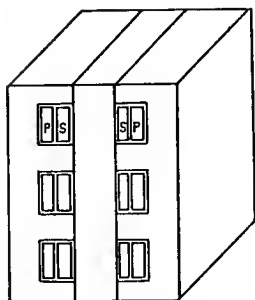


FIG. 1'14.—Three-Phase Shell Transformer (Schnckert).

In polyphase working, the primary and secondary of the same phase are invariably placed on one core only. The winding depth will consequently be greater for the single-phase shell and all polyphase transformers than for the single-phase core design with two-wound limbs, so that the mean length of a turn is greater. We save therefore no copper by using one design for two or more phases, but require a slightly increased amount as against single-phase core transformers.

In any particular case the considerations which govern the choice of single or polyphase transformers will be as follows:—(1) Total capital cost; (2) weight per kilowatt installed; (3) efficiency; (4) suitability as regards working conditions.

The average cost of a three-phase transformer is about 20 per cent. less than that of a single-phase one of the same output and type. On the other hand, a certain amount of spare plant will always have to be provided, and if three single-phase transformers be used, one spare will act as an efficient reserve to all three, whereas if three-phase transformers be installed, it would in most cases be as economical to completely duplicate the plant as to

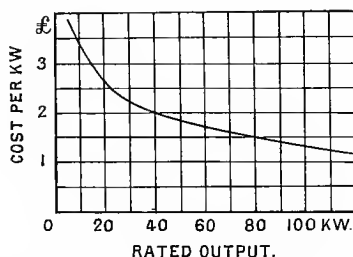


FIG. 1'15.—Cost per Kw. of Single-Phase Transformers.

multiply the number of transformers by using small ones of less efficiency and greater cost per kilowatt. Fig. 1'15¹ illustrates how rapidly the cost per kilowatt of transformers increases as the output is diminished. For instance, a 90-kw. design would cost about 28s. per kilowatt, whereas three 30-kw. transformers installed in place of it would cost 46s. per kilowatt.

Where a number of sub-stations are distributed over a large area, the question of weight is of some importance as regards transport and appliances

¹ See *Journal I.E.E.*, vol. xxxiii. p. 574.

for moving the transformers in the sub-station. The curves of fig. 1·16 show that the three-phase transformer has the advantage over the single-phase machine in this respect. The efficiency of three-phase and single-phase transformers should be about the same. It will be shown in a later chapter

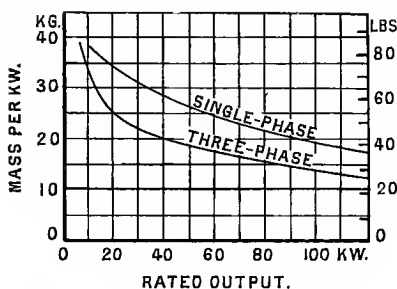


FIG. 1·16.—Mass per Kw. of Transformers.

that the three-phase transformer possesses the advantage of equalising the voltage upon an unbalanced load ; while, on the other hand, if a fault occurs in such a transformer, three or more times the amount of plant is thrown out of use than would be the case with a fault in a single-phase transformer. It is thus evident that whether single- or poly-phase transformers are best will depend on the circumstances of each particular case.

When the function follows a sine law,

$$f = 1.11, \text{ since } \bar{E} = \frac{2}{\pi} E \text{ and } E = \frac{1}{\sqrt{2}} \bar{E}. \quad 2.04.$$

For a flatter wave it is smaller, being 1 for a rectangular wave; and for a peaky wave it is greater.

While the magnetic flux through a coil is changing, an E.M.F. is induced in it, the value of which at any instant is

$$E = -N \frac{d\phi}{dT} = -N\dot{\phi} \quad 2.05,$$

where $\dot{\phi}$ or $\frac{d\phi}{dT}$ represents the rate of change of the flux through one turn, and N is the number of turns in the coil.

Let the flux be a periodic function of amplitude ϕ , period T_0 , and frequency f . The mean E.M.F.

$$\begin{aligned} &= \text{mean rate of cutting flux} \\ &= \text{total change of flux per cycle} \times \text{frequency} \\ &= 4N\phi f, \end{aligned}$$

since the flux per turn changes from 0 to ϕ , or *vice versa*, each quarter period. Thus the mean electromotive force depends on the maximum flux, but not on the shape of the flux wave.

The power of an alternating current is given by the equation

$$P = EI \cos \phi,$$

where E and I are the R.M.S. values of the E.M.F. and current respectively, and $\cos \phi$ is called the power-factor of the circuit. If both E and I follow sine waves, ϕ is the angle of phase difference between them. The R.M.S. value of the E.M.F. is consequently of greater importance than its mean value, and it is expressed by

$$E = 4fN\phi f \quad 2.06,$$

where f is the form-factor of the E.M.F. wave.

Shape of the Magnetising Current Curve.—Let us assume that the E.M.F. of the primary of a transformer follows a sine law, so that its value at any instant is

$$E_1 = -N_1 \frac{d\phi}{dT} = E_1 \sin \omega T = \sqrt{2} E_1 \sin \omega T \quad 2.07,$$

where $\omega = 2\pi f$ = the angular velocity of the vector in a clock diagram, or of a two-pole alternator of the given frequency. It follows that

$$d\phi = -\frac{\sqrt{2} E_1}{N_1} \sin(\omega T) dT,$$

or

$$\phi = -\frac{\sqrt{2} E_1}{N_1} \int \sin(\omega T) dT = \frac{\sqrt{2} E_1}{\omega N_1} \sin\left(\omega T + \frac{\pi}{2}\right) \quad 2.08.$$

It appears then that the flux also follows a sine law, leading the induced E.M.F. by 90° . At no-load, the primary current is extremely small, being only 1 to 2 per cent. of the full load current, and the resistance E.M.F. caused by such a current in the primary winding will be negligible. Hence the primary E.M.F. E_1 and the primary terminal P.D. V_1 are practically identical in magnitude. We may now write for sine waves,

$$\phi = \frac{\sqrt{2}V_1}{\omega N_1} \sin\left(\omega T - \frac{\pi}{2}\right) \quad 2.09$$

The change of sign, corresponding to a reversal of phase, is made here when changing from E to V , in order to agree with the usual practice which requires the P.D. vector to make an angle not exceeding 90° with the current vector when the transfer of energy is in the positive way. This involves one convention as to the sign of the P.D. when the part of the circuit dealt with is supposed to give out energy, such as the secondary coils of a transformer, and another when it is supposed to absorb energy, as in the primary. In the former case, the P.D. is the sum of all the E.M.F.'s in the part of the circuit dealt with, taken positive when helping the current; in the latter, it is minus that sum.

Although the flux follows a sine law, the current by which it is produced does not, and is not even symmetrical with regard to its maximum ordinate.

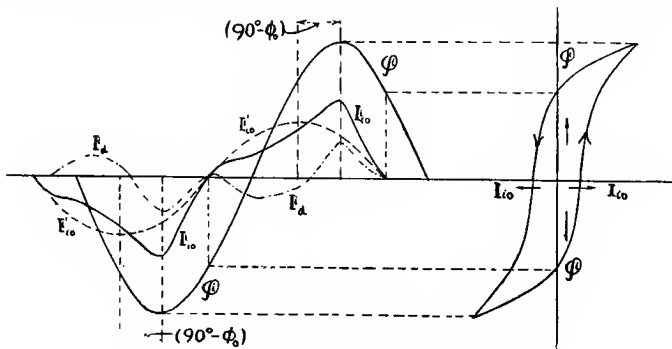


FIG. 2.01.—Shape of Magnetising Current Wave.

This is due to the fact that the magnetisation produced in a piece of iron by a given current is greater when that current is reached by coming down from a larger current than by rising from a smaller one. When iron is carried through a magnetic cycle, the curve showing the relation between the flux densities and the magnetising field forms a loop, the area of which is a measure of the energy which is converted into heat each time the cycle is performed. This phenomenon is called hysteresis. By means of this

magnetisation curve the magnetising current required for any flux may be found and the current wave plotted, as has been done in fig. 2.01. It will be seen that it does not even resemble a sine wave.

At no-load the transformer takes a certain power, which is expended almost entirely on hysteresis and eddy current losses, because the copper losses are then so small that they may be disregarded. The no-load power P_0 is therefore practically equal to the iron losses P_i , and we may write $P_0 = P_i = E_1 I'_{10} \cos \phi_0$, where I'_{10} is the R.M.S. value of a sine current which has the same R.M.S. value as the actual magnetising current I_{10} , and lags behind the P.D. by such an angle ϕ_0 as to take the same power. It is called the equivalent sine current, and is also shown in the figure. The difference between the ordinates of the two current curves will give the curve I_d , which represents the higher wattless harmonics of the magnetising current. It will be noticed that the flux lags with regard to the magnetising current by $90^\circ - \phi_0$, an angle called the "magnetic lag." The quantities may also be plotted in a vector diagram, as shown in fig. 2.02.

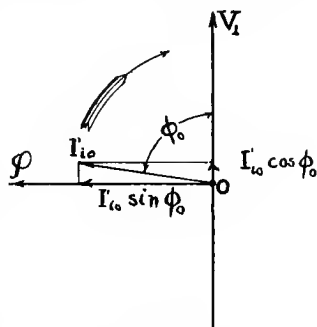


FIG. 2.02.—Vector Diagram for Unloaded Transformer.

Excitation for Iron.—Consider a single phase transformer, as in fig. 2.03, and make, as usual, the assumption that the primary terminal P.D., V_1 , is equal to the primary E.M.F., E_1 . It follows, then, from equation 2.06, that

$$\phi = \frac{E_1}{4\pi f N_1} = \frac{V_1}{4\pi f N_1} \quad \dots \quad 2.10.$$

The necessary current-turns to produce this flux will consist of those wanted for the iron part and those required to drive the flux across the joints. We may consequently write

$$\chi = \chi_{\text{joints}} + \chi_{\text{iron}} \quad \dots \quad 2.11.$$

The correct calculation of χ_{iron} would be difficult, as the magnetisation depends to a certain extent upon the frequency,

on account of the demagnetising effect of the eddy currents. For all practical purposes, we work with sufficient accuracy by employing the static

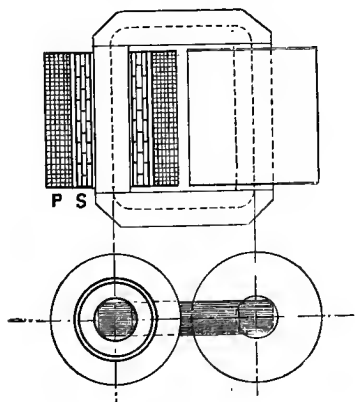


FIG. 2.03.—Single-Phase Core Transformer.

different flux densities, each part of the magnetic path must be considered separately, and we obtain

$$\chi_i = \frac{1}{\sqrt{2}} \left\{ \mathcal{H}_1 L_1 + \mathcal{H}_2 L_2 + \dots \right\} \quad 2.14.$$

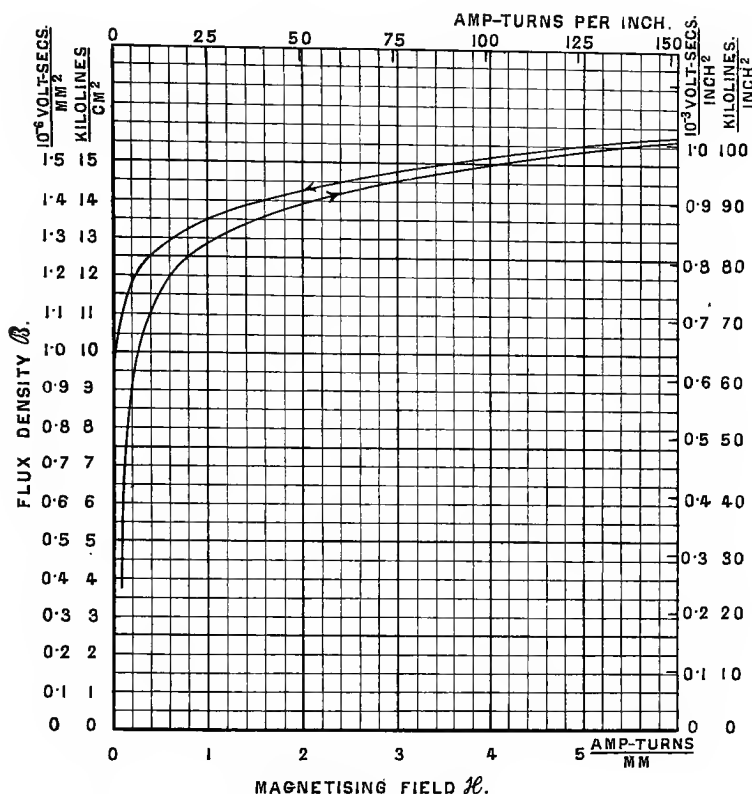


FIG. 2.05.—Magnetisation Curve for Alloyed Iron by Capito & Klein, Benrath-on-Rhine.

Influence of Joints.—The magnetic circuit is usually built up in sections in order to permit the use of former-wound coils, *i.e.* coils which have been wound in a lathe, for these are cheaper to manufacture and can be more easily and effectively insulated than hand-wound coils. The path of the flux is thus obstructed by joints, which may either be of the butt or overlapping type. From experiments made by Ewing¹ it appears that the reluctance of a turned butt joint in a solid iron rod is about the same as that of an air gap 0.05 mm., or 2 mils, thick, when there is no compression, and less when the surfaces are squeezed together. He also found that

¹ *Magnetic Induction in Iron*, p. 293.

scraping the faces to true planes reduced the reluctance to about half this value without pressure, and almost to zero with pressure.

As the iron part of a transformer is built up of insulated sheets, the butt joints will naturally be rough, and even if planed over, which would increase the eddy currents, they are not likely to be as good as in solid metal. Hence,

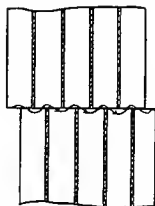


FIG. 2·06.—Eddy Currents at Joints.

we shall take the equivalent air-gap as 0·1 mm., or 4 mils, when there is no insulation at the joint, and equal to the thickness of the insulation when there is. A thin layer of insulating material is desirable to prevent the eddy

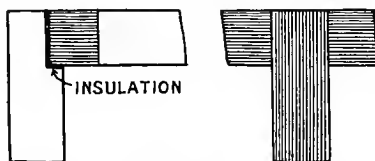


FIG. 2·07.—Joint with Crossed Laminations.

currents shown in fig. 2·06, and is essential when the laminations are crossed, as in fig. 2·07.

The reluctance of the joints can be considerably reduced by making them overlap, as in fig. 2·08; but the eddy currents are thereby increased, as the

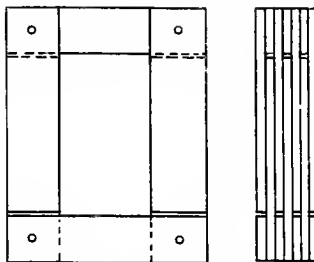


FIG. 2·08.—Transformer with Overlapping Joints.

flux at the joints is no longer parallel to the plane of the laminations, but cuts into them. The effect of the two kinds of joints is shown in fig. 2·09.

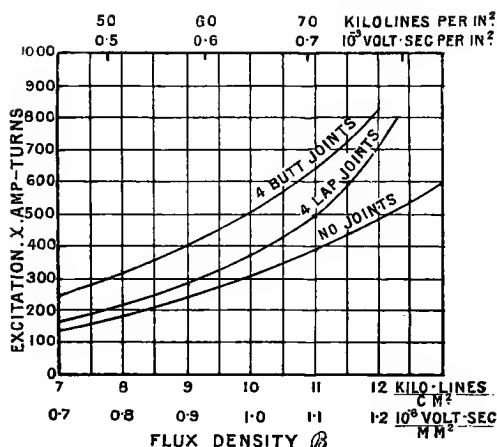


FIG. 2:09.—Comparison of Transformer Joints.

Taking 0.1 mm. as the equivalent air-gap, and the permeability of air as

$$\left. \begin{aligned} \mu_a &= 1.257 \frac{\text{lines per cm.}^2}{\text{amp.-turn per cm.}} \\ &= 1.257 \times 10^{-9} \frac{\text{volt-sec. per mm.}^2}{\text{amp.-turn per mm.}} = 1.257 \times 10^{-9} \frac{\text{Henry}}{\text{mm.}} \\ &= 31.9 \times 10^{-9} \frac{\text{volt-sec. per in.}^2}{\text{amp.-turn per in.}} = 31.9 \times 10^{-9} \frac{\text{Henry}}{\text{in.}} \end{aligned} \right\} \quad 2.15,$$

we have, for the R.M.S. excitation required for the joints :—

$$\left. \begin{aligned} X_J &= \frac{1}{\sqrt{2}} N_J B \times \frac{1 \text{ amp.-turn per cm.}}{1.257 \text{ lines per cm.}^2} \times 0.01 \text{ cm.} \\ &= 5.6 N_J B \frac{\text{amp.-turns}}{\text{Kilolines per cm.}^2} \\ &= 56 N_J B \frac{\text{amp.-turns}}{\text{volt-sec. per metre}^2} \\ &= 87 N_J B \frac{\text{amp.-turns}}{10^{-3} \text{ volt-sec. per in.}^2} \end{aligned} \right\} \quad 2.16,$$

where N_J is the number of joints in series.

Magnetising and No-Load Currents.—The idle component of the no-load current which magnetises the iron is now determined by

$$\text{idle, or wattless, component} = I_i = \frac{X_i + X_J}{N_1} \quad 2.17.$$

In order to find the working component of the no-load current we must know the iron losses, which are due to hysteresis and eddy currents. We

have $P_i = P_h + P_e$. At no-load, this loss is practically identical with the total power P_o taken in, since the copper loss is extremely small. If we make the further assumption that on open circuit the terminal P.D. is equal to the induced E.M.F., we may write $P_o \doteq P_i \doteq E_1 \times$ wattful component of current $= \bar{V}_1 \times$ wattful component. Hence,

$$\text{working, or wattful, component} = I_w \doteq \frac{P_i}{\bar{V}_1} \quad . \quad . \quad 2.18.$$

Wattless and wattful components differ in phase by 90° , and their resultant defines the no-load current, which is given by

$$I_o = \sqrt{I_i^2 + I_w^2} = \sqrt{\left(\frac{X_i + X_J}{N_1}\right)^2 + \left(\frac{P_i}{\bar{V}_1}\right)^2} \quad . \quad . \quad 2.19.$$

Example.—It is required to calculate the no-load current of a 50 K.V.A. (kilo-volt-amperes) single-phase core transformer. The mean length of the magnetic path is 1900 millimetres; the maximum flux density, $\mathcal{B} = 5000$ μ .C.G.S.; the number of primary turns, $N_1 = 1218$; the primary P.D., $V_1 = 5000$ volts; and the total iron losses, $P_i = 625$ watts.

The magnetising field which corresponds to the given flux density is 0.08 ampere-turns per mm., so that

$$X_i = \frac{0.08 \times 1900}{\sqrt{2}} = 109 \text{ R.M.S. ampere-turns.}$$

The transformer has four butt joints, so that

$$X_J = 4 \times 5.6 \times 5 = 112 \text{ R.M.S. ampere-turns.}$$

From this it follows that the idle component of the no-load current

$$I_i = \frac{X_i + X_J}{N_1} = \frac{109 + 112}{1218} = 0.183 \text{ ampere.}$$

The working component

$$I_w = \frac{P_i}{V_1} = \frac{625}{5000} = 0.125 \text{ ampere,}$$

hence the no-load current

$$I_o = \sqrt{0.183^2 + 0.125^2} = 0.222 \text{ ampere.}$$

As the transformer is made for 50 K.V.A., the full-load current

$$I_1 = \frac{50,000 \text{ watts}}{5000 \text{ volts}} = 10 \text{ amperes.}$$

The no-load current is therefore equivalent to 2.22 per cent. of the full-load current.

No-Load Currents of Two-Phase Transformers.—For two-phase currents we generally employ two single-phase designs, as the amount of material which is saved by using a single two-phase transformer instead of two single-phase ones is not considerable. Each of these latter is treated

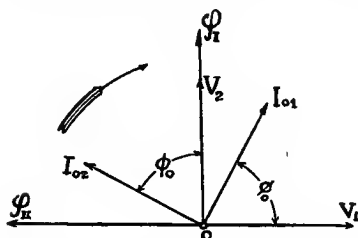


FIG. 2.10.—Vector Diagram for Pair of Unloaded Transformers on Two-Phase Supply.

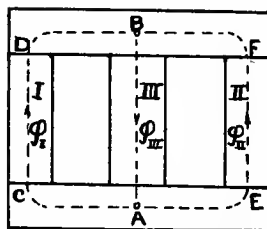


FIG. 2.11.—Magnetic Circuits of Two-Phase Transformer with Common Return.

separately, and their no-load current vectors are drawn perpendicular to one another as indicated in fig. 2.10.

The no-load currents of a two-phase transformer constructed on the lines of fig. 2.11, where cores I and II only carry windings, may be determined, with our usual assumptions, as follows:—

The maximum flux of each core is given by

$$\Phi_I = \Phi_{II} \doteq \frac{V_1}{4ffN_1} \quad . \quad . \quad . \quad 2.20;$$

where f is the frequency, N_1 the number of primary turns of either phase, and f the form-factor of the E.M.F. wave.

Since the flux of the common return is $\sqrt{2} \Phi_I$, we obtain

$$\Phi_{III} \doteq \frac{\sqrt{2}V_1}{4ffN_1} \doteq \frac{V_1}{2.83ffN_1} \quad . \quad . \quad . \quad 2.21.$$

We now divide the total magnetic circuit into three parts, and calculate the necessary excitation for each separately. We make the additional assumption that the flux density is constant for the whole transformer, which is necessary in order to make the iron losses a minimum.

We commence with the magnetic path ACDB. Let L_1 be the mean length of this path, \mathcal{H}_1 the magnetising field for a given flux density \mathcal{B} , N_{J_1} the number of joints in this path; then the total excitation required for this section of the magnetic circuit is

$$\chi_{i1} = \frac{\mathcal{H}_1 L_1}{\sqrt{2}} + 5.6 N_{J_1} \mathcal{B} \frac{\text{amp.-turns}}{\text{kilolines per cm.}^2} \quad . \quad . \quad . \quad 2.22.$$

But the path from A over E and F to B is identical with the circuit under consideration, and will consequently require the same excitation.

Similarly, for core III we have

$$\chi_{i3} = \frac{H_3 L_3}{\sqrt{2}} + 5.6 N_{J_3} \beta \frac{\text{amp.-turns}}{\text{kilolines per cm.}^2} \quad 2.23.$$

These determine the idle components of the no-load currents. To find the working components we must know the iron losses. Let these losses in the three different sections be denoted by P_I , P_{II} , and P_{III} respectively, and let $P_I + P_{II} + P_{III} = P_i$ be the total iron losses; then, since $P_I = P_{II}$, it follows that the working components of the no-load currents for sections I and II are expressed by

$$I_{w1} = I_{w2} = \frac{P_I}{V_1} \quad 2.24,$$

and the excitations due to them

$$\chi_{w1} = \chi_{w2} = N_1 I_{w1} = N_1 \frac{P_I}{V_1} \quad 2.25.$$

Similarly,

$$\chi_{w3} = N_1 I_{w3} = N_1 \frac{P_{III}}{V_1}.$$

The resultant ampere-turns for each section—

$$\left. \begin{aligned} \chi_I &= \chi_{II} = \sqrt{\chi_{i1}^2 + \chi_{w1}^2} \\ \chi_{III} &= \sqrt{\chi_{i3}^2 + \chi_{w3}^2} \end{aligned} \right\} \quad 2.26.$$

These quantities are not in phase, since the sections are magnetised by currents which themselves differ in phase, and it is best to proceed graphically to find the resultant current in each phase. The wattless components will coincide in direction with the respective fluxes, while the watt components

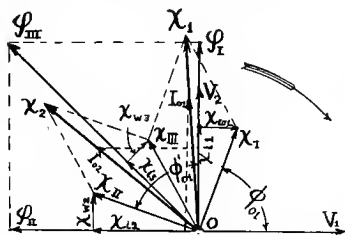


FIG. 2.12.—Vector Diagram for Unloaded Two-Phase Transformer.

are at right angles. In this way we are able to plot χ_I , χ_{II} , and χ_{III} . The resultant of χ_I and χ_{III} will then represent the current-turns which we must place on the core of phase I, and the resultant of χ_{II} and χ_{III} will give the current-turns required for phase II. The corresponding no-load currents coincide in direction with the vectors of the resultant excitation, and they are found by dividing the latter

by the number of primary turns. It will be noticed from the diagram that the no-load currents I_{01} and I_{02} are not perpendicular to each other, and that both phases do not even absorb the same energy. This is proved by projecting the current vectors upon the P.D. vectors. At first sight, this result seems wrong, because things are apparently symmetrical.

But they are actually not quite symmetrical, since the flux in one outer limb reaches its maximum before, and in the other limb after, that in the centre.

Two-phase transformers are but rarely built, as the net saving in material is more imaginary than real. For, although we reduce the weight of iron by using a common return, the amount of copper is slightly in excess of that for two single-phase transformers, on account of having to place the windings of one phase on one core only. The depth of the winding per core, and consequently the mean length of a turn, are greater in a two-phase design than when the turns of a phase are distributed on two limbs.

No-Load Currents of Symmetrical Three-Phase Transformers.—Three-phase transformers are more common. Their iron frames may be built so that all the phases have the same magnetic reluctance, or the limbs may be arranged in a line. The latter type is the more easily made, and is consequently often chosen. The magnetic reluctances of the three phases are then, however, no longer equal, and the no-load currents will differ.

In calculating the no-load currents for a symmetrical three-phase transformer, we shall assume that the windings of each phase are wound on one limb, that the terminal P.D. is the same for all three and follows a sine law, and that the resistance E.M.F.'s are negligible. If the windings are joined in star form, we must divide the line P.D. by $\sqrt{3}$ in order to obtain the phase P.D.—the P.D. per coil. For mesh connections, they are both alike.

The instantaneous values of the phase P.D.'s are denoted by

$$\left. \begin{aligned} \bar{V}_1 &= \sqrt{2}V_1 \sin(2\pi fT) \\ \bar{V}_2 &= \sqrt{2}V_1 \sin(2\pi fT - 120^\circ) \\ \bar{V}_3 &= \sqrt{2}V_1 \sin(2\pi fT - 240^\circ) \end{aligned} \right\} \quad . \quad . \quad . \quad 2.27,$$

and the instantaneous fluxes

$$\left. \begin{aligned} \phi_1 &\doteq \frac{V_1}{4ffN_1} \sin\left(2\pi fT - \frac{\pi}{2}\right) \\ \phi_2 &\doteq \frac{V_1}{4ffN_1} \sin\left(2\pi fT - \frac{\pi}{2} - 120^\circ\right) \\ \phi_3 &\doteq \frac{V_1}{4ffN_1} \sin\left(2\pi fT - \frac{\pi}{2} - 240^\circ\right) \end{aligned} \right\} \quad . \quad . \quad . \quad 2.28.$$

On account of symmetry,

$$\phi_1 + \phi_2 + \phi_3 = 0 \quad . \quad . \quad . \quad 2.29.$$

The amplitude of the flux in each core is

$$\phi \doteq \frac{V_1}{4ffN_1} \quad . \quad . \quad . \quad 2.30 ;$$

and, being $\sqrt{3}$ times smaller (equation 1.05), the flux in each yoke is

$$\phi_y = \frac{I}{\sqrt{3}} \phi = \frac{V_1}{6.9ffN_1} \quad . \quad . \quad . \quad . \quad 2.31.$$

If we are to have a constant maximum flux density along the whole path, the cross-sectional area of a yoke must be $\frac{I}{\sqrt{3}}$, or 0.577, times that of a core.

At the instant the flux in one core is a maximum, it will travel, say up, in core I, divide equally at the yoke, pass down the other two cores in parallel, and return through the lower yokes to core I. We must calculate the magnetising current for this circuit in steps, since the flux density will not be constant.

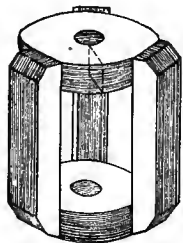


FIG. 2.13.—Symmetrical Three-Phase Core Transformer.

Consider core I from yoke to yoke. The excitation which is required for this portion is

$$X_{i1} = \frac{H_1 L_1}{\sqrt{2}} \quad . \quad . \quad . \quad . \quad 2.32,$$

where H_1 corresponds to the flux density \mathcal{B} in the core, while L_1 denotes the distance between two yokes. In the other two limbs the flux density is only equal to $\frac{\mathcal{B}}{2}$, since the flux divides through both. Consequently, for cores II and III we require

$$X_{i2} = \frac{H_2 L_1}{\sqrt{2}} \quad . \quad . \quad . \quad . \quad 2.33,$$

where H_2 refers to the flux density $\frac{\mathcal{B}}{2}$.

In the yokes the flux density has still another value, viz., $\mathcal{B}_y = \frac{\sqrt{3}}{2}\mathcal{B}$, so that for the yokes we must have

$$X_{i3} = \frac{H_3 L_3}{\sqrt{2}} \quad . \quad . \quad . \quad . \quad 2.34.$$

L_3 is here equal to twice the distance from one core to another, and H_3 corresponds to \mathcal{B}_y .

The excitation for the joints must also be determined step by step. The flux density at the joints between core I and the yokes will be practically equal to \mathcal{B} , so that for these joints we require (in R.M.S. values)

$$X_{J1} = 5.6 \times 2\mathcal{B} \frac{\text{amp.-turns}}{\text{kilolines per cm.}^2} \quad 2.35.$$

At the joints between the yokes and cores II and III the flux density is approximately equal to $\mathcal{B}_y = \frac{\sqrt{3}}{2}\mathcal{B}$; and for them

$$\mathcal{X}_{J_2} = 5.6 \times 2 \times \frac{\sqrt{3}}{2} \mathcal{B} \frac{\text{amp.-turns}}{\text{kilolines per cm.}^2} \quad . \quad . \quad 2.36.$$

The total excitation required is the sum of those for the different sections, or

$$\mathcal{X}_1 = \mathcal{X}_{i_1} + \mathcal{X}_{i_2} + \mathcal{X}_{i_3} + \mathcal{X}_{J_1} + \mathcal{X}_{J_2} \quad . \quad . \quad 2.37.$$

But the excitation per phase is smaller than this, because the currents in phases II and III also help. At the instant considered, these currents will have half their maximum value, and consequently the excitation which has actually to be supplied by each phase is only $\frac{2}{3}\mathcal{X}$, and the idle component of the no-load current per phase is

$$I_1 = \frac{2}{3} \frac{\mathcal{X}_1}{N_1} \quad . \quad . \quad . \quad 2.38.$$

In order to find the working component, we must have a knowledge of the iron losses, P_i . As the transformer is built symmetrically, we may assume that all phases absorb the same power, so that we find the watt component from

$$I_w = \frac{P_i}{3E_1} = \frac{P_i}{3V_1} \quad . \quad . \quad 2.39.$$

The no-load current per phase follows now from

$$I_0 = \sqrt{I_1^2 + I_w^2} \quad . \quad . \quad 2.40.$$

The above formulæ have been deduced on the assumption that the transformer has no leakage, but they will be found sufficiently accurate for all practical purposes.

No-Load Currents of Unsymmetrical Three-Phase Transformer.—The unsymmetrical type illustrated in fig. 2.14 is very popular on account of its simple construction. When running on open circuit it will be found that the no-load currents and the no-load power of the different phases vary because of their unequal magnetic reluctances. Even the induced E.M.F.'s in the secondaries of the three phases will not be exactly identical, that of the middle phase being smallest, owing to its leakage flux being greatest; but the differences are here so small (less than 1 per cent. as a rule) that for all practical purposes we may neglect them, and assume the fluxes \mathcal{P}_1 , \mathcal{P}_2 , and \mathcal{P}_3 to be all alike.

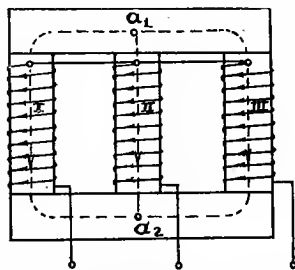


FIG. 2.14.—Magnetic Circuits of Unsymmetrical Three-Phase Core Transformer.

from the same diagram, that χ_{II} is larger, and χ_I and χ_{III} smaller, than might be anticipated from the reluctances of the different circuits. In the limit, when the magnetic reluctance of circuit II is negligible compared with that of I or III, the excitation χ_{II} will have the lowest possible value,

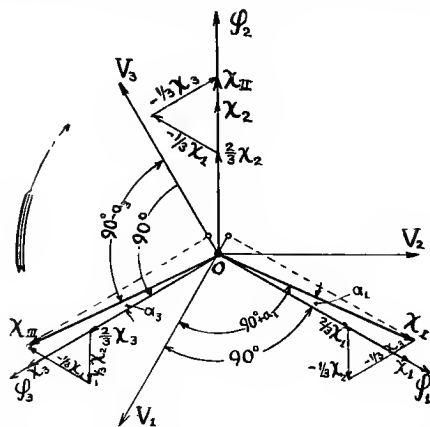


FIG. 2.15.—Vector Diagram for Unloaded Lossless Unsymmetrical Three-Phase Transformer.

and at the same time, the angle α will reach its limit with $\tan \alpha = \frac{1}{3} \sqrt{3}$ or $\alpha = 19.14^\circ$.

So far we have assumed that no dissipation of energy takes place. But even with the secondary on open circuit the transformer absorbs power on account of the hysteresis and eddy current losses (neglecting copper losses at no-load), and consequently the no-load currents will not be in phase with the fluxes. The actual displacement will depend upon the saturation of the iron and the magnetic reluctance. The magnetomotive forces have now wattful components, which are found in the usual way. We simply determine the losses between the points A_1 and A_2 for the three different circuits, and call them P_I , P_{II} , and P_{III} respectively. Each wattful component of the excitation is then given by

$$\left. \begin{aligned} \chi_{w1} &\doteq N_1 \frac{P_I}{E_1} \doteq N_1 \frac{P_I}{V_1} \\ \chi_{w2} &\doteq N_2 \frac{P_{II}}{E_2} \doteq N_1 \frac{P_{II}}{V_1} \\ \chi_{w3} &\doteq N_3 \frac{P_{III}}{E_3} \doteq N_1 \frac{P_{III}}{V_1} \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad 2.44.$$

These working components are at right angles to the respective idle components, and their resultants can be divided between the phases according to equation 2.43 to obtain the excitation which must actually be placed on

each limb. In fig. 2·16 the phase excitations are marked by χ_I , χ_{II} , and χ_{III} , which also represent to N_1 times the scale the no-load currents of the different phases. Their projections upon the P.D. vectors give a measure of the power received at no-load. The diagram indicates clearly that the

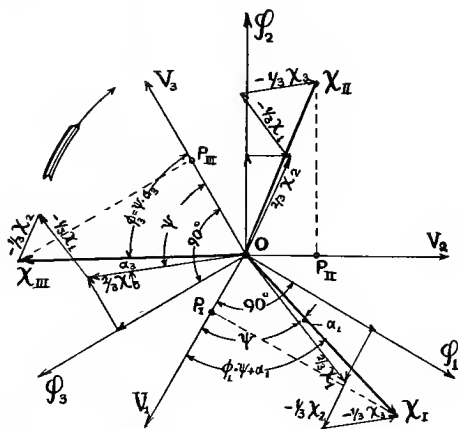


FIG. 2·16.—Vector Diagram for Unloaded Unsymmetrical Three-Phase Transformer.

no-load currents, the no-load power, and the phase angles of the three phases are all different. This will be understood when the transfer of power from one outer phase to the other of our ideal transformer is remembered. In our example, energy passes from phase III into phase I, *i.e.* from the one which is behind the central one in phase to the one which is ahead of it. By reversing the direction of rotation of the generator, or by changing two leads to the transformer, the two external phases change their actions.

CHAPTER III.

LOSSES IN TRANSFORMERS.

Losses in Transformers—Efficiency.—When we measure the energy which is given out by the secondary of a transformer, and compare it with that supplied to the primary terminals, we find that part has been lost in the conversion. This loss is due to several causes, and is conveniently divided into “iron” and “copper” losses. The process of magnetisation consists in putting the molecules of the material into a state of strain which requires the expenditure of energy for its production. In the case of iron and the other “magnetic” materials, very little of this energy is returned on removing the magnetism, but it is mostly frittered away in the molecular motions we call heat. The cause of this wastage is generally referred to as Magnetic Hysteresis. Consequently, when iron is subjected to cyclic magnetisation there is a continual conversion of energy into heat. Again, eddy currents are induced in the laminations because the main flux is continually changing, and these also waste energy by converting it to heat.

In the copper windings also there is a conversion of energy into heat by their resistance, and this exceeds that due to a steady current because an alternating current does not distribute itself uniformly over the cross-section. Additional losses are caused by dielectric hysteresis in the insulation of the windings, but these are so small that they do not affect the efficiency of the transformer. The ratio of the output to the input of power is termed the efficiency; it is always less than unity.

Hysteresis Loss.—An exact predetermination of the hysteresis loss is impossible, but an approximate calculation may be made by assuming that it is proportional to the 1·6th power of the maximum flux density. This empirical law was deduced by Steinmetz from a series of tests, and it has been found that for flux densities which are neither very high nor very low it usually gives very fair results.

Assuming Steinmetz’s law to be correct, we may express the waste of energy per unit volume per cycle by $\mathcal{S}\mathcal{B}^{1\cdot6}$, where \mathcal{B} is the amplitude of the flux density, and \mathcal{S} a constant which depends upon the quality of the iron,

and is called the hysteresis coefficient. The hysteresis loss in a volume V_i , or mass M_i , of iron with density D_i , at the frequency f , is thus

$$P_H = S B^{1.6} f V = \left(\frac{S}{D_i} \right) B^{1.6} f M_i \quad 3.01.$$

In ordinary cases we may take

$$\begin{aligned} S &= \frac{250 \text{ to } 500 \text{ watts per metre}^3}{(\text{cycles per sec.}) \times (\text{volt-secs. per metre}^2)^{1.6}} \\ &= \frac{6 \text{ to } 12.5 \times 10^{-6} \text{ watts per cm.}^3}{(\text{cycles per sec.}) \times (\text{kilolines per cm.}^2)^{1.6}} \\ &= \frac{8 \text{ to } 16 \times 10^{-3} \text{ watts per in.}^3}{(\text{cycles per sec.}) \times (10^{-3} \text{ volt-sec. per in.}^2)^{1.6}} \\ &= \frac{5 \text{ to } 10 \times 10^{-6} \text{ watts per in.}^3}{(\text{cycles per sec.}) \times (\text{kilolines per in.}^2)^{1.6}} \end{aligned} \quad 3.02$$

$$\begin{aligned} \frac{S}{D_i} &= \frac{32 \text{ to } 64 \times 10^{-3} \text{ watts per kilogram}}{(\text{cycles per sec.}) \times (\text{volt-secs. per metre}^2)^{1.6}} \\ &= \frac{0.8 \text{ to } 1.6 \times 10^{-3} \text{ watts per kilogram}}{(\text{cycles per sec.}) \times (\text{kilolines per cm.}^2)^{1.6}} \\ &= \frac{30 \text{ to } 60 \times 10^{-3} \text{ watts per lb.}}{(\text{cycles per sec.}) \times (10^3 \text{ volt-secs. per in.}^2)^{1.6}} \\ &= \frac{20 \text{ to } 40 \times 10^{-6} \times \text{watts per lb.}}{(\text{cycles per sec.}) \times (\text{kilolines per in.}^2)^{1.6}} \end{aligned} \quad 3.03.$$

The lower values are about the lowest which can be obtained at present, and a greater value than the highest should not be permitted for transformer iron.

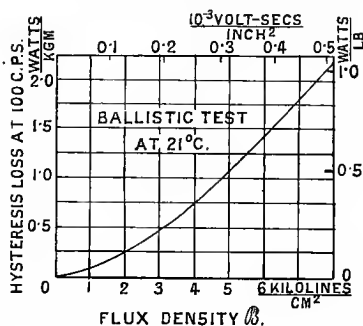


FIG. 3.01.—Hysteresis Losses in Good Transformer Iron.

The curve given shows the result of a test which agrees very well with Steinmetz's formula.

Eddy Current Loss.—The waste of energy caused by eddy currents may be reduced by finely laminating the core and by insulating the sheets. This increases the resistance in the paths of the eddies, and consequently

reduces them. Special brands of alloyed iron are now on the market which have a high resistivity and a consequent small eddy current loss, while their magnetic properties are as good, or nearly as good, as those of pure iron. Besides wasting energy the eddies cause an uneven distribution of the flux, since their demagnetising action is greatest near the middle, and consequently the flux density is smallest there. The exact calculation of the eddy current loss is further complicated by the inductance of the paths in which they flow. To obtain an approximate idea of these losses, we shall assume that this inductance is negligible, and that the flux is evenly distributed over the whole cross section, and is parallel to the sides of the sheet.

The R.M.S. E.M.F. induced round a symmetrically placed loop of width l_x and length L (see fig. 3.02) in a plane perpendicular to the flux is

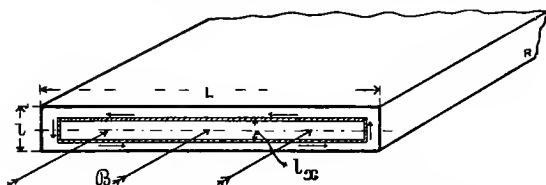


FIG. 3.02.—Eddy Currents in Transformer Plate.

$E = 4ff(l_x L \mathcal{B})$. The length of the perimeter of this loop is $2(L + l_x)$, but as the plate is thin we may neglect the second term and take the length of the eddy current path as $2L$. The R.M.S. E.M.F. per unit length is then

$$\dot{E} = \frac{E}{2L} = 2ffl_x \mathcal{B}.$$

And if the resistivity of the iron be ρ_I , the R.M.S. current density is

$$\dot{I} = \frac{\dot{E}}{\rho_I} = \frac{2ffl_x \mathcal{B}}{\rho_I},$$

and the power lost per unit volume at a distance $\frac{1}{2}l_x$ from the centre of the thickness is

$$\ddot{P}_{Ex} = \rho_I \dot{I}^2 = \frac{4f^2 f^2 l_x^2 \mathcal{B}^2}{\rho_I} \quad \dots \quad 3.04.$$

The mean loss per unit volume is obtained by integrating between the limits 0 and l and dividing by l , where l is the thickness of each lamination.

$$\begin{aligned} \therefore \ddot{P}_E &= \frac{1}{l} \int_0^l \frac{4f^2 f^2 l_x^2 \mathcal{B}^2}{\rho_I} \\ &= \frac{4}{3} \cdot \frac{f^2 f^2 l^2 \mathcal{B}^2}{\rho_I}. \end{aligned}$$

Or, we may write the total eddy current loss thus

$$P_E = \frac{4}{3} \cdot \frac{f^2 l^2 \mathfrak{B}^2}{\rho_I} V_I$$

$$= \frac{4}{3} \cdot \frac{f^2 l^2 \mathfrak{B}^2}{\rho_I D_I} M_I \quad . \quad . \quad . \quad . \quad 3.05,$$

where V_I is the total volume of the iron, M_I its mass, and D_I its density.

The resistivity of iron may be taken as 100×10^{-6} ohm - mms. (10,000 μ .C.G.S.), and its density 7800 kilograms per cubic metre (7.8 grams per c.cm.). Hence

$$\frac{4}{3\rho_I} = \frac{13.3 \text{ watts per metre}^3}{(\text{cycle per sec.})^2 \times (\text{mm.})^2 \times (\text{volt-sec. per metre}^2)^2},$$

and

$$\frac{4}{3\rho_I D_I} = \frac{1.7 \times 10^{-3} \text{ watts per kilogram}}{(\text{cycle per sec.})^2 (\text{mm.})^2 \times (\text{volt-sec. per metre}^2)^2} \quad . \quad . \quad 3.06.$$

In practice it is found that the eddy losses are usually larger than this, partly because the above assumptions are not fulfilled, and partly because the flux is not always parallel to the surface of the plates at the joints, especially with lap joints. The theory is, however, useful for showing how the loss depends on the several variables; but it should be noted that when the frequency is high the inductance of the eddy current paths will not be negligible, and the eddy current loss will then vary less than f^2 when \mathfrak{B} is kept constant, and will ultimately be constant when the frequency is high enough to make the impedance practically independent of the resistance. But with these limitations we may write

$$P_E = e f^2 l^2 \mathfrak{B}^2 V_I = \frac{e}{D_I} f^2 l^2 \mathfrak{B}^2 M_I \quad . \quad . \quad 3.07,$$

where e is a quantity depending on the resistivity of the material, and nearly independent of the other variables within ordinary working limits. We shall call it the eddy current coefficient. In ordinary cases we may take

$$e = \frac{16 \text{ to } 21 \text{ watts per metre}^3}{(\text{cycles per sec.})^2 \times (\text{mm.})^2 \times (\text{volt-secs. per metre}^2)^2}$$

$$= \frac{0.16 \text{ to } 0.21 \times 10^{-6} \text{ watts per cm.}^3}{(\text{cycles per sec.})^2 \times (\text{mm.})^2 \times (\text{kilolines per cm.}^2)^2}$$

$$= \frac{0.4 \text{ to } 0.5 \text{ watts per in.}^3}{(\text{cycles per sec.})^2 \times (\text{mils})^2 (\text{volt-secs. per in.}^2)^2}$$

$$= \frac{0.04 \text{ to } 0.05 \times 10^{-9} \text{ watts per in.}^3}{(\text{cycles per sec.})^2 (\text{mils})^2 \times (\text{kilolines per in.}^2)^2} \quad . \quad . \quad 3.08,$$

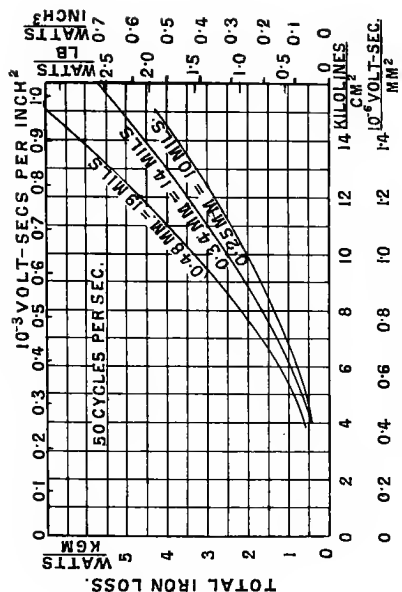


Fig. 3-04.—50 Cycles per Second.

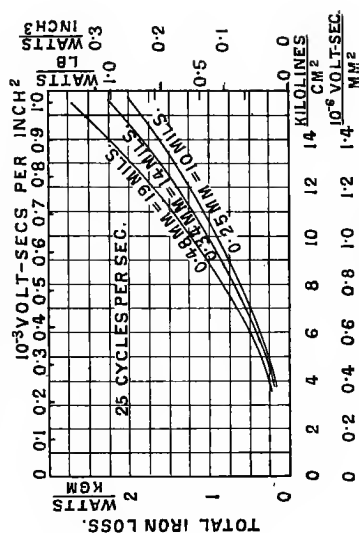


Fig. 3-06.—25 Cycles per Second.

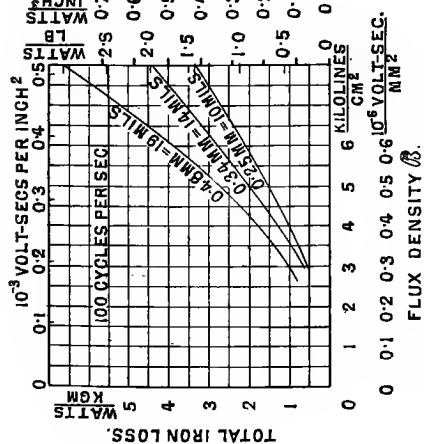


Fig. 3-03.—100 Cycles per Second.

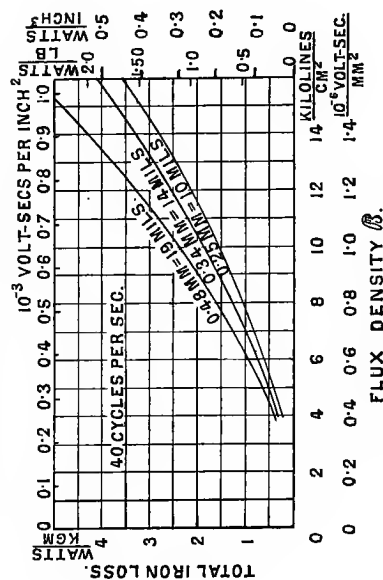


Fig. 3-05.—40 Cycles per Second.

Figs. 3-03-3-06.—Total Iron Losses in Best Unalloyed Transformer Sheets.

$$\begin{aligned}
 \frac{e}{D_I} &= \frac{2.1 \text{ to } 2.7 \times 10^{-3} \text{ watts per kilogram}}{(\text{cycles per sec.})^2 \times (\text{mm.})^2 \times (\text{volt-secs. per metre}^2)^2} \\
 &= \frac{21 \text{ to } 27 \times 10^{-6} \text{ watts per kilogram.}}{(\text{cycles per sec.})^2 \times (\text{mm.})^2 \times (\text{kilolines per cm.}^2)^2} \\
 &= \frac{1.5 \text{ to } 2 \times \text{watts per lb.}}{(\text{cycles per sec.})^2 \times (\text{mils})^2 \times (\text{volt-secs. per in.}^2)^2} \\
 &= \frac{0.15 \text{ to } 0.20 \times 10^{-9} \text{ watts per lb.}}{(\text{cycles per sec.})^2 (\text{mils})^2 \times (\text{kilolines per in.}^2)^2} \quad 3.09.
 \end{aligned}$$

Total Iron Losses.—The total iron losses are thus

$$\begin{aligned}
 P_I &= P_H + P_E = \underbrace{\frac{S}{D_I} \beta^{1.6} f V_I}_{\text{Hysteresis.}} + \underbrace{\frac{e}{D_I} f^2 l^2 \beta^2 V_I}_{\text{Eddy.}} \quad 3.10. \\
 &= \underbrace{\frac{S}{D_I} \beta^{1.6} f M_I}_{\text{Hysteresis.}} + \underbrace{\frac{e}{D_I} f^2 l^2 \beta^2 M_I}_{\text{Eddy.}}
 \end{aligned}$$

Figs. 3.03-3.06 show the total iron losses in one of the best brands of pure iron, and fig. 3.07 that in a brand of alloyed iron.

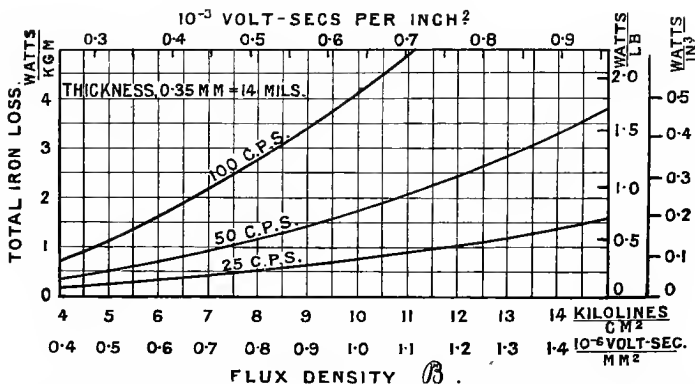


FIG. 3.07.—Total Iron Losses in Kapito & Klein's Alloyed Transformer Sheets.

As equation 3.10 is rather awkward, it will be convenient to neglect the fact that the eddy and hysteresis losses depend on different powers of the flux density, and to write for the total iron losses

$$P_I = K_I \beta^2 V_I = \frac{K_I}{D} \beta^2 M_I \quad 3.11.$$

The coefficient K_I is not a constant, but varies with the flux density as shown in figs. 3.08-3.12 for the irons whose losses have just been given. For some purposes, however, it is sufficiently accurate to treat K_I as a constant.

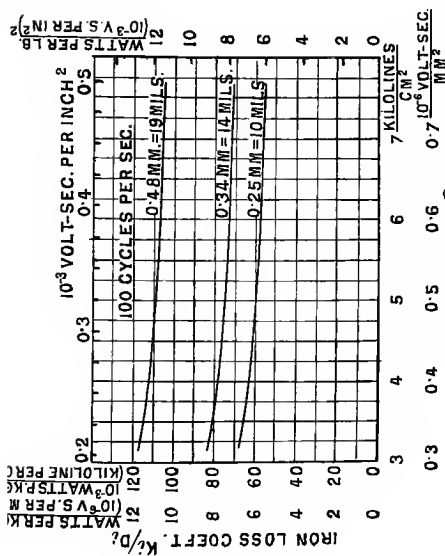


Fig. 3-08.—100 Cycles per Second.

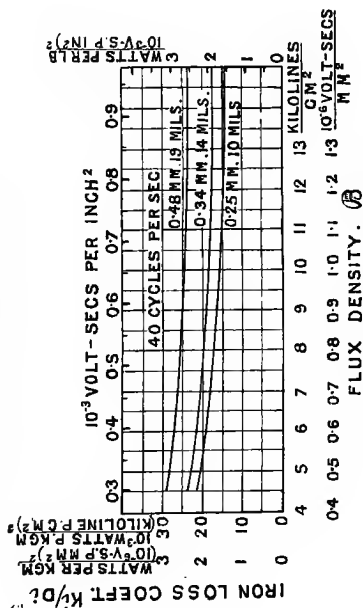


Fig. 3-10.—40 Cycles per Second.

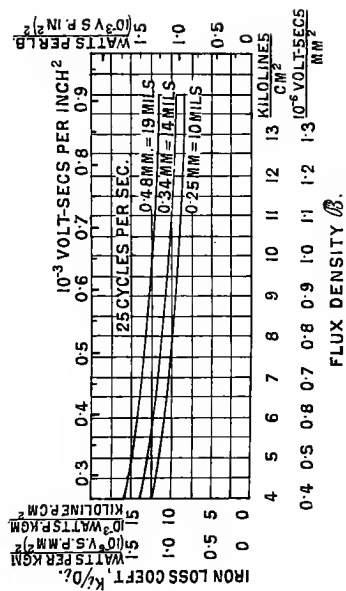


Fig. 3-11.—25 Cycles per Second.

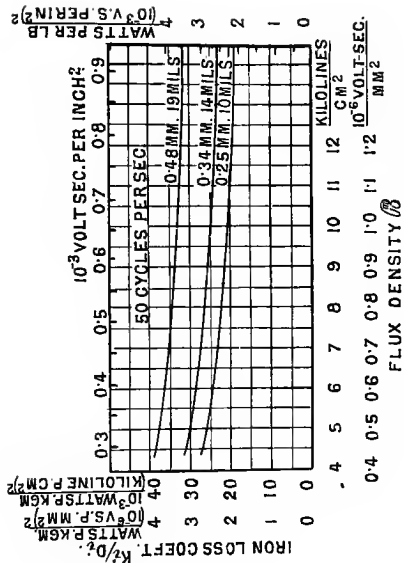


Fig. 3-09.—50 Cycles per Second.

Figs. 3-08-3-11.—Total Loss Coefficients for Best Unalloyed Transformer Sheets.

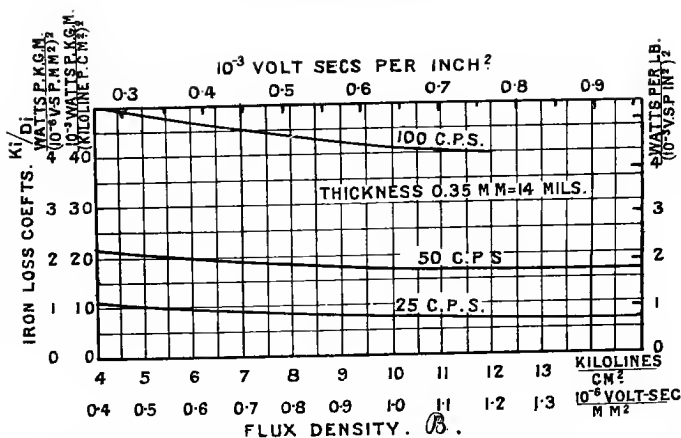


FIG. 3.12.—Total Loss Coefficients for Kapito & Klein's Alloyed Transformer Sheets.

Effect of Temperature on Iron Losses.—A very complete investigation of the effects of temperature on the magnetic qualities was made by Dr D. K. Morris, who showed that with constant limits of flux density the area enclosed within the cyclic curves of B and H becomes greatly reduced as the temperature approaches the critical point at which the metal loses its distinctive magnetic quality. Dr Morris gives the results shown in fig. 3.13

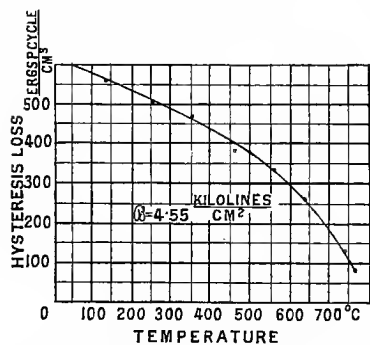


FIG. 3.13.—Variation of Hysteresis Losses with Temperature (Morris).

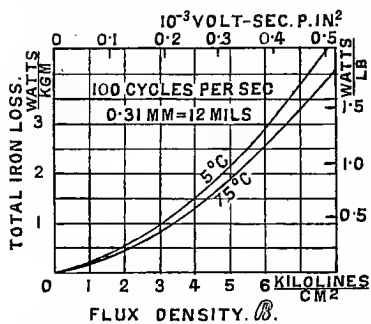


FIG. 3.14.—Iron Losses at Different Temperatures (Mordey and Hansard).

for a specimen of iron which had previously been annealed at 1150°C ., and which was then taken through cycles of magnetisation with a maximum flux density of $4550\ \mu\text{C.G.S.}$

Mordey and Hansard also give some interesting results, showing the influence of temperature on the iron losses, in their paper before the British Association. The iron used for the purpose was manufactured by Sankey & Co.; and it was tested at 5°C . and 75°C . (see fig. 3.14).

The eddy losses, again, form a large proportion of the total losses, being about 50 per cent. of the total loss at the higher values of B . On analysing the curves in this figure it will be found that the eddy currents—at least at the higher flux densities—apparently decrease rather more rapidly than would be expected from the increase in the resistance of their paths with rise of temperature, and this may possibly be due to a decrease of the hysteretic coefficient at high values of the flux density.

Of course, it must not be assumed that the considerable decrease in the losses with a rise in temperature would make it advisable to work transformers at a high temperature, as the increase of the resistance of the windings would counterbalance the gain in the iron.

Ageing of Transformer Iron.—It is a curious fact that after a transformer has been worked for some time, the iron losses are often found to have increased by a not inconsiderable amount, while the permeability has

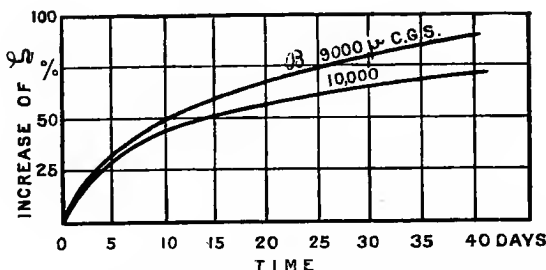


FIG. 3-15.—Ageing of Transformer Iron at Different Flux Densities (Stern).

at the same time been reduced. This phenomenon is usually called “ageing,” and was first brought prominently to the notice of engineers by a discussion in the technical press following upon an article which appeared in the *Electrician* of 7th December 1894. In this article Mr G. W. Partridge exhibited data showing that the core losses of three transformers experimented upon by him had increased from 35 to 50 per cent. in the course of one to two hundred days.

Since then, a number of articles have been written on this subject, the most noteworthy being those of Mr Mordey, “On Slow Changes in the Magnetic Permeability of Iron”;¹ Mr S. R. Roget on “Effects of Prolonged Heating on the Magnetic Properties of Iron”;² Mr A. R. Ford on “Hysteresis in Sheet Iron and Steel”;³ Mr J. A. Capp on “The Ageing of

¹ Paper read before the Royal Society, 19th Dec. 1894.

² Paper read before the Royal Society, 12th May 1898.

³ Paper presented at the 142nd Meeting of the American Institution of Electrical Engineers.

Transformer Steel";¹ and Dr G. Stern, "Über das Altern von Deutschen Eisenblechen."²

The results of all these investigations may be summarised as follows:—

Ageing may take place at any temperature; but it is the more rapid the higher the temperature at which the transformer is kept running.

Different makes of iron exhibit great differences in this respect, and as a rule the better the iron is otherwise the more likely it is to age. Soft steel seems to be less subject to ageing than soft sheet iron, and brands can be obtained which practically do not age at all at moderate temperatures.

Iron which has a tendency to age does so less quickly at high flux densities than at low ones (see fig. 3·15).

Additional annealing, after the plates have been cut, is often valueless, and sometimes even a disadvantage. The effects of this annealing are shown in the accompanying table:—

TABLE 3·01.—AGEING OF TRANSFORMER PLATES 0·5 MM. (20 MILS) THICK, AT 75° C. EFFECT OF ANNEALING (Stern).

Manu- facturer.	Hysteresis Coefficient (Original). μ.C.G.S.	Time of Heating. Hours.	Percentage Increase of the Hysteresis Coefficient.	Remarks.
1 B	121×10^{-5}	700	4·1	Annealed after cutting.
1 B	121×10^{-5}	"	4·1	Not annealed.
1 A	134×10^{-5}	"	8·9	Annealed.
1 A	131×10^{-5}	"	3·1	Not annealed.
2 B	119×10^{-5}	375	4·2	
3 B	139×10^{-5}	"	10·3	
2 A	154×10^{-5}	"	12·6	
1 D	114×10^{-5}	"	0	

The following three figures show some very interesting results obtained by Mr S. R. Roget. From the first one we see that at 50° C. the ageing is practically negligible, while it reaches its maximum at about 160° C. It will also be noticed that the peak at which the hysteresis reaches its maximum in each case comes sooner the higher the temperature, and that its height becomes reduced when the temperature is high.

The tests were executed by the ballistic method, using a ring of soft iron

¹ Report of the General Electric Company (British Thomson Houston).

² *Elektrotechnische Zeitschrift*, 1903, p. 407.

made by coiling up a long strip of sheet metal. This ring was annealed, and tested in the annealed state, and then baked. At 200°C . (figs. 3·17 and 3·18) the increase of hysteresis at first is very rapid, but after 19 hours there is a partial recovery in the permeability and a reduction of the losses.

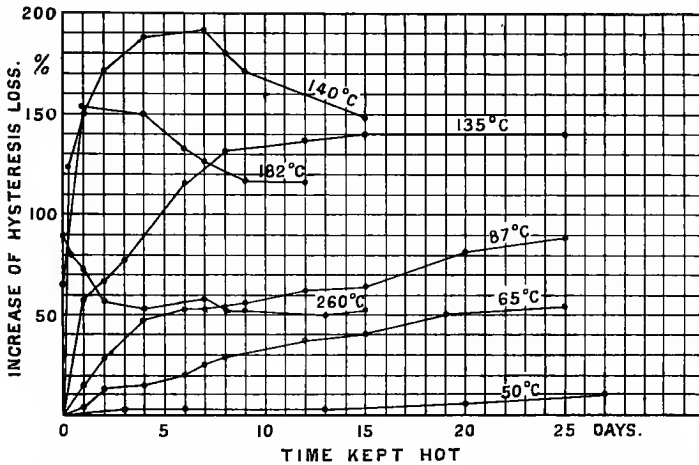


FIG. 3·16.—Ageing of Transformer Iron at Different Temperatures (Roget).

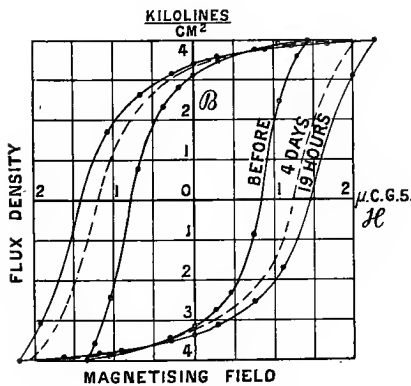


FIG. 3·17.

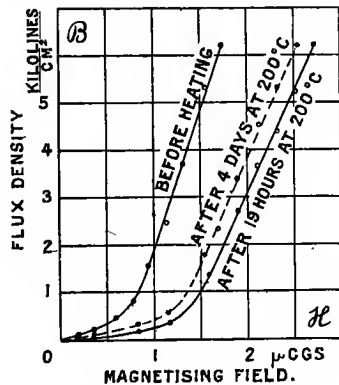


FIG. 3·18.

FIGS. 3·17 and 3·18.—Changes in Magnetisation Curve of Soft Iron produced by Baking at 200°C . (Roget).

The following table gives some results for “Tagger” plate, obtained by Mr Searle.¹ They show no appreciable ageing during ten months, the material being kept at the room temperature only.

¹ *Journ. I.E.E.*, vol. xxxiv. p. 55, 8th Dec. 1904.

TABLE 3·02.—HYSTERESIS OF TAGGER IRON (Searle).

Magnetising Field. μ . C. G. S.	Permeability.		\mathcal{H} μ . C. G. S.	μ (1902). μ . C. G. S.	μ (1903). μ . C. G. S.
	μ (1902). μ . C. G. S.	μ (1903). μ . C. G. S.			
0·2	579	589	1·6	4212	4243
0·4	905	876	2·0	4113	4128
0·6	1308	1258	3·0	3436	3450
0·8	1974	1929	4·0	2457	2479
1·0	2953	2901	5·0	2140	2162
1·2	3687	3714	6·0	1705	1714
1·4	4107	4107	8·0		

Fig. 3·19 gives the result of an ageing test executed by Professor Goldsborough on the H-type transformer, as sold by the British Thomson Houston Company.

The successful production of transformer sheets requires great skill and close watching during the process of manufacture. A treatment which will produce sheets of the low magnetic hysteresis desired will leave these sheets

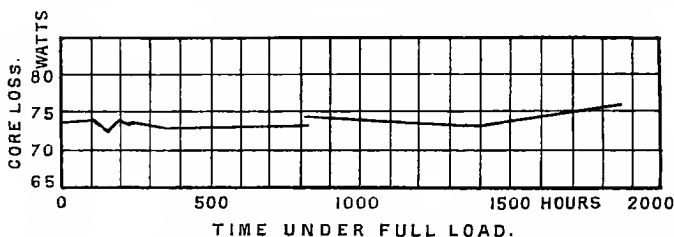


FIG. 3·19.—Ageing Test of an H-Type B.T.H. Transformer (Goldsborough).

in such an unstable condition that a slight elevation of their temperature may cause the hysteresis to rise rapidly until the once excellent sheets are no longer fit for use in a transformer. The chemical composition is of less importance than the proper treatment of the iron or steel, from the ingot to the rolling-mill. Specially designed furnaces have to be employed, and the whole process carried out under very rigid supervision.

Influence of Wave Form on Iron Losses.—The R.M.S. value of the induced E.M.F. is given by $E = 4f\beta S f N$,

$$\therefore \beta = \frac{E}{4f S f N} \quad \dots \quad 3·12.$$

We see that the maximum flux density for a given E.M.F. is inversely proportional to the form-factor. The latter depends upon the nature of the

stator winding of the alternator and the shape of the pole-shoes. It is now obvious that the greater we make f , the smaller will be β and the consequent hysteresis losses. For flat curves with a small form-factor the hysteresis losses are therefore larger than for pointed waves, which have a large form-factor.

This can be easily proved as follows. The mean square of any wave is the sum of the mean squares of all its harmonic components, and consequently its R.M.S. value is independent of the phase relations of these harmonics. An E.M.F. wave, being a symmetrical one, contains only odd components. If one of these has the same sign as the fundamental near the centre of the latter (fig. 3·20), it will add to the peakiness of the wave, but at the same time it will reduce the mean value, since it will have one more negative than

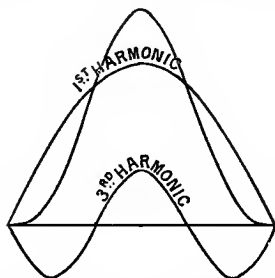


FIG. 3·20.—Peaky Wave.
Third Harmonic reduces Mean Value and
increases Form-Factor.

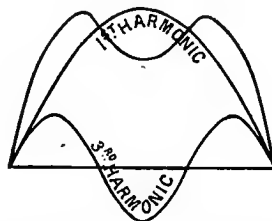


FIG. 3·21 —Flattened Wave.
Third Harmonic increases Mean Value and
decreases Form-Factor.

FIGS. 3·20 and 3·21.—Effect of Third Harmonic on Form-Factor.

positive half-waves included in the positive half of the given wave. On the other hand, if the sign of this harmonic is altered (fig. 3·21) it will flatten the wave and increase its mean value. Consequently, since the form-factor is $\frac{\text{R.M.S. value}}{\text{Mean value}}$, and the R.M.S. value is the same in both cases, the peaky wave will have a greater form-factor.

As an example, take two identical single-phase transformers, fed from 5000 volts (R.M.S.) networks, one of which has a rectangular E.M.F. wave, and the other a sine wave (see fig. 3·22). The frequency in each case is 50 cycles per second, the number of primary turns is 1218, and the cross-section of the magnetic path is 400 sq. cms. (64 in.²).

The maximum flux density in the first instance is

$$\beta_1 = \frac{E}{4fSfN_1} = \frac{5000 \times 10^8}{4 \times 1 \times 400 \times 50 \times 1218} = 5100 \text{ } \mu\text{.C.G.S.} = 33,000 \text{ lines per in.}^2,$$

and in the second

$$\beta_2 = \frac{5000 \times 10^8}{4 \times 1 \cdot 11 \times 400 \times 50 \times 1218} = 4600 \text{ } \mu\text{.C.G.S.} = 29,500 \text{ lines per in.}^2$$

In comparing these inductions and the hysteresis losses caused by them, it will be found that in the first case the loss is 18 per cent. higher than in the second one.

A perfectly flat curve is, of course, an extreme case, and such a wave form is never actually produced. The sharp corners are always rounded off (see curve III, fig. 3·22), since the magnetic field never changes suddenly from a

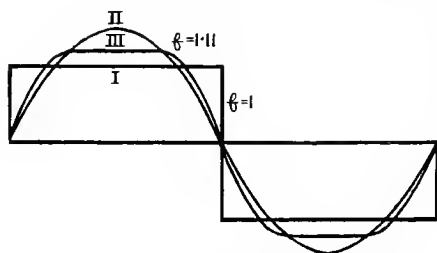


FIG. 3·22. —E.M.F. Waves with Different Form-Factors.

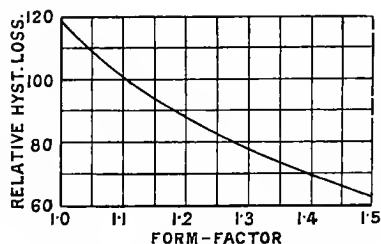


FIG. 3·23. —Variation of Hysteresis Loss with Form-Factor.

maximum value to zero, but has always a “fringe.” In fig. 3·23 the hysteresis loss has been plotted as a function of the form-factor, taking that with a sine wave as 100.

The eddy current loss is not greatly affected by the wave form, because the R.M.S. value of the E.M.F. causing them will be proportional to the R.M.S. E.M.F. in the coils, at least near the surface, since both are caused by the same change of flux. There will, however, be secondary effects, due to

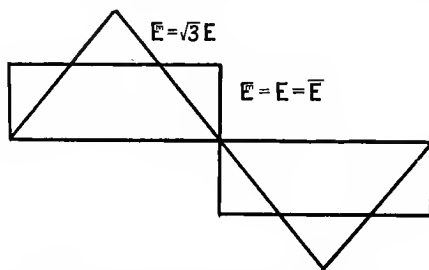


FIG. 3·24. —Rectangular and Triangular Waves.

the distortion of the flux wave in the interior by the demagnetising effect of the eddy currents, and by the inductance of the eddy current paths; but these cannot be calculated.

The advantages of a high form-factor are counter-balanced by disadvantages. Let us consider rectangular and triangular waves with an R.M.S. value of 5000 volts. Their form-factors are 1 and 1·16 respectively

and their amplitudes 5000 volts, and $\sqrt{3} \times 5000 = 8660$ volts. Thus, while the insulation of the first transformer has only to withstand 5000 volts, that of the other must be made for 8660 volts. Of course, the waves considered have extreme shapes, and the ratio between the amplitude and the R.M.S. value generally lies within these limits. But it is evident, nevertheless, that flat curves will produce less strain upon the insulation than waves with high form factors; and this disadvantage is of great importance, and may easily counter-balance the advantage of reduced iron losses, especially if the transformer is to be made for very high voltages. On the whole, it is best to have the E.M.F. follow a sine law as nearly as possible, and this is emphasised when the transmission system is being considered.

Effect of Frequency upon the Iron Losses.—Since $E = 4ff\beta SN_1$, the power lost by hysteresis is

$$P_H = \mathcal{L}f\beta^{1.6}V_I = \mathcal{L}f\left(\frac{E}{4ffSN}\right)^{1.6}V_I \quad . \quad . \quad . \quad 3.13.$$

We see from this that, when the E.M.F. and form-factor are constant, the hysteresis losses for a given transformer are inversely proportional to the (frequency)^{0.6}, and consequently decrease as the frequency is increased.

The eddy current losses are expressed by

$$P_E = ef^2l^2\beta^2V_I = el^2 \times \frac{E^2}{16S^2N^2}V_I \quad . \quad . \quad . \quad 3.14,$$

so that it appears that they are independent of the frequency as long as the inductance of the eddy current paths can be neglected. The effect of that inductance is to make the impedance of these paths increase with the frequency, and so make the losses less than they would otherwise be.

We may now say that for a given transformer fed from a constant terminal P.D., the iron losses are the smaller the higher the frequency.

Tests on the Variation of the Iron Losses.—In a paper read before the British Association in 1904, Mordey and Hansard give the results of tests on Sankey iron of different thicknesses, in which they found that for the 0.34 mm. (about 14 mils) and 0.48 mm. (19 mils) sheets the eddy current loss was practically proportional to the (thickness)²; but for the 0.65 mm. (25 mils) laminations the increase was less than that given by (thickness)² by about 30 per cent.

The tests also indicate that for this class of iron the eddy current loss is practically proportional to β^2 , but that it does not increase as rapidly as the square of the frequency.

The results of the tests are shown in the following two figures:—

The deviation of the eddy current loss from being proportional to (thick-

ness)² and (frequency)² is due to the inductance of the eddy current paths, which causes their impedance to increase with the frequency and so makes the current increase less rapidly than, and at very high frequencies be almost quite

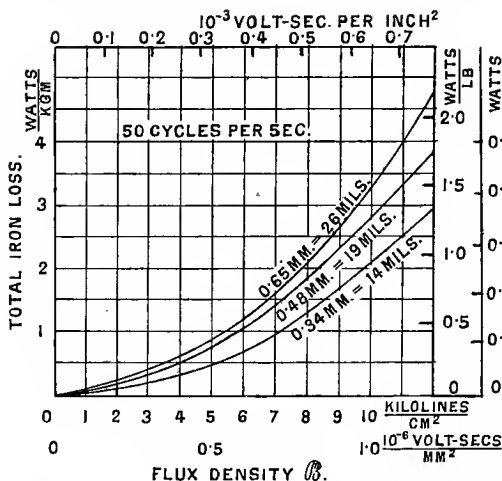


FIG. 3-25.—Losses in Sankey Iron at 50 Cycles per Second (Mordey and Hansard).

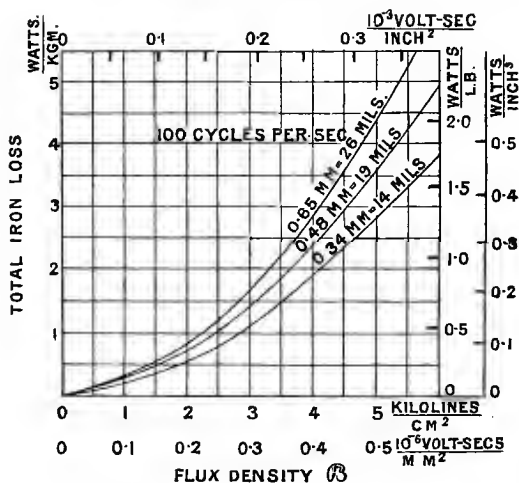


FIG. 3-26.—Losses in Sankey Iron at 100 Cycles per Second (Mordey and Hansard).

independent of, the frequency. Deviations from our formula also arise from the fact that Steinmetz's law for the hysteresis loss is not exactly true.

It is thus necessary to make frequent tests under working conditions on the actual brands of iron to be used, and from them to obtain values of the

constants in our theory, which will then yield reliable results when applied to other conditions not differing greatly from the tests.

Best Thickness of Laminations.—Since the hysteresis loss per unit volume with a given flux density is independent of the thickness of the plate, it would appear to be possible to reduce the total iron losses by diminishing the thickness of the laminations. In practice, however, improvement in this way is limited, because it is impossible to roll the sheets thinner than about 0.2 mm. (8 mils), and because, for a given core diameter, the effective area decreases with an increasing number of sheets owing to the increase in the space taken up by the insulation between them. The flux density must therefore rise if the total flux is to remain the same, and there will consequently be an increase in the hysteresis loss. Instead of the iron losses continually diminishing as the sheets are made thinner and thinner, they will be a minimum for a certain thickness. We can find this best thickness approximately as follows.

Let l_v be the thickness of the varnish or other insulation between a pair of sheets, supposed to be the same (usually about 0.05 mm., or 2 mils) for any thickness of sheet, l be the thickness of the sheets themselves, β_a the apparent flux density obtained by dividing the flux by the gross area of the core instead of by the net iron area, V_I the net volume of iron, and V_T the total volume of iron and lost space together. Then

$$\beta l = \beta_a(l + l_v), \quad \text{or} \quad \beta = \left(1 + \frac{l_v}{l}\right) \beta_a. \quad . \quad . \quad 3.15.$$

$$\text{Also} \quad V_I = \frac{l}{l + l_v} V_T = \frac{1}{1 + \frac{l_v}{l}} V_T. \quad . \quad . \quad . \quad 3.16.$$

Substituting these values in equation 3.10 we get

$$\begin{aligned} P_I &= \mathfrak{S} \left(1 + \frac{l_v}{l}\right)^{1.6} \beta_a^{1.6} f \frac{1}{1 + \frac{l_v}{l}} V_T + e f^2 f^2 l^2 \left(1 + \frac{l_v}{l}\right)^2 \beta_a^2 \frac{1}{1 + \frac{l_v}{l}} V_T \\ &= \mathfrak{S} \left(1 + \frac{l_v}{l}\right)^{0.6} \beta_a^{1.6} f V_T + e f^2 f^2 l^2 \left(1 + \frac{l_v}{l}\right) \beta_a^2 V_T. \end{aligned}$$

For a minimum, with a given V_T and β_a ,

$$\begin{aligned} 0 = \frac{dP_I}{dl} &= -0.6 \mathfrak{S} \left(1 + \frac{l_v}{l}\right)^{-0.4} \frac{l_v}{l^2} \beta_a^{1.6} f V_T + e f^2 f^2 (2l + l_v) \beta_a^2 V_T \\ \therefore (2l + l_v) l^2 \left(1 + \frac{l_v}{l}\right)^{0.4} &= \frac{0.6 \mathfrak{S}}{e f^2 f^2} l_v. \quad . \quad . \quad 3.17. \end{aligned}$$

We may, without very serious error, neglect l_v compared with l at this stage in order to simplify matters and get an approximate result. We then have

$$2l^3 = \frac{0.6\mathcal{S}}{ef^2\beta_a^{0.4}} l_v$$

$$\text{or } l = \sqrt[3]{\frac{0.3\mathcal{S}l_v}{ef^2\beta_a^{0.4}}} \quad \quad \quad 3.18$$

for least total iron losses.

The values of the best thickness as given by this equation are plotted in

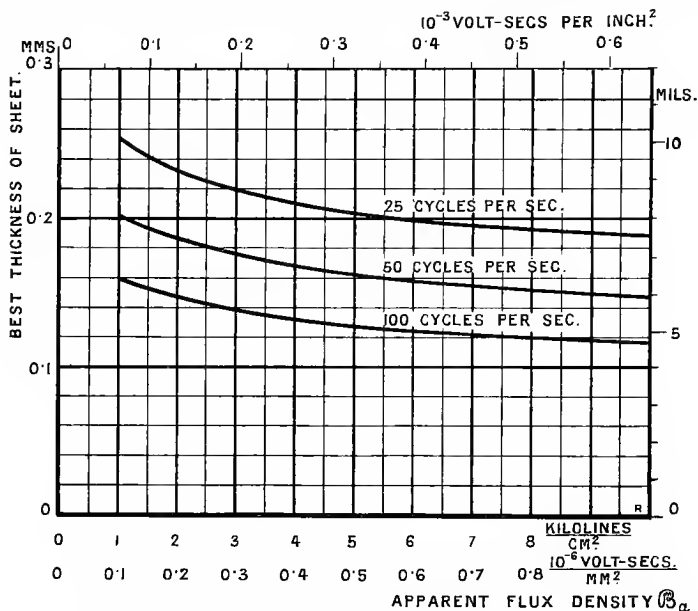


FIG. 3.27.—Most Favourable Thickness of Transformer Sheets.

fig. 3.27 as a function of the apparent flux density, the values assumed for the hysteretic and eddy current coefficients being

$$\mathcal{S} = 6 \times 10^{-6} \frac{\text{watts per cm.}^3}{(\text{cycles per sec.}) \times (\text{kilolines per cm.}^2)^{1.6}}$$

$$e = 0.18 \times 10^{-6} \frac{\text{watts per cm.}^3}{(\text{c.p.s.})^2 \times \text{mm.}^2 \times (\text{kilolines per cm.}^2)^2}$$

while the form-factor is taken as 1.11, and the thickness of the insulation 0.05 mm.

It will be observed that the best thickness varies comparatively little for ordinary frequencies and values of β_a , and that a mean value of 0.15 mm. (6 mils) may be taken. For such a thickness the space taken up by the

insulation is about $\frac{1}{4}$ of the total. At present sheets are not made commercially of less thickness than about 0.3 mm. (12 mils), so that a slight improvement is still possible. With sheets about 0.3 mm. (12 mils) about 15 per cent. of the space is taken up by the insulation.

With special alloyed irons the eddy current coefficient is considerably less, while the hysteresis loss is much the same. Consequently the best thickness of sheet with it is greater than that given in fig. 3.27.

Distribution of Iron for Least Iron Loss.—With a given flux, an increase of the cross-section will reduce the flux density and hence also the loss per unit volume, but at the same time it increases the volume. For any part of the magnetic circuit let the length be L , cross-section S , maximum flux \mathcal{P} , \mathcal{B} the maximum flux density, \mathcal{S} the Steinmetz hysteric constant, e the eddy current constant, and f the form-factor. The iron loss in that part at a frequency f is

$$P_I = \{ \mathcal{S} \mathcal{B}^{1.6} + e f^2 \mathcal{B}^2 l^2 \} f L S \quad . \quad . \quad . \quad 3.19$$

$$= \left\{ \mathcal{S} \mathcal{B}^{1.6} \frac{S^{1.6}}{S^{0.6}} + e f^2 \frac{\mathcal{B}^2 S^2}{S} l^2 \right\} f L$$

$$= \left\{ \mathcal{S} \frac{\mathcal{P}^{1.6}}{S^{0.6}} + e f^2 \frac{\mathcal{P}^2}{S} l^2 \right\} f L \quad . \quad . \quad . \quad 3.20.$$

Both terms are essentially positive, and decrease as S is increased. Hence, for a given flux the iron losses continually decrease as the cross-section is made greater, so long as the assumed laws for the hysteresis and eddy losses hold; but a practical limit is set by the cost and bulk of the transformer, and by the increased amount of copper and copper losses caused by the greater length of one turn.

Suppose, however, we are to have a given total volume V_I of iron in a single phase transformer of either the core or shell type, and wish to know how it should be distributed between the cores and yokes. Let L_c , L_Y , and S_c , S_Y be the total lengths and cross-sections of cores and yokes respectively; then the iron losses are, as before,

$$\begin{aligned} P_I &= \left\{ \mathcal{S} \frac{\mathcal{P}_c^{1.6}}{S_c^{0.6}} + e f^2 \frac{\mathcal{P}_c^2}{S_c} l^2 \right\} f L_c + \left\{ \mathcal{S} \frac{\mathcal{P}_Y^{1.6}}{S_Y^{0.6}} + e f^2 \frac{\mathcal{P}_Y^2}{S_Y} l^2 \right\} f L_Y \\ &= \left\{ \mathcal{S} \frac{\mathcal{P}_c^{1.6}}{S_c^{0.6}} + e f^2 \frac{\mathcal{P}_c^2}{S_c} l^2 \right\} f L_c + \left\{ \mathcal{S} \frac{L_Y^{0.6} \mathcal{P}_Y^{1.6}}{(V_I - L_c S_c)^{0.6}} + \right. \\ &\quad \left. e f^2 \frac{L_Y \mathcal{P}_Y^2}{(V_I - L_c S_c)} l^2 \right\} f L_Y \quad . \quad . \quad . \quad 3.21 \end{aligned}$$

since

$$V_I = L_c S_c + L_Y S_Y, \text{ or } S_Y = \frac{V_I - L_c S_c}{L_Y} \quad . \quad . \quad 3.22$$

$$\begin{aligned}
\frac{dP_I}{dS_c} &= \left\{ -0.6 \mathcal{S} \frac{\mathcal{P}_c^{1.6}}{S_c^{1.6}} - ef^2 f \frac{\mathcal{P}_c^2}{S_c^2} l^2 \right\} fL_c + \\
&\quad \left\{ 0.6 \mathcal{S} L_c \frac{L_Y^{0.6} \mathcal{P}_Y^{1.6}}{(V_I - L_c S_c)^{1.6}} + ef^2 f L_c \frac{L_Y \mathcal{P}_Y^2}{(V_I - L_c S_c)^2} l^2 \right\} fL_Y \\
&= \{ -0.6 \mathcal{S} \beta_c^{1.6} - ef^2 f \beta_c^2 l^2 + 0.6 \mathcal{S} \beta_Y^{1.6} + ef^2 f \beta_Y^2 l^2 \} fL_c \\
&= \{ 0.6 \mathcal{S} (\beta_Y^{1.6} - \beta_c^{1.6}) + ef^2 f l^2 (\beta_Y^2 - \beta_c^2) \} fL_c \quad . \quad . \quad . \quad 3.23 \\
&= (\beta_Y - \beta_c) \{ 0.6 \mathcal{S} (\beta_Y^{0.6} + \beta_Y^{-0.4} \beta_c + \beta_Y^{-1.4} \beta_c^2 + \text{etc.}) \\
&\quad + ef^2 f l^2 (\beta_Y + \beta_c) \} fL_c \quad . \quad . \quad . \quad . \quad 3.24
\end{aligned}$$

This is zero if $\beta_Y = \beta_c$, and only then, since β_Y and β_c must have positive values. Also, it then corresponds to a minimum and not a maximum of P_I , because $\frac{dP_I}{dS_c}$ is negative when S_c is so small as to make $\beta_c > \beta_Y$, but positive when S_c is great enough to make $\beta_c < \beta_Y$. Hence, if the total volume of iron is fixed, the iron losses are a minimum when the flux density is the same throughout the whole circuit; but when the volume is not fixed, the iron losses can be reduced without affecting the copper required by increasing the section of the yoke, and this will have the additional advantage of reducing the magnetising current.

Copper Losses.—When an electric current flows through a conductor there is a generation of heat in it at a rate proportional to the square of the current. That is, the power absorbed by the resistance of the conductor is

$$P_R = RI^2 \text{ (Joule's law)} \quad . \quad . \quad . \quad 3.25,$$

where R is the resistance of the conductor. Since

$$R = \rho \frac{L}{S} \quad . \quad . \quad . \quad . \quad 3.26,$$

where L is the length of the conductor, S its cross-section, and ρ its resistivity, we may write

$$P_R = RI^2 = \rho \frac{L}{S} I^2 = \rho L S \frac{I^2}{S^2} = \rho V_c \dot{I}^2 \quad . \quad . \quad . \quad 3.27,$$

or

$$\frac{P_R}{V_c} = \ddot{P}_c = \rho \left(\frac{I}{S} \right)^2 = \rho \ddot{I}^2 \quad . \quad . \quad . \quad 3.28,$$

where $\ddot{I} \left(= \frac{I}{S} \right)$ is the current density in the conductor. Thus (\ddot{P}_R) the power lost per unit volume due to the resistance of the conductor depends only on the current density and its resistivity.

The power lost per unit mass is

$$\frac{P_R}{M_c} = \frac{P_R}{D_c V_c} = \left(\frac{\rho}{D_c} \right) \dot{I}^2, \text{ or } P_R = M_c \left(\frac{\rho}{D_c} \right) \dot{I}^2 \quad . \quad . \quad 3.29,$$

where D_c is the density of the conductor.

For soft copper wires the resistivity at 0°C. is 16×10^{-6} ohm-mms., or 0.63×10^{-6} ohm-ins. ; and at other temperatures it is

$$\left. \begin{aligned} \rho &= 16 \times 10^{-6} \text{ ohm-mms.} \times \left(1 + \frac{4 \times 10^{-3}}{1^{\circ}\text{C.}} t\right) \\ &= 0.63 \times 10^{-6} \text{ ohm-ins.} \times \left(1 + \frac{4 \times 10^{-3}}{1^{\circ}\text{C.}} t\right) \\ &= 0.63 \times 10^{-6} \text{ ohm-ins.} \times \left(1 + \frac{2.3 \times 10^{-3}}{1^{\circ}\text{F.}} t\right) \end{aligned} \right\} \quad . \quad . \quad 3.30.$$

The density of copper may be taken as 8.9×10^{-6} kilograms per cubic mm., or 0.318 lb. per cubic in. Hence, at 0°C. ,

$$\begin{aligned} \frac{\rho_0}{D} &= 1.80 \text{ ohms} \times \text{mms.}^4 \text{ per kilogram} \\ &= 1.98 \times 10^{-6} \text{ ohms} \times \text{ins.}^4 \text{ per lb.} \end{aligned}$$

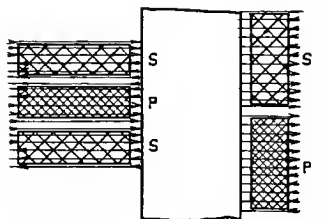
For convenience, the values of ρ and $\frac{\rho}{D}$ are given in the table for a number of temperatures.

TABLE 3.03.—RESISTIVITY OF COPPER AT DIFFERENT TEMPERATURES.

Temperature.		Resistivity, ρ .		Resistivity = $\frac{\rho}{D}$.	
$^{\circ}\text{C.}$	$^{\circ}\text{F.}$	10^{-9} ohm-metres. 10^{-6} ohm-mms.	10^{-6} ohm-ins.	$\frac{\text{Ohms} \times \text{mms.}^4}{\text{Kilograms.}}$	$\frac{10^{-6} \text{ ohms} \times \text{ins.}^4}{\text{Lbs.}}$
0	32	16.0	0.630	1.80	1.98
10	50	16.7	0.656	1.88	2.06
20	68	17.3	0.683	1.95	2.15
30	86	18.0	0.709	2.03	2.23
40	104	18.7	0.736	2.10	2.31
50	122	19.4	0.762	2.18	2.40
60	140	20.0	0.788	2.26	2.48
70	158	20.7	0.815	2.33	2.56
80	176	21.4	0.841	2.41	2.64
90	194	22.0	0.868	2.48	2.73
100	212	22.7	0.894	2.55	2.81

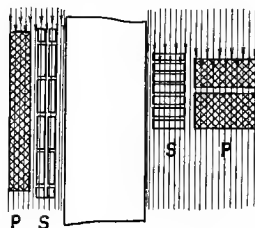
Eddies in Conductors.—There is a further loss due to the leakage flux, for it cuts across both the primary and secondary conductors, and induces E.M.F.'s which cause the current to be unevenly distributed over the cross-section and produce eddy currents in them. The result is equivalent to an increase in the resistances of the two windings, and gives rise to additional waste of energy. These supplementary losses cannot be pre-determined, as they depend upon the construction of the transformer, its

insulation, the arrangements of its windings, and the shape of the copper conductors. We may, however, keep them within reasonable limits by laminating the copper conductors, and by arranging them in such a way that the leakage flux cuts across their narrow sides, as shown in figs. 3·28 and 3·29. At the same time, the windings should be arranged so as to produce a small leakage flux, either by using long and thin concentric coils, or by subdividing them into a large number of sections. Further copper losses may be caused by joining in parallel conductors which have different E.M.F.'s. Internal currents will then circulate between them in addition to the load current.



Right Way. Wrong Way.

FIG. 3·28.—Subdivided Coils.



Right Way. Wrong Way.

FIG. 3·29.—Concentric Coils.

FIGS. 3·28 and 3·29.—Laminations of Conductors.

On account of supplementary losses, the actual copper loss will be greater than that calculated from the resistivity. But if we take a value for ρ about 5 to 25 per cent. greater than the actual resistivity we shall get about the right result.

Distribution of Copper for Least Copper Loss.—It will now be interesting to see how we must distribute the copper losses between the primary and secondary so that for a given total volume of copper the copper losses may be a minimum.

Let V_1, V_2 = volume of copper in primary and secondary,

L_1, L_2 = lengths „ „ „ supposed constant,

S_1, S_2 = cross-sections „ „ „

Then the power wasted by their resistance is

$$P_R = \rho \left\{ \frac{I_1^2}{S_1^2} V_1 + \frac{I_2^2}{S_2^2} V_2 \right\} \quad . \quad . \quad . \quad 3\cdot31$$

$$= \rho \left\{ \frac{I_1^2}{S_1^2} V_1 + \frac{L_2^2 I_2^2}{L_1^2 S_2^2} V_2 \right\}$$

$$= \rho \left\{ \frac{I_1^2}{S_1^2} L_1 S_1 + \frac{L_2^2 I_2^2}{V_2} \right\}$$

$$= \rho \left\{ \frac{L_1 I_1^2}{S_1} + \frac{L_2^2 I_2^2}{(V_C - L_1 S_1)} \right\} \quad . \quad . \quad . \quad 3\cdot32.$$

Equation 3.35 shows that $\frac{dP_R}{dS_C}$ is always positive, and consequently $\frac{dP_I}{dS_C}$ must be negative if the losses are to be least. By referring back to 3.23 it will be seen that this requires β_C to be greater than β_Y .

An accurate solution of the equation is scarcely necessary, for a small variation in the ratio of the flux densities does not greatly affect the total loss. From a series of examples it has been found that, on the average, the flux density in the core should be about twice as high as that in the yoke. In core transformers in which the length of the yoke is small as compared with that of the core, we do not introduce a serious error by making $\beta_C = \beta_Y$, which has the additional advantage of increasing the cooling surface. In shell transformers, however, the length of the yoke forms a large part of the magnetic circuit, so that for this type of transformer it is advisable to make the flux density in that part of the frame which is surrounded by the windings higher, if not twice as high, as that in the unwound portion, as long as the cooling surface is not reduced too much.

Maximum Efficiency of a Given Transformer.—The iron losses of a transformer working with a constant P.D. may be taken as approximately constant at all loads, while the copper losses are proportional to the square of the current. The former keep down the efficiency at small loads, and the latter make it small at large loads. To get the point of maximum efficiency we may put:—

$$\begin{aligned}\eta &= \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}} \\ &= \frac{P - P_I - P_R}{P} = 1 - \frac{P_I + RI^2}{VI \cos \theta} = 1 - \frac{P_I}{VI \cos \theta} - \frac{RI}{V \cos \theta} \quad \dots \quad 3.37.\end{aligned}$$

Where P is the input, V the P.D., I the current, $\cos \theta$ the power-factor, and η the efficiency. For a maximum,

$$\begin{aligned}0 &= \frac{d\eta}{dI} = \frac{P_I}{VI^2 \cos \theta} - \frac{R}{V \cos \theta} \\ \therefore \quad P_I &= RI^2 = P_R \quad \dots \quad \dots \quad \dots \quad \dots \quad 3.38.\end{aligned}$$

That is to say, the efficiency is a maximum with any given P.D. for that current which makes the copper loss the same as the iron loss. Whether this is to be the normal full load of the machine or not will depend on circumstances. Sometimes it is purposely made smaller than full load in order to get a high average efficiency throughout the year; at other times the copper resistance has to be kept small in order to keep the voltage drop on load within prescribed limits. In any case, when comparing the

efficiencies only of different designs, the load at maximum efficiency may be taken as the criterion, but it is to be remembered that the temperature rise and voltage drop may be even more important than efficiency when fixing the practical value of any fresh design.

It is interesting to note that the efficiency remains practically constant when working with variable voltage but constant impedance. For if the P.D. be raised in the ratio x , the current will also be increased in the same proportion, and the output as x^2 . But the iron loss will be increased approximately as x^2 , and the copper loss exactly as x^2 . The total losses are thus increased in nearly the same ratio as the load, and so the efficiency is unchanged. If we take account of the fact that the hysteresis loss increases less rapidly than the square of the flux, or P.D., we shall find that the efficiency is slightly higher at the greater P.D., and the maximum efficiency at that P.D. greater still.

CHAPTER IV.

TEMPERATURE RISE.

Rise of Temperature with Time. — The energy which is lost in a transformer is converted into heat, and causes its temperature to rise above that of the surrounding medium. Heat will then be given off by the transformer and absorbed by the surrounding air or oil, the interchange taking place by convection, radiation, and conduction. The rate of dissipation of heat is practically proportional to the difference between the temperatures

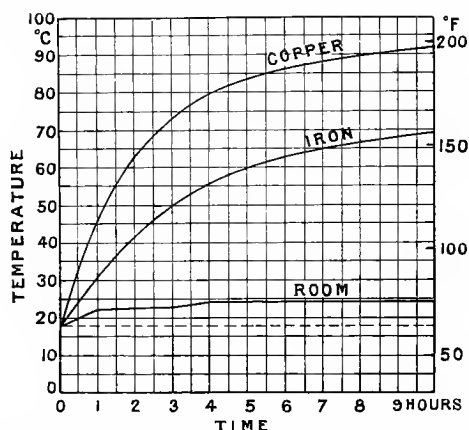


FIG. 4.01.—Temperature-Time Test of a P.D.M. Transformer at 45 per cent. Overload.

of the heat producer and the heat absorbers, and approximately also to the area of the heat-emitting surface. If no dissipation of heat were to take place, the temperature of the transformer would rise continually, when working, until it got high enough to destroy the insulation and melt the conductors.

If we plot the rise of temperature as a function of the time, we find that the resulting curve starts nearly straight, then bends away from the axis of temperature, getting flatter and flatter, and finally becomes horizontal. Fig. 4.01 shows such a curve for an air-cooled transformer by the Phoenix

Dynamo Company when heavily overloaded. If we plot such curves for different transformers we find that all have approximately the same shape, and can be made to fit one another very nearly by a suitable choice of scales. Thus the same law is applicable to them all.

Let everything to begin with be at one temperature, and suppose that the external conditions, the rate of generation of heat, and the specific heats of the materials remain constant. Let

P_H = Power spent in heating a mass M whose specific heat is S ,

t = Temperature at any time T after the generation of heat has commenced,

t_0 = Constant temperature of surroundings.

t_r = Final temperature rise above surroundings.

At first there will be no loss of heat, since everything is at the same temperature. Hence all the heat generated is available for warming the material.

$$\therefore \left. \begin{array}{l} \text{Initial rate of} \\ \text{rise of temp.} \end{array} \right\} = \frac{\text{rate of generation of heat}}{\text{heat required for unit rise of temp.}} = \frac{P_H}{MS} \quad 4.01.$$

But, as the temperature rises, heat begins to escape by conduction, convection, and radiation at a rate approximately proportional to the excess temperature $(t - t_0)$ and to the cooling surface S_C . Thus,

$$\text{Rate of dissipation of heat} = P_D = AS_C(t - t_0) \quad 4.02;$$

where A is a quantity depending on the nature of the surface and its surroundings, but practically independent of its temperature within ordinary ranges. It may be called the emissivity of the surface. Only the remaining portion of the heat generated is now available for raising the temperature whose rate of increase is consequently reduced to

$$\frac{dt}{dT} = \frac{P_H - P_D}{MS} = \frac{P_H - AS_C(t - t_0)}{MS} \quad 4.03.$$

The rate of warming thus gets less and less as the temperature rises, until finally a temperature is reached at which the rate of dissipation of heat is equal to the rate of generation, and then no further change occurs. This steady temperature is attained when

$$P_H = P_D = AS_C(t_\infty - t_0),$$

or

$$t_r = t_\infty - t_0 = \frac{P_H}{AS_C} \quad 4.04.$$

It thus depends only on the rate of generation of heat and the facilities for cooling, but not on the thermal capacity.

Equation 4·03 must be reduced to a form in which both sides are numerical quantities, thus:—

$$\frac{dt}{dT} = \frac{P_H - P_D}{MS} = \frac{AS_C \{t_r - (t - t_0)\}}{MS} \quad . \quad . \quad 4\cdot05.$$

$$\therefore \frac{-dt}{t_r - (t - t_0)} = -\frac{AS_C}{MS} dT \quad . \quad . \quad . \quad 4\cdot06.$$

$$\therefore \log \{t_r - (t - t_0)\} - \log t_r = -\frac{AS_C}{MS} T \quad . \quad . \quad . \quad 4\cdot07,$$

since $t = t_0$ when $T = 0$.

$$\therefore \frac{t_r - (t - t_0)}{t_r} = \epsilon^{-\frac{AS_C}{MS} T} \quad . \quad . \quad . \quad 4\cdot08,$$

or

$$t - t_0 = t_r \left(1 - \epsilon^{-\frac{T}{T_0}}\right) \quad . \quad . \quad . \quad 4\cdot09;$$

where

$$\begin{aligned} \epsilon &= \text{Base of Napierian logarithms} \\ &= 2\cdot71828 \dots \end{aligned}$$

and

$$\begin{aligned} T_0 &= \frac{MS}{AS_C} = \frac{MS t_r}{AS_C t_r} = \frac{\text{heat reqd. to reach final temp.}}{\text{rate of generation of heat}} \\ &= \text{Time required to reach actual final temperature if there} \\ &\quad \text{were no cooling.} \quad . \quad . \quad . \quad . \quad . \quad 4\cdot10. \end{aligned}$$

Owing to the loss by cooling, the actual temperature rise only reaches $\left(1 - \frac{1}{\epsilon}\right)$, or 63·4 per cent. of its steady value in the time T_0 .

An infinite time is required to actually reach the steady temperature, but after four or five times the time constant T_0 has elapsed, the further change is so small as to be quite negligible for practical purposes.

For comparison with the actual test of fig. 4·01 an exponential curve with the final rise taken as 100 per cent. is given in fig. 4·02, and the time required to reach various fractions in table 4·01. The curves for actual transformers do not agree exactly with the simple exponential law because of the gradual rise of temperature of the immediate surroundings (*e.g.* insulation, oil, etc.) of the material in which the heat is actually generated, and the variations of the loss density and thermal capacity from part to part.

TABLE 4·01.—VALUES OF EXPONENTIAL FUNCTIONS $\epsilon^{-\frac{T}{T_0}}$ AND $(1 - \epsilon^{-\frac{T}{T_0}})$.

$1 - \epsilon^{-\frac{T}{T_0}}$	0·10	0·20	0·30	0·40	0·50	0·60	0·70	0·80	0·90	0·95	0·99	0·995	0·999
$\epsilon^{-\frac{T}{T_0}}$	0·90	0·80	0·70	0·60	0·50	0·40	0·30	0·20	0·10	0·05	0·01	0·005	0·001
$\frac{T}{T_0}$	0·105	0·223	0·357	0·511	0·693	0·916	1·204	1·609	2·303	2·996	4·606	5·30	6·91

Since the ratio of volume to surface increases with size, and T_o is the ratio of the thermal capacity to the cooling facilities (see eqn. 4.10), the time constant is greater the larger the apparatus if the method of cooling is kept the same. It is also increased by anything which hinders the cooling, such as enclosing in a case, but reduced by anything, such as forced circulation, which promotes the removal of the heat.

In most electric machines the output is chiefly limited by the heating of

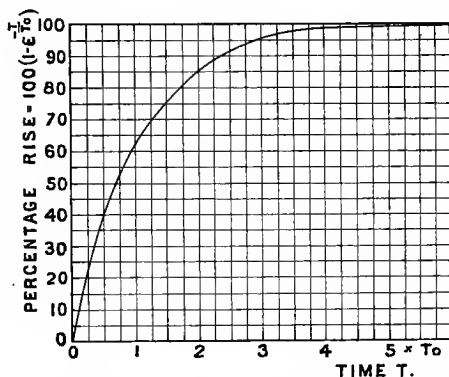


FIG. 4.02.—Exponential Curve. $100(1 - e^{-\frac{T}{T_0}})$.

the copper windings, and so the time of a proper heating test will be determined by the temperature rise of the copper.

As we have already seen (eqn. 3.28), the rate of generation of heat by resistance is

$$P_B = \rho V \dot{I}^2 \quad . \quad . \quad . \quad . \quad . \quad 4.11;$$

where ρ is the resistivity and V the volume of the conductor, while \dot{I} is the current density in it. Hence the initial rate of rise of temperature is

$$\begin{aligned} \dot{i} &= \frac{\text{rate of generation of heat}}{\text{thermal capacity}} \\ &= \frac{\rho \mathbf{V} \dot{\mathbf{I}}^2}{S D} = \frac{\rho}{SD} I^2 . \end{aligned} \quad 4.12.$$

S is the specific heat, or thermal capacity per unit mass, and \mathbf{D} the density of the material.

For copper we may take $\frac{\rho}{D} = 2.14 \frac{\text{ohms} \times \text{mms.}^4}{\text{kilograms}}$ (at 45° C. or 113° F., see

table 3·03), and S as $0·093 \times 4184 =$ joules per kilogram-°C. Hence

$$\frac{\rho}{SD} = \frac{2·14}{0·093 \times 4184} \times \frac{\text{ohms} \times \text{mms.}^4}{\text{kilograms}} \times \frac{\text{kilogram-}^\circ\text{C.}}{\text{joule}}$$

$$= 5·50 \times 10^{-3} \frac{\text{ohms} \times \text{mm.}^4 \times ^\circ\text{C.}}{\text{amp.}^2 \times \text{ohm} \times \text{sec.}} \times \frac{60 \text{ secs.}}{1 \text{ min.}}$$

or

$$\left. \begin{aligned} i &= 0·33 \text{ C.}^\circ \text{ per min.} \times \left(\frac{\bar{I}}{\text{amps. p. mm.}^2} \right)^2 \\ &= 0·80 \times 10^{-6} \text{ C.}^\circ \text{ per min.} \times \left(\frac{\bar{I}}{\text{amps. p. in.}^2} \right)^2 \\ &= 1·43 \times 10^{-6} \text{ F.}^\circ \text{ per min.} \times \left(\frac{\bar{I}}{\text{amps. p. in.}^2} \right)^2 \end{aligned} \right\} \quad 4·13.$$

In these formulæ the influence of the cotton covering has not been taken into account. Since its weight varies from about 3 to 5 per cent. of the total weight of the winding, and cotton has a specific heat about four times that of copper, the values obtained by the above formulæ are 12 to 20 per cent. too large.

TABLE 4·02.—INITIAL RATE OF TEMPERATURE RISE IN COPPER
FOR DIFFERENT CURRENT DENSITIES.

Current Density.		Pure Copper.		Copper with 4% Insulation.	
Amps. per mm. ²	Amps. per in. ²	C.° per min.	F.° per min.	C.° per min.	F.° per min.
0·25	162	0·02	0·036	0·017	0·031
0·50	324	0·08	0·144	0·069	0·124
0·75	486	0·18	0·324	0·155	0·28
1·0	645	0·33	0·59	0·284	0·51
1·25	806	0·50	0·90	0·43	0·77
1·50	968	0·75	1·35	0·645	1·16
1·75	1119	1·0	1·80	0·86	1·7
2·0	1290	1·3	2·34	1·12	2·0
2·25	1452	1·5	2·70	1·29	2·3
2·50	1613	2·0	3·60	1·72	3·1
2·75	1764	2·5	4·50	2·15	3·9
3·00	1935	3·0	5·40	2·58	4·65

When the final temperature and current density are known, the time constant for the copper can be calculated from table 4·02, and thus the time which will be required to raise the temperature of a transformer to any fraction of its final or specified temperature can be estimated.

In the temperature test of the P.D.M. transformer the current density was equal to 1.6 amperes per mm.² (1080 amperes per in.²), and so the initial rate of warming the copper was 0.73° C. (1.31° F.) per minute. Assuming that the final rise would have been 85° C. (153° F.), the value of the time constant would be 116 minutes. To reach a temperature rise within 2 per cent. of the steady value, the test would have to last for 7½ hours. These results are approximately confirmed by the test, the difference being due to the fact that the transformer is not a homogeneous mass of one material with the heat generated uniformly throughout it, as was supposed when deducing the formulæ.

Final Temperature Rise. — We have already seen (eqn. 4.04) that the final rise of temperature is

$$t_r = \frac{P_H}{A S_C}$$

where P_H is the total rate of generation of heat, and consequently the sum of all the losses, and S_C is the cooling surface. The value of A varies with the circumstances, and should be determined experimentally for each design of transformer. In the absence of such data we may take, for open coils,

$$\left. \begin{aligned} \frac{1}{A} &= 0.15 \frac{\text{C.}^\circ \text{ (by resis.)}}{\text{watt per metre}^2} \\ &= 240 \frac{\text{C.}^\circ \text{ (by resis.)}}{\text{watt per in.}^2} \\ &= 430 \frac{\text{F.}^\circ \text{ (by resis.)}}{\text{watt per in.}^2} \end{aligned} \right\} \dots \dots \dots 4.14,$$

and $\frac{1}{3}$ of these for enclosed ones.

Fig. 4.03 shows graphically the relation between the temperature rise and specific cooling surface for open and enclosed transformers, $\frac{1}{A}$ being taken as 0.15 and 0.20 °C. per $\frac{\text{watt}}{\text{metre}^2}$ respectively. These curves are rectangular hyperbolæ.

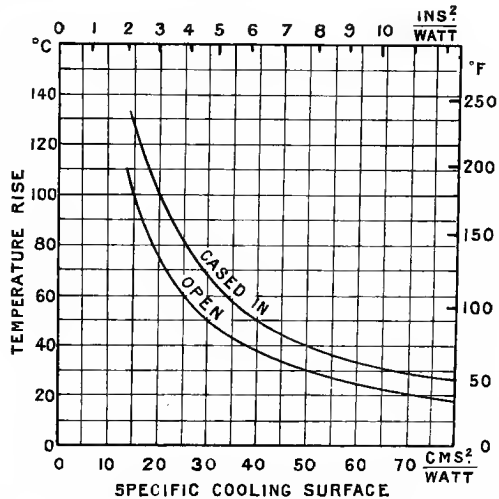


FIG. 4.03. — Variation of Temperature Rise of Transformers with Cooling Surface.

The greatest permissible temperature rise depends on the insulating material used. It varies with circumstances, but table 4.03 may be taken as a guide.

TABLE 4.03.—GREATEST PERMISSIBLE TEMPERATURE
RISE IN TRANSFORMERS.

Insulating Material.	Greatest Temperature Rise.	
	C.°	F.°
Cotton	60	108
Paper	70	126
Mica	90	162

It is best, however, to take even smaller values, as a high temperature increases the tendency of the iron to age, and causes an increased resistance loss in the copper.

The maximum temperature rise of a transformer is usually found in the copper, while the cooling area in the above illustration was obtained from that of the copper and iron.

If we consider each part separately, we may reckon for the iron about 15 cms.² (2.25 in.²) cooling surface per watt lost if the temperature rise has not to exceed 50° C. (90° F.), and about twice as much for the copper coils. It is, however, more convenient to deal with the transformer as a whole, since part of the heat generated in the copper coils will be discharged through the cores and yokes of the iron frame.

For use with the above constants and curves, the cooling surface is to be reckoned by adding together the following :—

(a) *Core Transformer*.—All the sides of the cores, three sides and two ends of each yoke, the cylindrical surface and both ends of each coil, and both sides of every ventilating duct which is not less than one centimetre ($\frac{3}{8}$ " wide in cores, yokes, and coils.

(b) *Shell Transformer*.—The external surface of the iron, the surface of the coils exposed outside the shell, and the ducts as above.

These formulæ will give approximate results if sufficient space is allowed between the cores and the windings, but when the low-voltage coil is wound directly on the limb, it is hardly correct to assume that the total surface of all the limbs emits heat. For every particular design, the coefficient should be found experimentally.

Oil-Cooling.—For large transformers, natural-draught cooling has been

found insufficient, unless we use more active material than a rational method of manufacture would justify. The reason is easily given. Suppose we increase a transformer in all its linear dimensions: the cooling surface then increases proportionally to the second, and the losses, being proportional to the volume, to the third power. Consequently, the temperature rises of two transformers with the same current and flux densities and similar construction are not equal, but that of the larger transformer is the greater. For equal temperature rises we would consequently have to reduce the densities in the iron and the copper for the larger transformer, and this would entail an increased expense for the active material.

The rate of emitting heat is augmented by placing the transformer into a bath of suitable oil, which conveys the heat much more readily to the

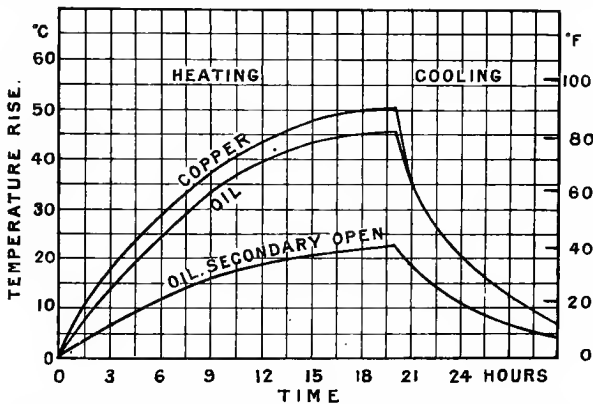


FIG. 4·04.—Temperature Test of an A.E.G. Oil-Cooled Transformer.

walls of the casing than air would do. This not only makes it possible to reduce the quantity of active material, but produces also a good influence upon the efficiency, since with otherwise equal conditions a transformer in oil keeps cooler than a design without it. Fig. 4·04 gives the heating and cooling curves of a transformer in oil, and can be compared with fig. 4·01, which was for a natural-draught air-cooled transformer. It is, however, necessary to remember that the transformers are of different sizes, and would therefore have different time constants even if the methods of cooling had been the same.

The heat which is emitted by the transformer and absorbed by the oil causes the latter to expand and to rise to the surface, its place being filled with comparatively cool oil from near the walls of the tank. The liquid will therefore be in a constant state of circulation, conveying the heat from the transformer to the walls of the vessel, whence it is removed by radiation and

convection by the air. For small transformers it will be found that a smooth cast-iron tank is usually sufficient, but for large designs the vessel

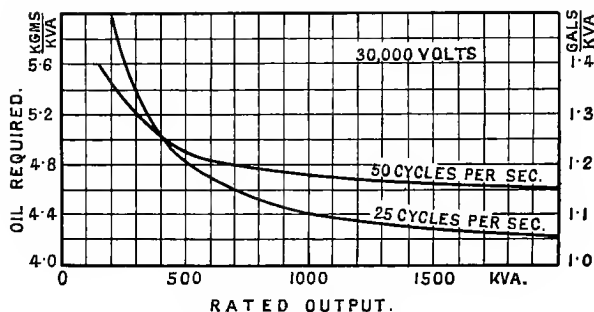


FIG. 4'05.—30,000 Volts.

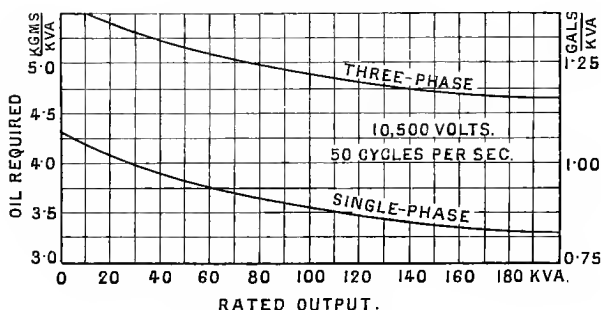


FIG. 4'06.—10,000 Volts.

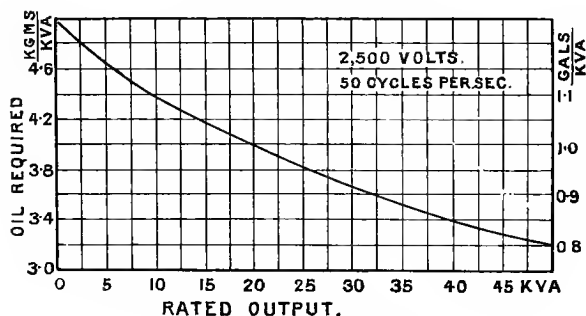


FIG. 4'07.—2500 Volts.

FIGS. 4'05-4'07.—Quantity of Oil for Transformers.

must be corrugated in order to expose a large enough surface to the atmosphere. The emission of heat may be further accelerated by adding special side tanks joined by pipes near the top and bottom to the main tank.

These keep cooler than the main tank, and so promote the circulation of the oil. Another way is to employ a bucket-shaped lid which dips down into the oil, and which may be kept cool by circulating water through it. Examples of these will be found along with other constructional details in Chapter IX.

The quantity of oil used depends on the P.D. as well as on the size. Large transformers require slightly less per unit of power than small ones. Figs. 4·05–4·07 give the average practice of the leading manufacturers.

Besides facilitating the escape of heat, oil prevents moisture working into the windings by the alternate heating and cooling to which they are subjected in use, and greatly improves the insulation by preventing brush discharges from the terminals and surfaces of the coils. Oil is necessary for

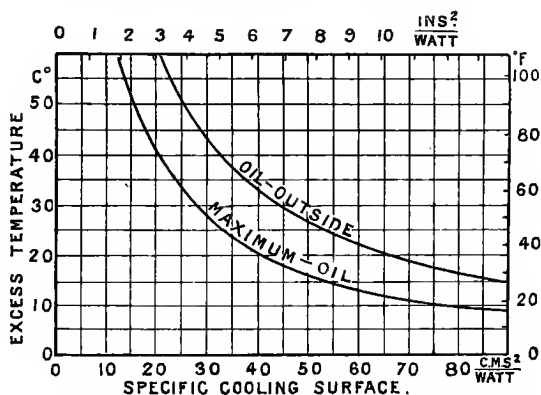


FIG. 4·08.—Temperature Rise of Oil-Cooled Transformer.

the former reason alone when the transformer is to be used in damp places, and for the latter reason when the voltage exceeds about 10,000, quite irrespective of heating considerations.

On the other hand, the use of oil limits the choice of insulating materials, as many—*e.g.* rubber, guttapercha, and even to a small extent mica—are dissolved or attacked by it. Good results are obtained by using presspahn which has been dried in a vacuum and afterwards kept in transil oil at 120° C. (248° F.) for about twelve hours. Cotton, linen, and tape are also suitable for use with oil. Another disadvantage of oil-cooling is the fact that if a fault occurs necessitating the withdrawal of the oil and the removal of the transformer it is frequently necessary to entirely rewind the coils, as the insulation may have got so soft as to be easily damaged by any movement between them. For this reason, the oil should not be withdrawn or the transformer moved unless it is unavoidable. The oil tank adds considerably

to the cost, especially if the conditions of use are such as to permit the use of an unenclosed transformer.

The contact surface between the oil and the tank, and the external cooling surface necessary for any specified temperature limit, vary considerably with the nature of the oil and the surrounding conditions. In the absence of constants obtained by experience with the actual oil and general design to be used, we may employ the curves of fig. 4·08, which show how the excess temperature of the windings above the oil, and of the oil above that outside, vary with the surface. If the temperature rise is not to exceed 50° C. (90° F.), the specific cooling surface of the transformer iron should be at least 11 cms.² ($1\frac{3}{4}$ ins.²) per watt dissipated, and about twice that for the windings.

Water-Cooling.—Simple oil-cooling is insufficient for transformers over 500 K.V.A., if they are to be made of a reasonable size and cost. It is then necessary to cool the oil by artificial means. The oil tank may be surrounded by a jacket through which cold water is kept circulating, often serpentine fashion, or the water may be passed through a coil or coils of pipe fixed in the upper part of the tank. In the latter case, precautions are necessary to prevent any moisture which may condense from the atmosphere on the inlet pipe from getting into the oil.

The greater the amount of cooling-water available, the smaller can be the bulk and cost of the transformer, but against this we must put the cost of the piping, etc., of the water, and of the energy required to circulate it. The quantity of water required is usually found by actual trial, but approximate data may be obtained by a simple calculation. With the allowed temperature rise t_r , the transformer will get rid of heat at a certain rate $P_D = A_t S_t t_r$ (see eqn. 4·02). The remainder of the total losses P_L must be carried off by the cooling-water. We may write:—

$$\text{Convection of heat by water} = P_w = \frac{M_w S_w t_{rw}}{T} \quad . \quad 4\cdot15,$$

and also

$$\text{Convection of heat by water} = P_w = A_p S_p (t_t - t_w) \quad . \quad 4\cdot16 ;$$

where M_w = Mass of water passed in time T ,

S_w = Specific heat of water,

D_w = Density of water,

t_{rw} = Rise of temperature of water in passing through pipe,

S_p = Effective transmitting surface of pipe,

t_t, t_w = Mean temperatures of transformer oil and water in pipe,

and A_p is a quantity depending on the nature and thickness of the pipe.

Hence the flow of water required is

$$\frac{M_w}{T} = \frac{P_w}{S_w t_{rw}} = \frac{P_L - P_D}{S_w t_{rw}} \quad 4.17,$$

or

$$\frac{V_w}{T} = \frac{M_w/D_w}{T} = \frac{P_w}{D_w S_w t_{rw}} = \frac{P_L - P_D}{D_w S_w t_{rw}} \quad 4.18,$$

and the surface of pipe necessary

$$S_p = \frac{P_w}{A_p(t_t - t_w)} = \frac{P_L - P_D}{A_p(t_t - t_w)} \quad 4.19.$$

We may take

$$\left. \begin{aligned} \frac{1}{S_w} &= \frac{1 \text{ kilogram-C.}^\circ}{4184 \text{ joules}} = \frac{1 \text{ kilogram-C.}^\circ}{70 \text{ watt-mins.}} \\ &= \frac{1 \text{ lb.-C.}^\circ}{1900 \text{ joules}} = \frac{1 \text{ lb.-C.}^\circ}{32 \text{ watt-mins.}} = \frac{1 \text{ lb.-F.}^\circ}{17.5 \text{ watt-mins.}} \end{aligned} \right\} \quad 4.20,$$

$$\left. \begin{aligned} \frac{1}{D_w S_w} &= \frac{1 \text{ litre-C.}^\circ}{4184 \text{ joules}} = \frac{1 \text{ litre-C.}^\circ}{70 \text{ watt-mins.}} = \frac{0.86 \text{ litre-C.}^\circ}{\text{watt-hours}} \\ &= \frac{1 \text{ gal.-C.}^\circ}{19,000 \text{ joules}} = \frac{1 \text{ gal.-C.}^\circ}{316 \text{ watt-mins.}} = \frac{0.19 \text{ gal.-C.}^\circ}{\text{watt-hours.}} \\ &= \frac{1 \text{ gal.-F.}^\circ}{175 \text{ watt-mins.}} = \frac{0.34 \text{ gal.-F.}^\circ}{\text{watt-hours.}} \end{aligned} \right\} \quad 4.21,$$

$$\left. \begin{aligned} \frac{1}{A_p} &= 3 \text{ to } 6 \frac{\text{cms.}^2 \times \text{C.}^\circ}{\text{watt}} \\ &= 0.5 \text{ to } 1 \frac{\text{ins.}^2 \times \text{C.}^\circ}{\text{watt}} \end{aligned} \right\} \quad 4.22.$$

The velocity of the water should not exceed 45 metres per minute (150 feet per min.).

Forced-Draught Air-Cooling.—When the conditions as to location and voltage permit it, forced-draught air-cooling will generally be found more economical than oil- and water-cooling for large transformers. The transformer is designed with numerous ventilating ducts through which cold air is continually forced by a fan or blower. The transformer is usually fixed over a chamber into which the air is delivered, and from which several transformers may be supplied. The blower is coupled directly to an induction motor which absorbs only a small fraction of 1 per cent. of the total output. It does not matter if the air is damp, for it is heated in passing through, and so will not condense on the windings. Forced draught, however, has the disadvantage of spreading the fire when a coil burns out, and may cause it to be entirely destroyed in a short time.

The quantity of air required for effective cooling may be calculated in the

same way as the water, by using equations 4·17 or 4·18, with appropriate constants. For air at ordinary atmospheric pressure (760 mm. Hg.) and temperature (15° C.), we may take

$$\frac{1}{S} = \frac{1 \text{ kilogram-C.}^\circ}{1000 \text{ joules}} = \frac{1 \text{ kilogram-C.}^\circ}{16.6 \text{ watt-mins.}} = 3.6 \frac{\text{kilograms}}{\text{hour}} \text{ per } \frac{\text{watt}}{\text{C.}^\circ} \left\{ \begin{array}{l} \\ \\ \end{array} \right. \quad 4.23,$$

$$= \frac{1 \text{ lb.-C.}^\circ}{450 \text{ joules}} = \frac{1 \text{ lb.-C.}^\circ}{7.5 \text{ watt-mins.}} = \frac{1 \text{ lb.-F.}^\circ}{4.2 \text{ watt-mins.}}$$

$$\frac{1}{DS} = \frac{1 \text{ litre-C.}^\circ}{1.23 \text{ joules}} = \frac{50 \text{ litre-C.}^\circ}{\text{watt-min.}} = \frac{3000 \text{ litre-C.}^\circ}{\text{watt-hour}} \left\{ \begin{array}{l} \\ \\ \end{array} \right. \quad 4.24.$$

$$= \frac{1 \text{ cub. ft.-F.}^\circ}{19.2 \text{ joules}} = \frac{3.2 \text{ cub. ft.-F.}^\circ}{\text{watt-min.}} = \frac{190 \text{ cub. ft.-F.}^\circ}{\text{watt-hour}}$$

Examples of the application of the various methods of cooling are given in Chapter IX.

CHAPTER V.

MAGNETIC LEAKAGE.

Equations for a Single-Phase Transformer.—When the primary of a transformer is joined to a source of supply, and the secondary to a resistance, we have two currents, I_1 and I_2 , circulating through the windings, which both set up magnetic fields.

Consider the primary circuit of a single-phase core transformer, and assume at present that the two windings are placed on different limbs (see fig. 5·01). Its current produces a flux ϕ_1 which has its course entirely through the iron, and which is consequently interlinked with both windings; and a flux ϕ_{L1} , interlinked only with the primary turns, and taking its course chiefly through air. We shall call the latter the primary leakage flux. In a similar manner, the secondary current causes the fluxes ϕ_2 and ϕ_{L2} , the latter being termed the secondary leakage flux.

Each flux induces an E.M.F. in the windings with which it is interlinked, and the terminal P.D., which is always the sum of all the E.M.F.'s in the winding, is obtained by adding to these induced E.M.F.'s the resistance E.M.F.'s caused by the resistances of the windings.

Let

$$\left. \begin{aligned} \mathcal{L}_{L1} &= \frac{N_1 \phi_{L1}}{I_1} \\ \mathcal{L}_{L2} &= \frac{N_2 \phi_{L2}}{I_2} \end{aligned} \right\} \dots \dots \dots 5\cdot01,$$

and

$$\mathcal{L}_M = \frac{N_1 \phi_2}{I_2} = N_1 N_2 \frac{\phi_2}{N_2 I_2} = \frac{N_1 N_2}{\mathcal{R}} = N_1 N_2 \frac{\phi_1}{N_1 I_1} = \frac{N_2 \phi_1}{I_1} \dots 5\cdot02;$$

where N_1 , N_2 , are the primary and secondary turns, \mathcal{L}_{L1} , \mathcal{L}_{L2} are called

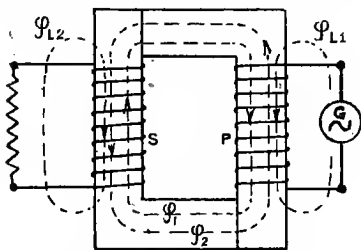


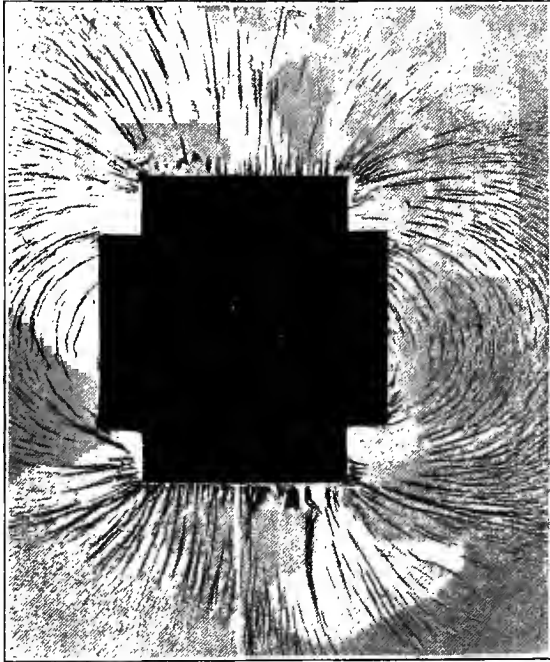
FIG. 5·01.—Leakage Fluxes in Core Transformer.

The ratio

$$\nu_1 = \frac{\mathcal{L}_{T1}}{N_1 \mathcal{L}_M} = 1 + \frac{N_2}{N_1} \cdot \frac{\mathcal{L}_{L1}}{\mathcal{L}_M} \quad . \quad . \quad . \quad 5.10$$

is termed the leakage coefficient, and is always greater than unity.

Filing Pictures of Magnetic Leakage.—The nature of magnetic leakage can be shown by taking filing pictures of the external field in the well-known manner. In a single-phase transformer having its windings



¹ FIG. 5.02.—External Field in Plane of Axes of Loaded Core Transformer with Primary and Secondary on Separate Limbs.

arranged according to fig. 5.01 and with its secondary circuit open, the primary current will cause excitation in the interior of the coil which is carried forward and distributed by the induction of free polarity on the surface of the iron. But this free polarity also magnetises the surrounding air and causes part of the flux to leave the iron. The flux distributes itself between the various paths in the ratio of their permeances; but the greater part keeps to the iron, since its reluctance is low, and only a small portion completes its circuit through the air. Of course, all of the latter is not

¹ The authors are indebted to Mr Görner of Messrs Hartmann & Braun for this interesting photograph.

necessarily leakage, but only such of it as is not linked with both coils; that is, only that part of it which returns *between* the coils.

The course of the flux is altered when the secondary circuit is closed, for in it we have a current which is nearly opposite in phase to the primary current. Thus there are now two excitations in different places which are opposed to one another, and the net field is due to their difference. The free polarity at the ends of the primary core, and the consequent leakage into the air, are much greater than before; because, in addition to the small excitation actually required to magnetise the iron, that required to balance the back excitation of the secondary has to be transnitted as well. Fig. 5.02 shows a filing picture for this case.

When both coils are placed on the same limb and the secondary is open, the character of the leakage is practically the same as before; but when its secondary is closed, the flux going from yoke to yoke is small, and it is no longer a leakage flux, as it is now linked with both coils. But there is a large leakage from the core between the two coils; for the free polarity between the yokes has now only to magnetise the other limb, while that between the coils has to balance the back excitation of the secondary as well.

The leakage may be much reduced by arranging the coils concentrically. The excitations then act on the same part of the iron, and require no free polarity to connect them together. The leakage flux is now due entirely to the direct effect of the coils on the space between them.

The reduction of the magnetic leakage may be carried still further by dividing one of the coils and placing one part inside and one outside the other coil. If the concentric arrangement is not suitable, we may split each winding into a large number of sections, and arrange primary and secondary sections alternately.

By dividing a coil into two sections, the excitation is split into two halves, each giving half the flux. Both halves of the useful flux are linked with both sections of the winding, and so produce the same effect as before, but the leakage flux produced by one section cannot be linked with the other section without also being linked with the other coil which comes between, and then it would not be leakage. Thus, being only linked with half the turns, the same total leakage flux, half linked with each coil, only produces one-half of the effect that it would with an undivided coil. The result of division is even better than this, because the total leakage flux is actually diminished owing to the reduction of the mean distance between the primary and secondary windings. Against this, however, we must place the fact that the flux which is linked with one turn only is not affected by subdivision.

Leakage Inductance of Long Thin Concentric Coils.—An exact calculation of the leakage inductance in the general case lies beyond the range of practical mathematics, but an exact solution can be obtained in certain ideal cases which will serve as a guide when dealing with real apparatus.

Consider a transformer with concentric coils whose length is infinitely great, whose winding depth is small enough to allow us to ignore differences in perimeter, whose turns are wound in a large number of thin layers, and in which a negligible part of the winding space is occupied by the insulation.

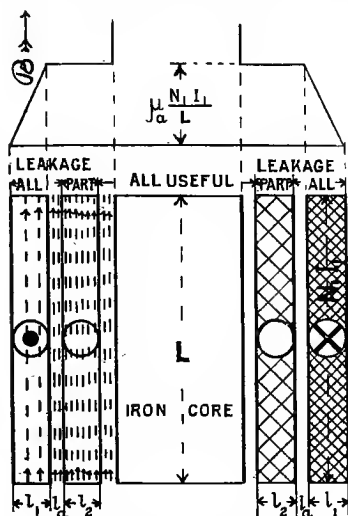


FIG. 5'03.—Flux due to Outer Coil only.

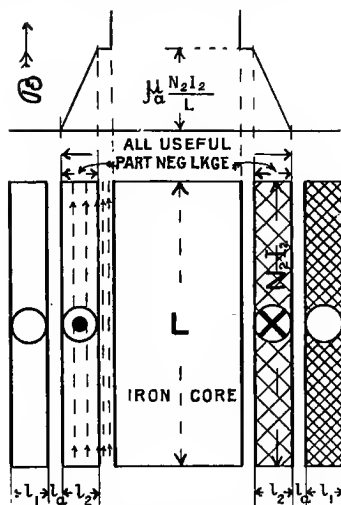


FIG. 5'04.—Flux due to Inner Coil only.

Leakage of Long Concentric Coils.

The equivalent of infinite length can be very nearly attained by winding the coils uniformly round a ring-shaped core whose diameter is large compared with that of the coils.

The leakage flux for the outer, supposed to be the primary, of such a pair of concentric coils due to a current in it is made up of three portions, viz., that within the winding space itself, that between the coils, and that within the winding space of the other coil. The space between is most effective, because all the flux there is linked with all the turns of the outer and with none of those of the inner coil. The flux in the inner winding space is only leakage in respect of those turns with which it is not linked.

The magnetic field due to a current in one of the coils is everywhere parallel to the axis and proportional to the current-turns outside a circle

drawn concentric with the coils and passing through the point considered. It is thus zero outside the coil, proportional to the distance within the winding space, and constant inside the coil, as shown in fig. 5·03. An iron core will not affect the leakage, as it will have no free polarity.

At a fraction x of the total primary winding depth from the outside, the flux density is x times that between the coils, and a line there is linked only with x of the total turns in the coil. A strip there is thus only equivalent to one of x^2 the width between the coils so far as producing inductance is concerned, and the whole primary winding space to $\int_0^1 x^2 dx$, or $\frac{1}{3}$ its actual width.

A line at a fraction x of the inner winding space out from its inner circumference is not linked with the fraction x of the turns; and as the flux density here due to the primary current is uniform, the effectiveness of this space is $\int_0^2 x dx$, or $\frac{1}{2}$. This is easily seen otherwise; for, if the secondary turns are brought to their mean position in pairs from symmetrical positions, the reduction of the flux linked with one of a pair is compensated by the increase through the other.

The flux density between the coils is

$$\mathcal{B} = \mu_a \frac{N_1 I_1}{L} \quad . \quad . \quad . \quad . \quad 5.11;$$

where N_1 is the number of primary turns on a length L of the core, and μ_a is the permeability of the air.

$$\left. \begin{aligned} \mu_a &= 1.257 \frac{\text{lines per cm.}^2}{\text{amp.-turn per cm.}} \\ &= 1.257 \times 10^{-9} \frac{\text{volt-sec. per mm.}^2}{\text{amp.-turn per mm.}} = 1.257 \times 10^{-9} \frac{\text{henry}}{\text{mm.}} \\ &= 31.9 \times 10^{-9} \frac{\text{volt-sec. per in.}^2}{\text{amp.-turn per in.}} = 31.9 \times 10^{-9} \frac{\text{henry}}{\text{inch}} \end{aligned} \right\} \quad 5.12.$$

The leakage inductance of the primary coil is then

$$\begin{aligned} \mathcal{L}_{L_1} &= \frac{N_1}{I_1} \mathcal{B} S = \frac{N_1}{I_1} \mu_a \frac{N_1 I_1}{L} L_P \left(\frac{1}{3} l_1 + l_a + \frac{1}{2} l_2 \right) \\ &= \mu_a N_1^2 \frac{L_P}{L} \left(\frac{1}{3} l_1 + l_a + \frac{1}{2} l_2 \right) \quad . \quad . \quad . \quad . \quad 5.13; \end{aligned}$$

where L_P is the mean perimeter of the coils, supposed to be constant for all depths, l_1 and l_2 the winding depths, and l_a the distance between the windings.

The whole of the flux caused by a current in the inner coil is linked with all the turns of the outer coil, but part of it is only linked with some of those

of the inner coil itself. Consequently the mutual inductance is greater than the self-inductance of the inner coil, and the leakage is negative.

At a fraction x of the secondary winding depth out from its inner surface the flux density due to a current in it is $(1-x)$ times that inside, and it fails to link with x of the total turns, as that fraction lies inside it. Hence the mean effectiveness for leakage is

$$\int_0^1 (1-x)x dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6},$$

and the leakage inductance

$$\begin{aligned} \mathcal{L}_{L_2} &= -\frac{N_2}{I_2} \mathcal{B}S = -\frac{N_2}{I_2} \cdot \frac{\mu_a N_2 I_2}{L} \cdot L_P \frac{l_2}{6} \\ &= -\mu_a N_2^2 \frac{L_P}{L} \times \frac{1}{6} l_2 \quad . \quad . \quad . \quad 5.14. \end{aligned}$$

If we suppose the magnetising current to be negligible, the primary excitation will be equal and opposite to that of the secondary, and any E.M.F. may be transferred from one to the other by multiplying by the ratio of the number of turns. Consequently we can suppose the secondary to have no leakage inductance if we add $\frac{N_1^2}{N_2^2} \mathcal{L}_{L_2}$ to that of the primary. The total leakage inductance referred to either coil, the other having none, is

$$\begin{aligned} \mathcal{L}_{TL} &= \mu_a N^2 \frac{L_P}{L} \left\{ \left(\frac{1}{3} l_1 + l_a + \frac{1}{2} l_2 \right) - \frac{1}{6} l_2 \right\} \\ &= \mu_a N^2 \frac{L_P}{L} \left(\frac{1}{3} l_1 + l_a + \frac{1}{3} l_2 \right) \quad . \quad . \quad . \quad 5.15; \end{aligned}$$

where N is the number of turns in the coil to which it is referred.

Leakage Inductance of Long Thick Concentric Coils.—When the combined thickness of the coils is not small compared with their diameter the variations in the length of one turn will not be negligible.

Let

$$\begin{aligned} L_{OP1}, L_{OP2} &= \text{Outer perimeters of primary and secondary.} \\ L_{IP1}, L_{IP2} &= \text{Inner} \quad \quad \quad , \quad \quad \quad , \\ L_P &= \frac{1}{2}(L_{OP1} + L_{IP2}) \\ &= L_{OP1} - \pi(l_1 + l_a + l_2) \\ &= L_{IP2} + \pi(l_1 + l_a + l_2) \\ &= L_{OP2} + \pi(l_1 + l_a - l_2) \end{aligned}$$

Then at a fraction x of the primary winding depth in from its outer circumference the flux density due to a current in it is x of that within the coil, it

is linked with \mathbf{x} of the total turns, and the circumference is $(1 - \frac{2\pi l_1}{L_{OP1}} \mathbf{x})$ of the outer one. Hence the effective area of the primary winding space is $L_{OP1} l_1 \int_0^1 \mathbf{x}^2 \left(1 - \frac{2\pi l_1}{L_{OP1}} \mathbf{x}\right) d\mathbf{x}$, or $L_{OP1} l_1 \left(\frac{1}{3} - \frac{1}{4} \cdot \frac{2\pi l_1}{L_{OP1}}\right)$.

At a fraction \mathbf{x} of the space between the coils from its inside the circumference is $\left(1 + \frac{2\pi l_a}{L_{OP2}} \mathbf{x}\right)$ times its inner one. The effective area of this part is thus $L_{OP2} l_a \int_0^1 \left(1 + \frac{2\pi l_a}{L_{OP2}} \mathbf{x}\right) d\mathbf{x}$, or $L_{OP2} l_a \left(1 + \frac{1}{2} \cdot \frac{2\pi l_a}{L_{OP2}}\right)$.

At a fraction \mathbf{x} of the secondary winding depth out from its inner periphery the flux is not linked with \mathbf{x} of the total turns, and the perimeter is $\left(1 + \frac{2\pi l_2}{L_{IP2}} \mathbf{x}\right)$ of the inner circumference. The effective area is thus $L_{IP2} l_2 \int_0^1 \mathbf{x} \left(1 + \frac{2\pi l_2}{L_{IP2}} \mathbf{x}\right) d\mathbf{x}$, or $L_{IP2} l_2 \left(\frac{1}{2} + \frac{1}{3} \times \frac{2\pi l_2}{L_{IP2}}\right)$.

The primary leakage inductance is thus

$$\begin{aligned} \mathcal{L}_{L1} &= \frac{N_1^2}{I_1} \beta S \\ &= \frac{\mu_a N_1^2}{L} \left\{ \frac{1}{3} l_1 (L_{OP1} - \frac{3}{4} \times 2\pi l_1) + l_a (L_{OP2} + \frac{1}{2} \times 2\pi l_a) \right. \\ &\quad \left. + \frac{1}{2} l_2 (L_{IP2} + \frac{2}{3} \times 2\pi l_2) \right\} \quad . \quad . \quad . \quad 5.16. \end{aligned}$$

The perimeters here represented by the expressions in the brackets are those one-quarter way out from the inside of the outer coil, midway between the coils, and one-third in from the outside of the inner coil.

With a current in the inner coil only, the whole flux is linked with the other coil; but at a point in the winding space at a fraction \mathbf{x} out from the inside the flux density is $(1 - \mathbf{x})$ times that inside, and fails to link with \mathbf{x} of the total secondary turns. Also, the perimeter there is $\left(1 + \frac{2\pi l_2}{L_{IP2}} \mathbf{x}\right)$ of the innermost circumference. Hence the secondary leakage inductance is

$$\begin{aligned} \mathcal{L}_{L2} &= -\mu_a N_2^2 \frac{L_{IP2}}{L} l_2 \int_0^1 (1 - \mathbf{x}) \mathbf{x} \left(1 + \frac{2\pi l_2}{L_{IP2}} \mathbf{x}\right) d\mathbf{x} \\ &= -\mu_a N_2^2 \frac{L_{IP2}}{L} l_2 \left\{ \frac{1}{2} - \frac{1}{3} + \frac{2\pi l_2}{L_{IP2}} \left(\frac{1}{3} - \frac{1}{4}\right) \right\} \\ &= -\frac{\mu_a N_2^2}{L} \cdot \frac{1}{6} l_2 (L_{IP2} + \frac{1}{2} \times 2\pi l_2) \quad . \quad . \quad . \quad 5.17. \end{aligned}$$

transformer with concentric coils and rectangular core. Bunching the winding thus, introduces free polarity in the core to magnetise the iron outside the coils, diminishes the reluctance of the leakage paths by allowing the lines to spread out at the ends of the coils and some to return outside the windings, and it reduces the linkage by making part of the flux spring out before reaching the end of the coil. As the reluctance of the iron is always small, the effect of the first may be neglected. The diminution of reluctance increases the inductance, but the decrease of linkage lessens it. With fairly long coils we shall not be far wrong if we set the one against the other, and use the formula we have just deduced. A shell transformer which has all the coils on one limb is a further step in concentration of the winding, and our formula will apply a little less accurately to it, but still will not be far wrong. We may thus write for the leakage inductance E.M.F. of any transformer with concentric coils, when it is all transferred to one coil—

$$E_{TL} = 2\pi f \mathcal{L}_{TL} I$$

$$= 2\pi \mu_a f N^2 \frac{L_P}{L} \left(\frac{1}{3} l_1 + l_a + \frac{1}{3} l_2 \right) I \quad . \quad . \quad 5.20 ;$$

where f = Frequency,

N = Number of turns in winding in which inductance is supposed to be collected,

I = Current in same winding,

L_P = Mean of outermost and innermost perimeters,

L = Length of centre line of whole iron circuit,

l_1, l_2 = Winding depths of outer and inner coils,

l_a = Radial distance between windings,

$2\pi \mu_a = 8 \times 10^{-9}$ henries per mm.,

$= 0.2 \times 10^{-6}$ henries per in.

The E.M.F. in the same coil due to the useful flux ϕ is

$$E = 4ffN\phi$$

$$\therefore \frac{E_{TL}}{E} = \frac{2\pi \mu_a f N^2 \frac{L_P}{L} \left(\frac{1}{3} l_1 + l_a + \frac{1}{3} l_2 \right) I}{4ffN\phi}$$

$$= \frac{\pi \mu_a}{2f} \cdot \frac{NI}{\phi} \cdot \frac{L_P}{L} \left(\frac{1}{3} l_1 + l_a + \frac{1}{3} l_2 \right) . \quad . \quad 5.21.$$

For sine waves $f = 1.11$, and then

$$\frac{\pi \mu_a}{2f} = 1.8 \times 10^{-9} \frac{\text{volt-secs. per mm.}}{\text{amp.-turn}}$$

$$= 45 \times 10^{-9} \frac{\text{volt-secs. per in.}}{\text{amp.-turn}}.$$

The results of experiment agree fairly well with these formulæ.

It will be seen from this equation that the effect of the leakage can be kept within limits by using a large flux with small numbers of turns, by using small winding depths and long coils, and by keeping the space between them as small as the requirements of ventilation and insulation will permit. It should not exceed 3 or 4 per cent.

The frequency has cancelled out of the ratio $E_{TL} : E$. But it must be remembered that if the P.D. is kept constant, a rise of frequency involves a diminution of the flux and a consequent increase in the importance of the leakage.

Leakage Inductance of Divided Concentric Coils.—With the same assumptions as we first made, we can best calculate the leakage for

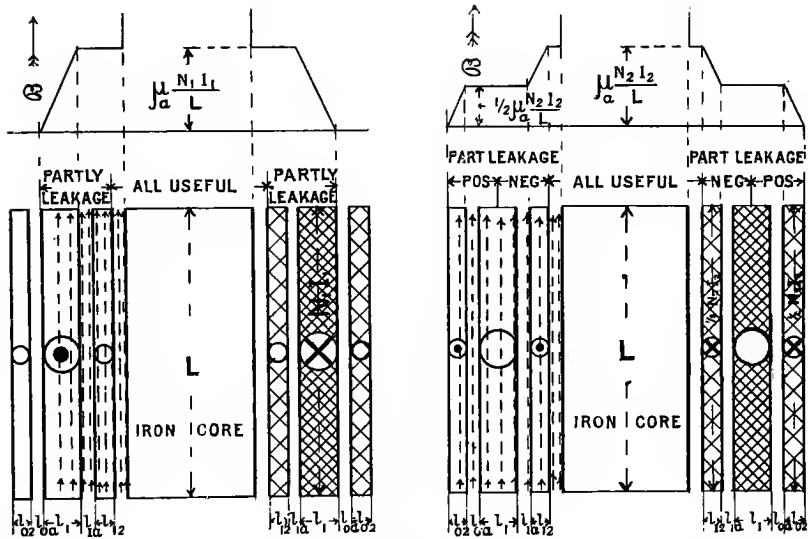


FIG. 5-07.—Flux due to Primary Coil only. FIG. 5-08.—Flux due to Secondary Coil only.

Leakage with Divided Concentric Coils.

divided concentric coils by reducing the various depths to equivalent ones inside. Let half the secondary turns be wound outside the primary, and half inside, and suppose first that the primary alone carries current. The flux density will be as represented in fig. 5-07.

So far as the primary itself is concerned, its winding space is equivalent to $\frac{1}{3}$ its depth inside, and the total effective width for self-induction outside the innermost circumference is $(\frac{1}{3}l_1 + l_{1a} + l_{12})$. But as far as mutual inductance with the secondary is concerned, this space is only equivalent to

$\frac{1}{2}\{(\frac{1}{2}l_1 + l_{Ia} + l_{I2}) + \frac{1}{2}l_{I2}\}$, since each coil has half the turns. The difference is $(\frac{1}{12}l_1 + \frac{1}{2}l_{Ia} + \frac{1}{4}l_{I2})$, and the primary leakage inductance

$$\mathcal{L}_{L1} = \frac{1}{2}\mu_a N_1^2 \frac{L_P}{L} (\frac{1}{6}l_1 + l_{Ia} + \frac{1}{2}l_{I2}) \quad . \quad . \quad . \quad 5.22.$$

With a current in the secondary only, the flux density varies as shown in fig. 5.08. The effectiveness of the outer winding space for self-induction is $(\frac{1}{2})^2$ or $\frac{1}{4}$ what it would be with all the turns in it. Hence it is $\frac{1}{4} \times \frac{1}{3}$ or $\frac{1}{12}$ that of an equal space inside all the coils. The space between the two secondaries is equivalent to $\frac{1}{4}$ as much inside where the flux density is twice as great and is linked with all instead of half the turns. At a fraction x within the innermost winding space the flux density is $\frac{1}{2}(1+x)$ of that inside, and is linked with $\frac{1}{2}(1+x)$ of the total secondary turns. Its effectiveness is thus

$$\frac{1}{4} \int_0^1 (1+x)^2 dx = \frac{1}{4}(1 + 1 + \frac{1}{3}) = \frac{7}{12}.$$

The effectiveness of the middle winding space for mutual inductance is $\frac{1}{4}$, since there is only half the inside density there. That of the innermost air space is $\frac{1}{2}$, and that of the innermost winding space $\frac{3}{4}$. Hence the net equivalent leakage space is

$$(\frac{1}{12}l_{o2} + \frac{1}{4}l_{oa} + \frac{1}{4}l_1 + \frac{1}{4}l_{Ia} + \frac{7}{12}l_{I2}) - (\frac{1}{4}l_1 + \frac{1}{2}l_{Ia} + \frac{3}{4}l_{I2}) = (\frac{1}{12}l_{o2} + \frac{1}{4}l_{oa} - \frac{1}{4}l_{Ia} - \frac{1}{6}l_{I2}).$$

It should be noted that the part to be subtracted as effective for mutual inductance is the same as before. This also follows from the fact that the mutual inductance of two coils is the same whichever is considered as the primary. The secondary leakage inductance is thus:—

$$\mathcal{L}_{L2} = \frac{1}{2}\mu_a N_2^2 \frac{L_P}{L} (\frac{1}{6}l_{o2} + \frac{1}{2}l_{oa} - \frac{1}{2}l_{Ia} - \frac{1}{3}l_{I2}) \quad . \quad . \quad . \quad 5.23.$$

This will be negative with the usual proportions.

The total leakage inductance of concentric coils with divided secondary is thus

$$\mathcal{L}_{TL} = \frac{1}{4}\mu_a N^2 \frac{L_P}{L} \{ \frac{1}{3}l_1 + (l_{oa} + l_{Ia}) + \frac{1}{3}(l_{o2} + l_{I2}) \} \quad . \quad . \quad . \quad 5.24,$$

and the corresponding E.M.F.

$$E_{TL} = 2\pi f \mathcal{L}_{TL} I$$

$$= \frac{\pi}{2}\mu_a f N^2 \frac{L_P}{L} \{ \frac{1}{3}l_1 + (l_{oa} + l_{Ia}) + \frac{1}{3}(l_{o2} + l_{I2}) \} \quad . \quad . \quad . \quad 5.25$$

and

$$\frac{E_{TL}}{E} = \frac{\pi\mu_a}{8f} \cdot \frac{NI}{\phi} \cdot \frac{L_P}{L} \{ \frac{1}{3}l_1 + (l_{oa} + l_{Ia}) + \frac{1}{3}(l_{o2} + l_{I2}) \} \quad . \quad . \quad . \quad 5.26 ;$$

where f = Frequency,

N = Number of turns in winding in which inductance is supposed to be collected,

I = Current in same winding,

L_P = Mean of outermost and innermost perimeters,

L = Length of centre line of whole iron circuit,

l_1, l_{o2}, l_{I2} = Winding depths of primary and outer and inner secondaries,

l_{oa}, l_{Ia} = Radial distance between primary and outer and inner secondaries,

ϕ = Useful flux,

$$\frac{\pi}{2}\mu_a = 2 \times 10^{-9} \text{ henries per mm.}$$

$$= 50 \times 10^{-9} \text{ henries per in.,}$$

$$\frac{\pi\mu_a}{8f} = 0.45 \times 10^{-9} \frac{\text{volt-secs. per mm.}}{\text{amp.-turn}}$$

$$= 11 \times 10^{-9} \frac{\text{volt-secs. per in.}}{\text{amp.-turn}}.$$

With the same total winding space as before, the leakage is reduced to one-fourth. It is halved by the reduction of the field in the leakage spaces, as already explained, and halved again by the reduction of the mean space between the windings.

It should be noted that the same result would have been obtained by considering half the primary to belong to each section of the secondary, and adding together the leakages of the pairs so obtained, remembering that each pair now has half the total turns.

Leakage Inductance of Plane Sheets.—If we have a uniformly distributed current flowing in parallel streams in an infinite plane sheet, the magnetic lines are all parallel to the sheet and perpendicular to the current. Since the current has to magnetise both sides of the sheet instead of one only, as in the cases already considered, the flux density is only half as great as before.

At any point outside it is $\mu_a \frac{NI}{2L}$, where NI is the total flow of current across a length L taken at right angles to the streams. The field has opposite directions on the two sides of the sheet, varies uniformly within it from one to the other, and is zero at the centre.

As the flux coming down on one side must be regarded as the return of that going up on the other, we must either integrate the flux over one side of the centre only when reckoning the linkage, or, in symmetrical cases, integrate over both sides (neglecting sign) and halve the result; or we may count the turns on one side only of the zero flux density as linked with the flux on that side. The effectiveness of the thickness of

the sheet is thus one-sixth of that of an equal space entirely on one side of the sheet.

If two such sheets be placed close together the effective leakage space for the first is thus $(\frac{1}{6}l_1 + l_a + \frac{1}{2}l_2)$, for the second $(\frac{1}{2}l_1 + l_a + \frac{1}{6}l_2)$, and for the two together $(\frac{2}{3}l_1 + 2l_a + \frac{2}{3}l_2)$. Consequently, the total leakage inductance for the two, for a length L_P parallel to the stream, is

$$\mathcal{L}_{TL} = \frac{N\beta S}{I} = \mu N^2 \frac{L_P}{L} (\frac{1}{3}l_1 + l_a + \frac{1}{3}l_2) . \quad . \quad . \quad 5.27.$$

This result is similar to that obtained for concentric coils, the fact that

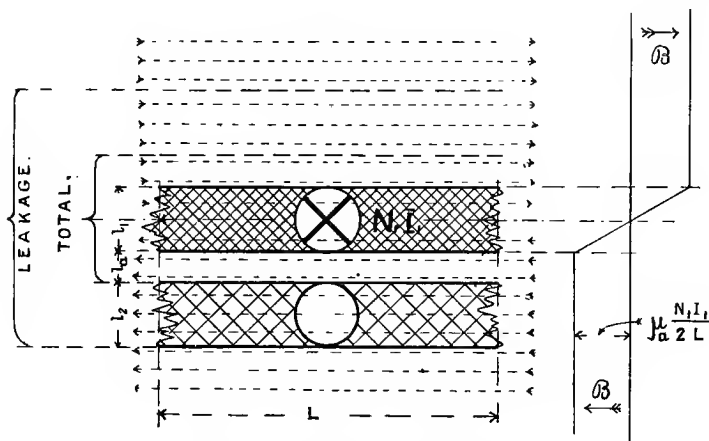


FIG. 5.09.—Leakage of Infinite Sheets.

both leakages are here positive having compensated for the reduced flux density.

Leakage of Subdivided Sandwiched Coils.—If the sheets in the last example be each divided into N_s equal sections placed alternately, with the outside one put half on one side and half on the other to make things symmetrical, the total leakage inductance is the sum of those of the $2N_s$ entirely separate systems formed by division at the centre lines of the sections. This result is not by any means self-evident, for all the sheets have mutual inductance with one another before being divided into entirely separate systems. But an examination of table 5.01, which has been compiled to show the relative effectiveness of the different parts in a case where each winding is divided into eight sections, will convince the student that the result is right, no matter how many sections there may be, so long as they are symmetrically placed. With an odd number of sections, the halved outermost one will not belong to the same winding as the central one. The table is constructed thus:—The first line shows the effectiveness for primary

self-inductance of the different places with a current in the central primary coil only, the second line with a current in the next pair of primary coils only, and so on. The second set shows the effectiveness for mutual inductance in the same way. The effectiveness for leakage is found by subtracting the total for mutual from the total for self inductance, and that of the secondary is worked out in a similar manner.

The effectiveness is taken as unity where the flux density is that due to one section only and is linked with half the turns of that section, this being the effectiveness just outside one section when it is removed from the others. The numbers at the top of the columns show the relative numbers of turns, that of a half section being the standard; the central one is written 1 + 1 instead of 2 because only one half is to be reckoned with the flux on each side. The outermost ones are marked 1 because they are only half sections. The right-hand column shows the relative flux densities outside the symmetrically placed current-carrying sections due to the current in those sections. It is zero between them.

The effectiveness of any place outside the current-carrying sections is found by multiplying this flux density by the total primary or secondary turns contained between the centre of the space considered and the middle of the central section. That of the current-carrying space for self induction is obtained by taking that flux density with one-third the turns in the section and half those of the same winding between it and the centre. For mutual induction take half the turns of the other winding between the section considered and the centre. The effectiveness of the space between the two current-carrying sections is zero.

The leakage inductance due to an area **S** at the standard effectiveness is

$$\mathcal{L}_{TL} = \frac{\beta S}{l} \frac{N}{2N_s} = \mu_a \frac{N^2}{4N_s^2} \cdot \frac{S}{L} \quad . \quad . \quad . \quad 5.28.$$

Hence, taking the effectiveness of the different spaces from the table, we have

$$\mathcal{L}_{TL} = \mu_a \frac{N^2}{4N_s^2} \frac{L_P}{L} \left\{ \frac{1}{3} \Sigma l_1 + \Sigma l_a + \frac{1}{3} \Sigma l_2 \right\} \quad . \quad . \quad . \quad 5.29;$$

where

$\Sigma l_1, \Sigma l_2$ = Total thickness parallel to axis of primary and secondary windings,

Σl_a = Sum of all spaces between the windings,

L = Depth dealt with perpendicular to current,

L_P = Length dealt with parallel to current,

N_s = Number of equal sections into which each winding is divided,

N = Number of wires across the depth **L** in that winding in which all the leakage inductance is supposed to be collected.

Although the windings of actual subdivided transformers can hardly be called infinite sheets, this result can still be applied without very great error. L is then the radial depth of the coils, and L_P their mean perimeter.

When subdivided coils are employed, it is always advisable to use the symmetrical arrangement of fig. 5.10, as it prevents extensive leakage around the extreme coils. When the coils are all of the same thickness, the leakage of the sections near the yokes is approximately twice as large as in the

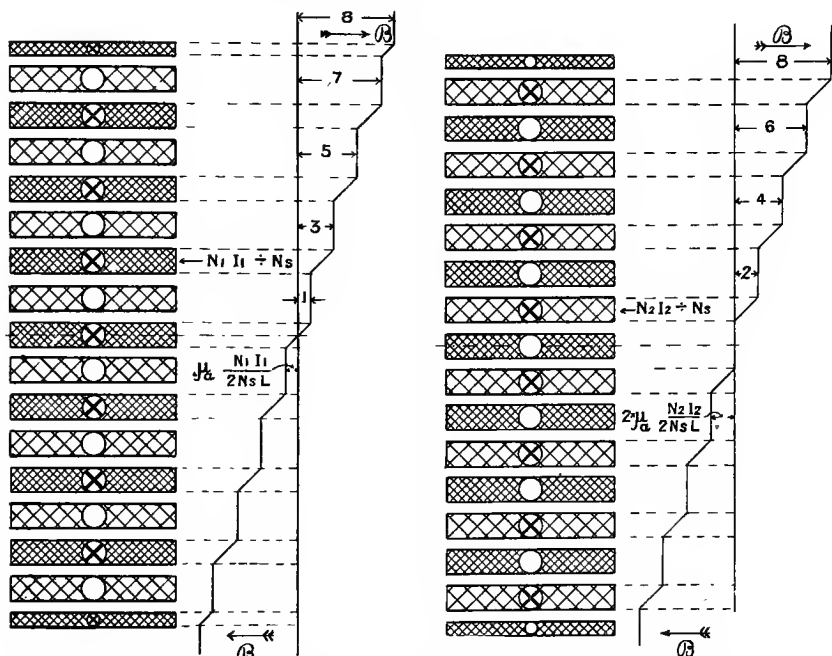


FIG. 5.10.—Flux due to Primary Coil only. FIG. 5.11.—Flux due to Secondary Coil only.

Leakage of Subdivided Sheets.

symmetrical arrangement. If the conductors of a coil are partly joined in parallel, the symmetrical type is almost essential, as otherwise internal currents are set up on account of the uneven flux distribution.

Concentric coils are more popular than subdivided ones, since a better insulation may be obtained with less insulating material, and a low reactance E.M.F. is usually more easily procurable with concentric than with subdivided sandwiched coils. In order to reduce the P.D. per coil, each concentric high-voltage section is often divided into a number of parts, but this does not influence the reactance E.M.F. to any appreciable extent.

CHAPTER VI.

TRANSFORMER VECTOR DIAGRAMS.

Vector Diagrams.—In the previous chapter we represented the various E.M.F.'s whose resultant gives the primary or secondary P.D. by formulæ which, besides being complicated, do not show at a glance the phase relations between the various members. Some fifteen years ago Kapp showed how

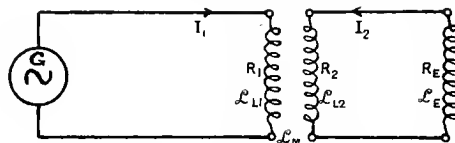


FIG. 6·01.—Actual Connections of Transformer.

these equations could be solved by simple geometrical constructions in vector diagrams.

In order to simplify matters, it is convenient to reduce all the secondary quantities to equivalent ones in the primary circuit, or *vice versa*. If we

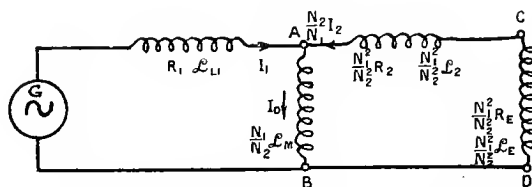


FIG. 6·02.—Equivalent Connections of Transformer.

then substitute for the given transformer one whose ratio of transformation is unity, the two circuits may be joined together. Thus, the actual connections of fig. 6·01 are replaced by the equivalent ones of fig. 6·02, where \mathcal{L}_{L_1} , \mathcal{L}_{L_2} represent the leakage inductances of the primary and secondary windings, and R_1 , R_2 their resistances. \mathcal{L}_E and R_E are those of the external secondary circuit C D. The shunt circuit A B takes the magnetising current,

for which the mutual inductance, \mathcal{L}_M , is uncanceled, and has a resistance equal to $\frac{V_1^2}{P_I}$, where P_I is the power lost in the iron.

Neglecting the magnetising current, which is small, the equivalent circuit has a resistance and inductance

$$R = R_1 + \frac{N_1^2}{N_2^2}(R_2 + R_E) \quad . \quad . \quad . \quad 6.01,$$

$$\mathcal{L} = \mathcal{L}_{TL} + \frac{N_1^2}{N_2^2}\mathcal{L}_E \quad . \quad . \quad . \quad 6.02.$$

This transformation makes no essential difference to the action of the transformer, for the same power is absorbed, transmitted, and lost as in the actual case.

In the diagrams which follow we shall therefore assume a unit ratio of transformation. The diagram will not be altered if this is not true, but the scales will then be different for primary and secondary quantities. The vectors represent R.M.S. values to one scale and maximum values to another scale, and are supposed to rotate clockwise.

Vector Diagram for Unloaded Transformer.

—Suppose that the no-load current I_o , and its working and idle components I_w , I_b , have been calculated by the methods already explained in equations 2.17–2.19. Set off a line, say horizontally and to the right, to represent I_o , and with the aid of a semicircle on it as diameter draw in I_w and I_b in their proper phase relations as shown, I_w ahead of and I_b behind I_o . The useful flux φ is in phase with I_b , and the leakage flux with I_o , since the latter flux, being practically all in air, causes no hysteresis and eddy losses. E_1 is the E.M.F. induced by the useful flux, and E_{L1} by the leakage flux, each lagging 90° behind the corresponding flux. E_{R1} is the resistance E.M.F. due to the primary resistance, and opposes that current,

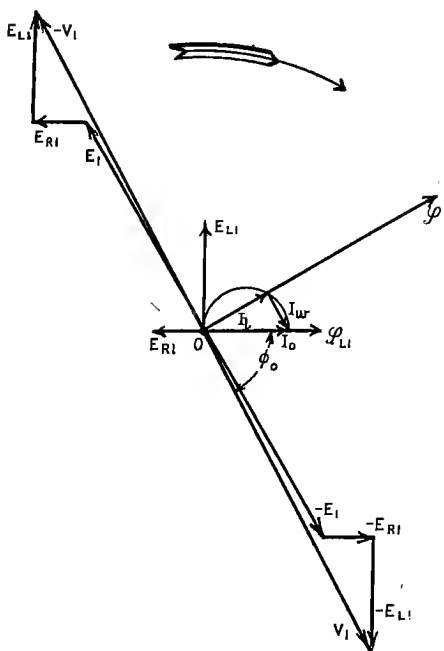


FIG. 6.03.—Clock Vector Diagram for Unloaded Transformer.

The P.D. between two points is the sum of all the E.M.F.'s in any one path by which we can go from one to the other in the direction of the current, the signs being reversed if the particular branch chosen is supposed to be receiving energy with the assumed positive current. Thus, as we are supposing the primary to take energy from the mains, the resultant of the three E.M.F.'s just mentioned is $-V_1$, or the resultant reversed is V_1 . We may look upon $-E_{R1}$, $-E_L$, and $-E_{L1}$ as the E.M.F.'s which must be given by the mains to overcome the resistance, and useful and leakage inductance respectively.

Vector Diagrams for Transformer on Non-Reactive Load.—

When the transformer is loaded, begin by setting off I_2 , say to the left. It

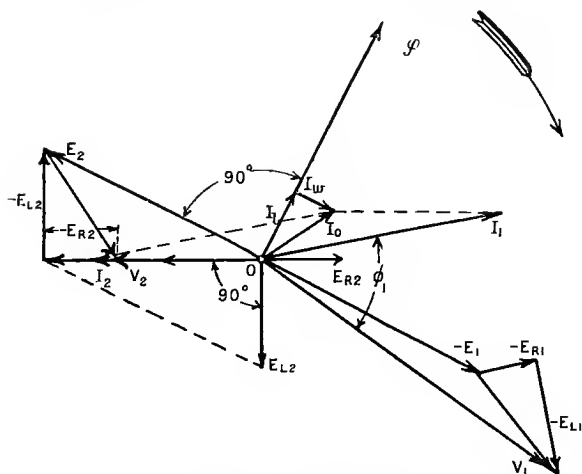


FIG. 6'04.—Clock Diagram.

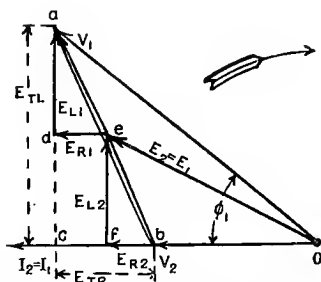


FIG. 6'05.—Transformed Diagram.

Vector Diagrams for Transformer on Non-Reactive Load.

produces a leakage flux Φ_{L2} in phase with it, and a consequent leakage inductance E.M.F. 90° behind it. The resistance E.M.F. E_{R2} is exactly opposite to, and the P.D. V_2 in phase with, the secondary current, since the load has unit power-factor. E_2 , the E.M.F. induced by the useful flux, is the resultant of V_2 , $-E_{R2}$, and $-E_{L2}$. We can therefore draw it, and Φ 90° ahead of it. The magnetising current I_0 is assumed constant at its no-load value, and can be filled in by making I_1 in phase with Φ , and I_w 90° ahead of it. It is also the resultant of the primary and secondary currents, and so I_1 can be obtained by completing the parallelogram of which I_0 is the diagonal and I_2 one side. The primary resistance and leakage inductance E.M.F.'s corresponding to this current can be calculated and plotted as before to get the primary P.D. V_1 .

It will be seen that this construction cannot be started from the primary circuit. If V_1 be given, it will be necessary to alter the scale of the diagram to suit. If the external circuit has a given resistance or its equivalent the problem is then solved, but if it is to take a given current, fresh diagrams must be drawn until one is found by trial for which that is also right. The magnetising current has been greatly exaggerated in the diagrams in order to show its effect, but in practice we may, without serious error, neglect it altogether at full load. The primary current will then be equal and opposite to the secondary current.

If, in addition to making this simplification, we also reverse the vectors for the primary circuit, we transform fig. 6.04 into fig. 6.05, which is more convenient to use. Ob is the secondary terminal P.D., bf , fe the E.M.F.'s used in overcoming the resistance and leakage inductance of the secondary winding, and ed , da those in the primary; eb and ae are their respective impedance E.M.F.'s. The total internal leakage inductance, resistance, and impedance E.M.F.'s are ac , cb , and ab respectively. Oe is the E.M.F. induced in both windings by the useful flux, and Oa the primary P.D. Since the corresponding E.M.F.'s are proportional to the current, Ob , bc , ca are proportional to the equivalent resistance of the load and the total internal resistance and leakage reactance of the transformer respectively. This diagram can consequently be drawn commencing with V_1 when these are known. That is, because the magnetising current has been neglected.

Vector Diagrams for Transformer on Reactive Load.—When the load is reactive the secondary current is no longer in phase with the P.D., but lags behind it by an angle whose cosine is the power factor of the load when the reactance is positive (inductive load), or leads by that amount when it is negative (capacity load). V_2 must consequently be set that much ahead of or behind I_2 , as the case may be. The secondary resistance and leakage inductance E.M.F.'s are then added, and the rest of the diagram constructed exactly as before.

It will be noticed that the triangle abc in the transformed diagram does not alter its shape with a change of the load, and, since its sides are equal to the current multiplied by the resistance, leakage reactance, and impedance of the transformer, which we are assuming to be constant, its size does not vary with the power factor of the load if the current is kept constant. It is consequently termed the characteristic triangle of the transformer.

Whatever may be the nature of the load, it may be replaced by one having resistance combined with either inductance or capacity only, which

will take the same current in the same phase. Ok, kb represent the resistance and reactance of this equivalent load to the same scale as bc, ca repre-

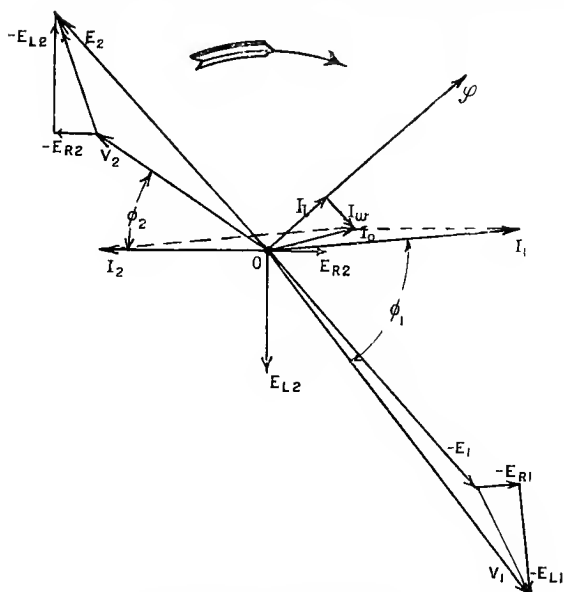


FIG. 6.06.—Clock Diagram

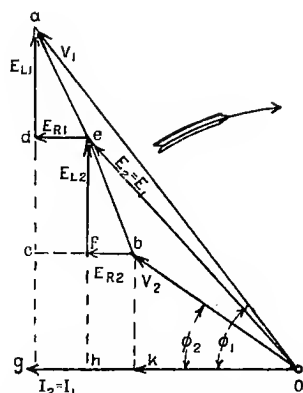


FIG. 6.07.—Transformed Diagram.

Vector Diagrams for Transformer on Load with Positive Reactance (Inductive Load).

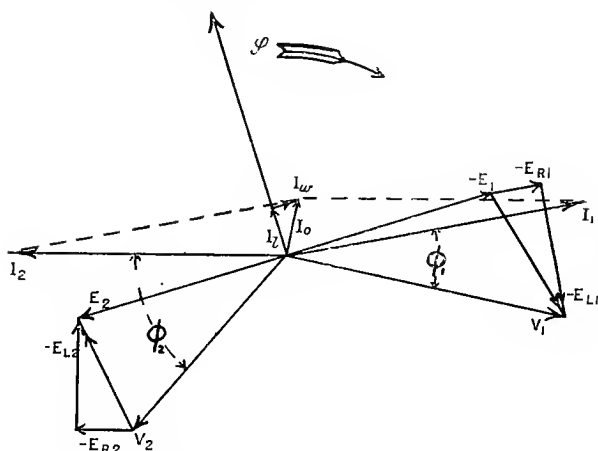


FIG. 6.08.—Clock Diagram.

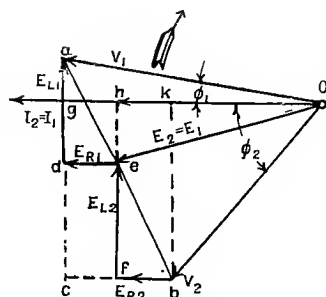


FIG. 6.09.—Transformed Diagram.

Vector Diagrams for Transformer on Load with Negative Reactance (Capacity Load).

sent those of the transformer when reduced to the secondary circuit. The diagram can be drawn when these are known.

Vector Diagrams for Short-Circuited Transformer. — If the secondary of the transformer be short-circuited by a short thick copper bar, V_2 will be zero, and the transformed diagram will consist of the characteristic triangle only. A short-circuit test thus enables this triangle to be determined. In carrying out the test the primary P.D. is gradually raised until the transformer takes the normal full load current, when its value is noted. The primary P.D. necessary to give this current is very much less than the normal, and consequently the iron loss is negligible in the short-circuit test. A low-reading wattmeter connected in the primary circuit, after correction for the losses in any of the primary instruments which it may include, will

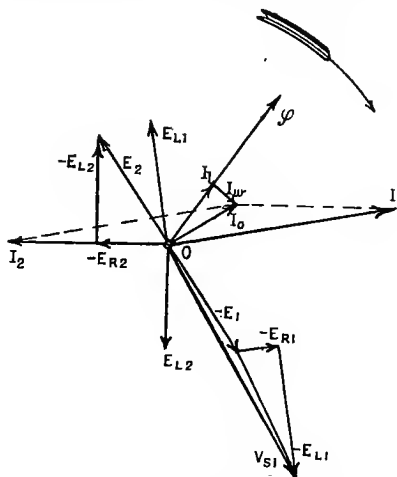


FIG. 6-10.—Clock Diagram.

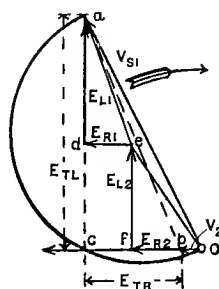


FIG. 6-11.—Transformed Diagram.

Vector Diagrams for Short-Circuited Transformer.

thus give the resistance loss in the transformer. The total internal resistance E.M.F. is obtained by dividing this power by the primary current, and the equivalent resistance reduced to the primary circuit by dividing it by the square of this current. It is best to determine the resistance of a transformer in this way, as its effective value is always greater than that measured with steady currents in the ordinary way, owing to the uneven distribution of alternating currents across the conductors. The test should, of course, be carried out when the transformer is as hot as it will be in use.

Set off Oa to represent the observed primary P.D. to any convenient scale; on it draw a semicircle, and insert a chord Oc to represent, to the same scale, the resistance E.M.F. just found. Oac is the characteristic triangle.

If an ammeter be put in the secondary circuit, and its impedance be not

sloping diameter a_0b_0 may be drawn by constructing the characteristic triangle with its resistance side horizontal as in figs. 6.12 and 6.13. It makes an angle $\tan^{-1} \frac{\omega L_{TL}}{R_T}$ with the horizontal. The diameter ab is then drawn, making the required phase angle with a_0b_0 . The horizontal distance of a or b from the vertical line gives the rise of primary or drop of secondary P.D. required, to the same scale as the radius of the circle represents the total internal impedance E.M.F. when all transferred to the winding considered.

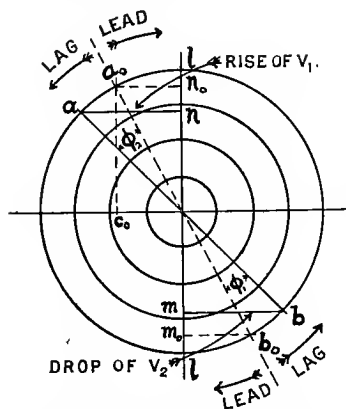


FIG. 6.14.—Bragstad's Circle Diagram for Transformer Regulation.

The radius of the circle is proportional to the current. For other currents than the one assumed in constructing the diagram, we must either alter our scale of E.M.F. to suit, or make fresh circles for each current. The latter has been done in fig. 6.14, in which circles are drawn for $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and full load. When the inclination of the P.D. vectors can be neglected the voltage change is proportional to the current, and the intercepts may be taken as representing an impedance by which

the current must be multiplied to get it.

It will be observed that the greatest rise or fall of P.D. occurs when the vectors V_1 , V_2 coincide, and that it is then equal to the internal impedance E.M.F., which, as we have seen, is the same as the primary P.D. required to give the load current when the secondary is short-circuited.

Kapp's Circle Diagram.—In fig. 6.15 $OacbO$ is part of fig. 6.07 turned round so as to make the internal impedance line ab horizontal. If the power-factor of the load is altered, but not the current or primary P.D., a will move in a circle whose centre is at O . If $O'b'$ be made equal and parallel to ab , $b'b$ is equal and parallel to Oa . Consequently, b moves in a circle about b' as centre. The intercept bm gives the secondary voltage drop when the load is switched on.

Hence, to find the voltage drop for any load, draw the characteristic triangle $a'b'c'$, and with a' and b' as centres describe semicircles whose radii represent the primary P.D. to the same scale as the long side represents the internal impedance E.M.F. with the given current. Draw the current vector parallel to the resistance side $b'c'$ of the triangle, and the secondary P.D. making an angle ϕ_2 with it. The intercept on the secondary P.D.

vector between the circles gives the drop required. The intersection of the two circles gives the position corresponding to no drop in the secondary P.D.

This diagram is the best to use for poor or very small transformers in which the regulation is bad. But for ordinary ones difficulties arise owing to the closeness of the two circles and the consequent large size required. Bragstad's method is then to be preferred.

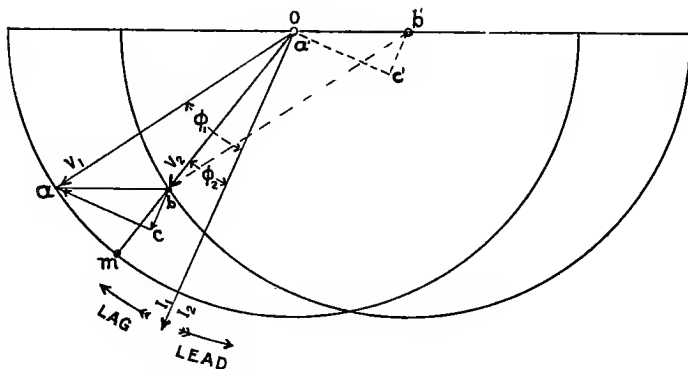


FIG. 6·15.—Kapp's Circle Diagram for Voltage Drop.

Polyphase Connections.—The windings of polyphase apparatus may be joined to form a star radiating from a point to the different mains, or to form a mesh joining main to main. With star connections the line currents are the same as in the corresponding windings, and the P.D. between two lines is the vectorial difference of the E.M.F.'s in the two rays connected to them. With mesh connections, the line P.D. is that of one side of the mesh, and the line current the vectorial difference of those in the two sides of the mesh connected to it. Star connections for high-voltage windings have the advantage that the coils consist of a smaller number of turns of larger wire; that the centre of the star gives a symmetrical point which can be earthed, and to which an extra conductor can be connected in order to give a lower voltage for lighting loads; and that part of the winding is more nearly at earth potential than with a mesh. On the other hand, a supply can be kept up to all the mains through the remaining phases of the mesh should one break down, whereas the breakdown of one ray of the star would throw one main out entirely and seriously disturb the potentials of the others. This advantage of the mesh, however, can only be utilised where all the phases have separate transformers separately controlled, or four-limb three-phase transformers with separate switches for each, which is

not the usual practice. Again, we shall see that star-connected primaries are quite unsuitable for use along with star-connected secondaries, unless there is a fourth conductor connecting the neutral points of the primary system.

Neglecting the voltage drop, the phases of all the E.M.F.'s in the coils on one limb will be the same, or opposite, according to which end is taken as the positive one, since they are all caused by the same flux. Hence it follows that when the primary and secondary are both connected in star, or both connected in mesh, the primary and secondary line P.D.'s are in the same (or opposite) phases. But when one is mesh- and the other star-connected, the line P.D. of the former is in phase with the winding E.M.F., while that of the latter is not (*e.g.*, see figs. 6·30 and 6·31). Consequently, the primary and secondary line P.D.'s differ in phase by an amount depending on the number of phases. When both the primary and secondary are connected in the same way, the connections are known as "pure"; when one is star and the other mesh, they are "mixed." Where the transformers have to run in parallel on both the primary and secondary sides, all must have pure connections, or all have mixed. If some were mixed and some pure, the one lot would form a partial short-circuit on the other.

Three-Phase Star Connections.—In the three-phase system with star connections (see figs. 6·16–6·18) the line currents are the same as those in the windings, and the line P.D.'s, being the vectorial difference between the E.M.F.'s in two windings, $\sqrt{3}$ times one of the latter and $\frac{1}{\sqrt{2}}$ period out of phase with it. When everything is symmetrical, the system is "balanced," and the potential of the centre of the star, which is termed the neutral point, is the centre of the potentials of the mains. The neutral point is often earthed, to limit the normal P.D. between any main and earth to that of one ray of the star, the earth connection being made through a resistance short-circuited by a fuse in order to prevent an interruption of the supply being caused by too large a current flowing through any fault outside. The potentials to earth will, of course, be abnormal while the fault remains on.

In the vector diagram fig. 6·17 the secondary currents are first set out at 120° , and then the secondary P.D.'s, which are equal to the currents multiplied by the impedance of one phase of the load, the angle ϕ_2 being the one whose cosine is the power-factor of the load. We then add the internal impedance E.M.F.'s, ZI , and thus get the P.D. of each ray of the primary. The angle that ZI makes with I is given by the characteristic triangle for the transformer. To simplify matters, the exciting current is neglected throughout, and all the secondary vectors are shown heavy. The line P.D.'s

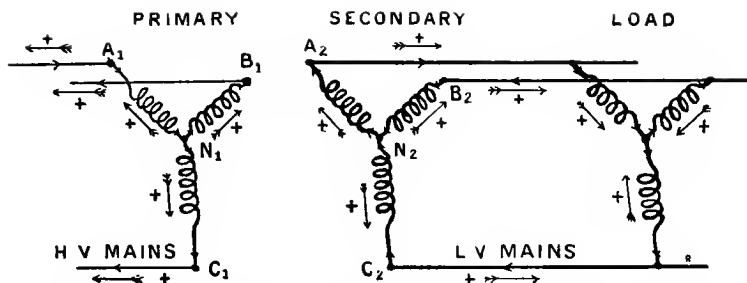


FIG. 6.16.—Connections. (The arrows outside show the positive way ; those on the windings, that of the current at one particular instant.)

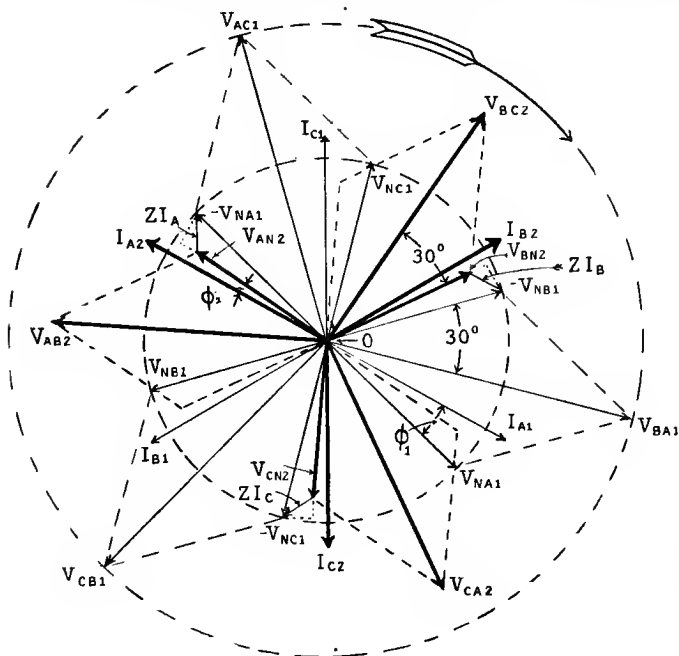


FIG. 6.17.—
Clock Vector
Diagram.

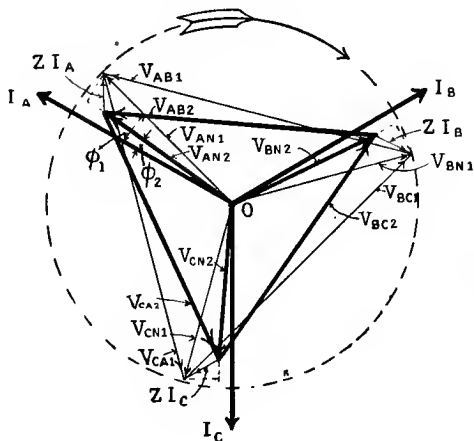


FIG. 6·18.—Transformed Vector Diagram.

Three-Phase Star Connections, Symmetrically Loaded.

are then obtained from the first of the vectorial equations for this case, which apply to either the primary or secondary :—

$$\left. \begin{aligned} V_{AB} &= V_{AN} - V_{BN} \\ V_{BC} &= V_{BN} - V_{CN} \\ V_{CA} &= V_{CN} - V_{AN} \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad 6\cdot03.$$

$$V_{AB} + V_{BC} + V_{CA} = 0 \quad . \quad . \quad . \quad . \quad 6\cdot04.$$

$$I_A + I_B + I_C = 0 \quad . \quad . \quad . \quad . \quad 6\cdot05.$$

Circles have been drawn through the extremities of the primary P.D. vectors ; the radial distance of the ends of the corresponding secondary P.D. vectors gives the voltage drop.

In fig. 6·18 the primary quantities are reversed, and the diagram is compressed by joining the points of the ray P.D. vectors to get the line P.D.'s. A comparison of the two diagrams will show that this gives the right result.

If the primary P.D. be given, all the vectors (current as well as P.D.) must be altered in proportion to make it right. In other words, we draw the diagram first, assuming any current, and choose the scales afterwards so as to give the desired primary P.D. It is, however, generally sufficiently accurate to estimate the magnitude and phase of the load currents from the unchanged P.D., for in practice the voltage drop is only a small fraction of the total P.D.

When the loading is unsymmetrical, the system is “out of balance,” and the potentials of the mains are no longer equidistant from the neutral point. So long as there is not a fourth conductor connected to the secondary neutral point, the above equations will still apply, and whatever the load may be, it can be replaced by others taking exactly the same currents from the mains at every instant and consisting of a balanced star together with a single-phase load across two mains. The one main takes the star current, another takes the sum, and the third the difference of the star current and the single-phase one. The voltage drop due to the balanced load may be obtained as before, and can be added to that due to the single-phase load to get the total.

Fig. 6·19 shows a load across one pair of mains only. Obviously, at every instant, the same current flows in N_2A_2 and B_2N_2 , but in the positive way through one and in the negative way through the other. Consequently, $I_{A_2} = -I_{B_2}$. The vector diagrams figs 6·20 and 6·21 are constructed in the same way as before, except that now there is no voltage drop in one ray of the star, and that the phases of the impedance E.M.F.'s in the others are opposite instead of differing by $\frac{1}{2}$ period. It will be noticed that the P.D.'s

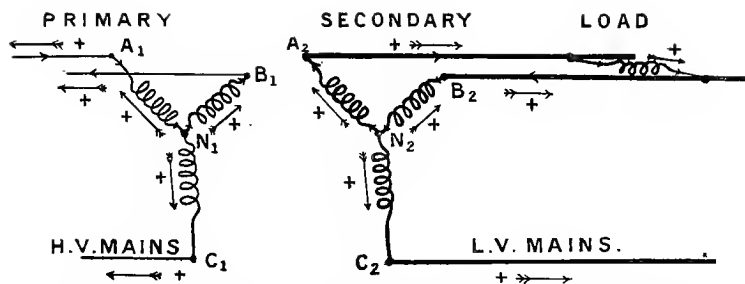


FIG. 6.19.—Connections. (The arrows alongside show the positive way ; those on the windings, that of the current at one particular instant.)

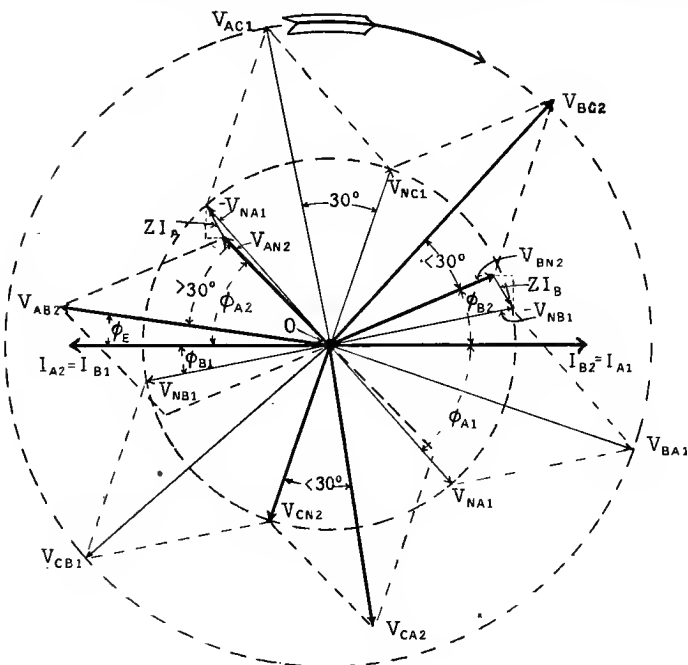


FIG. 6.20.—Clock Vector Diagram.

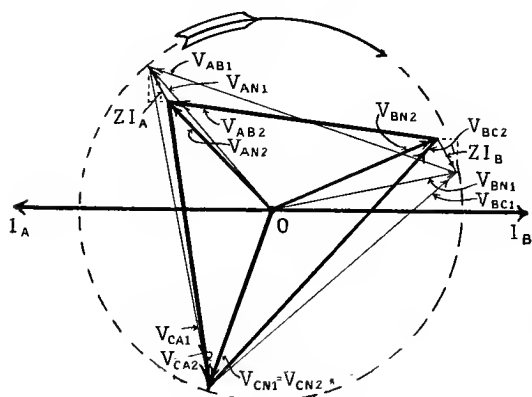


FIG. 6.21.—Transformed Vector Diagram.

Three-Phase Star Connections with Load across One Pair of Mains.

between two pairs of mains are reduced, while that of the other is raised. ϕ_E is the angle of lag for the load; ϕ_{A_2} and ϕ_{B_2} for the secondary rays; and ϕ_{A_1} and ϕ_{B_1} for the primary. Those for the B ray are both negative, since the current in these leads the P.D. The secondary P.D.'s are neither equal nor make equal angles with one another.

If a fourth conductor is added between the neutral points of the secondary and the load, and also between those of the primary and generator, a third load, taking the vectorial sum of the three line currents, must be added to the representative load between one main and the neutral point. It is, however, simpler to replace the given load by separate loads between each main and the neutral point, and to treat each ray by itself as a single-phase transformer independent of the others.

The double-star connection must not be employed if there is no fourth wire in the primary system. For then, the primary current required to

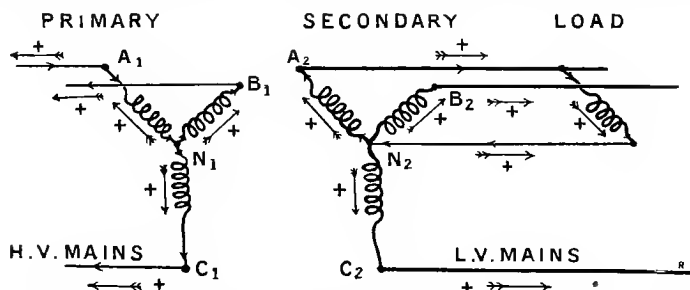


FIG. 6·22. —Three-Phase Double Star without Primary Neutral Wire and Loaded on One Secondary Ray only.

supply a load on one ray only of the secondary (see fig. 6·22) must also divide between the other rays of the primary; since these have no opposing secondary excitation, they will produce an abnormal flux in their limbs which will cause them to take practically the full line voltage. The load would thus only get a fraction of its proper voltage, sufficient only to pass enough current to excite the other limbs to about the full line voltage. A comparatively small current in the neutral wire thus throws the secondary P.D.'s entirely out of balance. Even if there be no neutral wire on the secondary, this condition is liable to be produced by faulty insulation of the distributing mains or load if, as is usual, the neutral point of the secondary be earthed.

In the above, and also in what follows, it is assumed that fluxes are quite independent, so that a current in the coils of one phase will not affect the E.M.F.'s in the others. With separate single-phase transformers, and with four-limb three-phase transformers, this is true, but with three-limb three-

phase transformers it is not. In the extreme limit of the last case, all the flux of one limb must return by the other two, and the sum of the three is zero. Sufficient current will be taken by the primary of the unloaded phase to make the fluxes satisfy this relation, with the result that the load is more evenly divided between the mains on the primary than on the secondary side, and the voltage drop is reduced to some extent thereby. The three-limb transformer thus helps to balance the load between the mains, but in actual practice, owing to the fact that part of the flux of one limb may return by the surrounding air, this effect is small.

Three-Phase Mesh Connections. — With a symmetrically loaded mesh connection (see figs. 6·23–6·25), the line P.D. is the same as that of one phase, but the line current, being the vectorial difference between those in the windings, is $\sqrt{3}$ times one of these, and differs in phase by $\frac{1}{12}$ period from it. The division of the current between the windings is determined by the condition that the sum of the line P.D.'s must remain zero when the load is put on. Consequently, the sum of the three impedance E.M.F.'s must be zero, and therefore, if the impedances are alike for all three phases, the sum of the three mesh currents must be zero. The equations for this case are :—

$$V_{AB} + V_{BC} + V_{CA} = 0 \quad . \quad . \quad . \quad . \quad 6\cdot06,$$

$$I_A + I_B + I_C = 0 \quad . \quad . \quad . \quad . \quad 6\cdot07,$$

$$I_{AB} + I_{BC} + I_{CA} = 0 \quad . \quad . \quad . \quad . \quad 6\cdot08,$$

$$\left. \begin{aligned} I_A &= I_{CA} - I_{AB} \\ I_B &= I_{AB} - I_{BC} \\ I_C &= I_{BC} - I_{CA} \end{aligned} \right\} . \quad . \quad . \quad . \quad 6\cdot09.$$

Whether the load be star- or mesh-connected, or both, it may be replaced by a balanced mesh together with a single-phase load across one pair of mains, which together take exactly the same currents from the mains at every instant: one line takes the balanced current, another the sum, and the third the difference of the balanced and single-phase currents. The voltage drops can be determined for each separately, and then added together vectorially.

In fig. 6·24, which applies to the balanced load, the line currents I_{A_2} , I_{B_2} , I_{C_2} are first set out at 120° to one another; each is then divided into components at 30° to satisfy equations 6·09. Then the P.D.'s V_{AC_2} , V_{BA_2} , V_{CB_2} can be put in, ϕ_2 being the angle of lag for the load across one pair of mains; the P.D.'s are equal to the assumed line currents multiplied by the load impedance. The internal impedance E.M.F.'s are then added, and the primary P.D.'s obtained as before. It is to be noted that the load power-factor is determined by the mesh current and the line P.D., and not by the line current. If the load were star-connected and took exactly the same

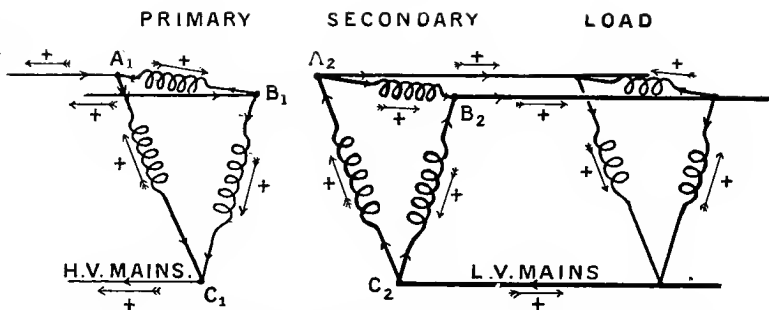


FIG. 6·23.—Connections. (The arrows alongside show the positive way; those on the windings, that of the current at one particular instant.)

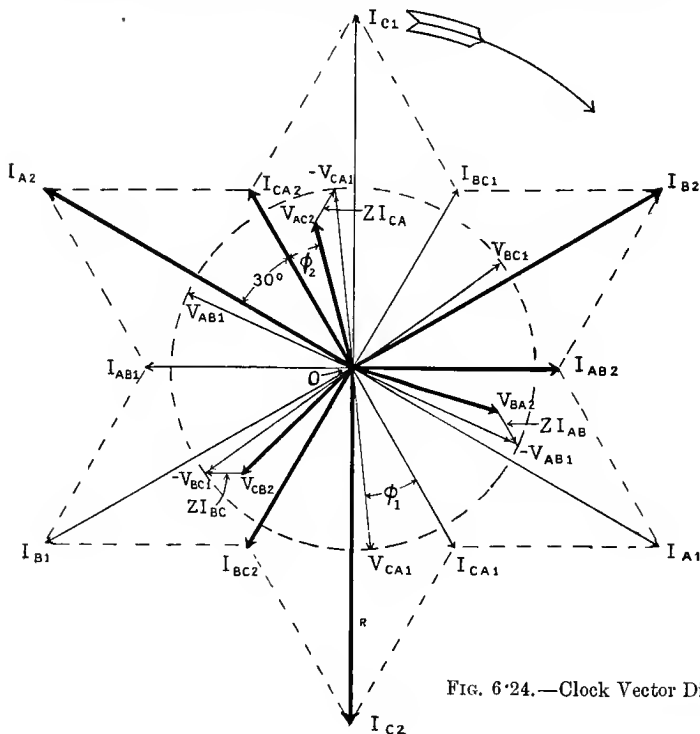


FIG. 6·24.—Clock Vector Diagram.

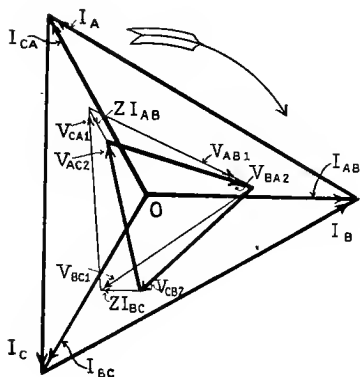


FIG. 6·25.—Transformed Vector Diagram.

Three-Phase Mesh Connections with
Balanced Load.

currents from the lines at every instant, exactly the same power-factor would be given by the ray current with the ray P.D.

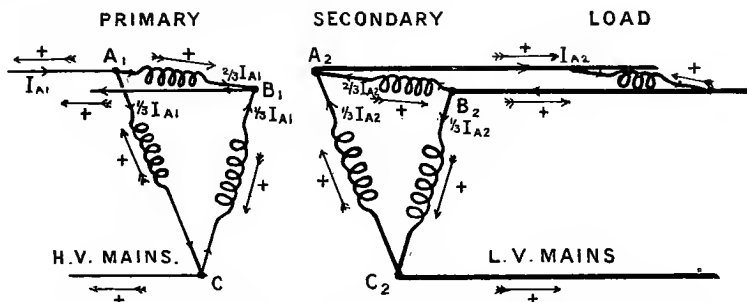


FIG. 6.26.—Connections. (The arrows alongside show the positive way; those on the windings, that of the current at one particular instant.)

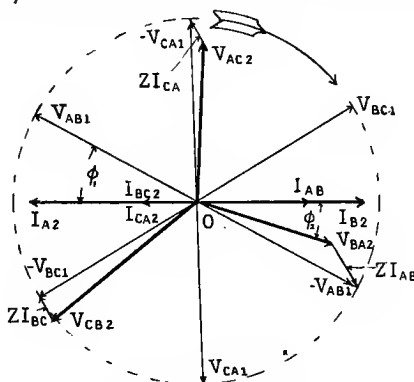


FIG. 6.27.—Clock Vector Diagram.

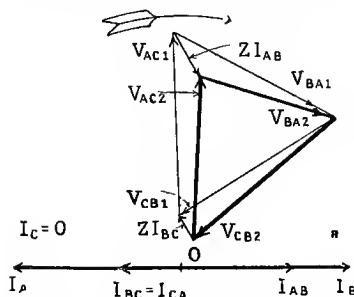


FIG. 6.28.—Transformed Vector Diagram.

Three-Phase Mesh Connections with Load across One Pair of Mains only.

In fig. 6.25 the three line currents form the three sides of a triangle; the *mesh* currents are the three lines drawn from the centre of the triangle to the points. It will be seen that this satisfies equations 6.09. The triangle

of primary P.D.'s is then drawn, but not necessarily superimposed on the current vectors. Lastly, the internal impedance E.M.F.'s of two phases are subtracted to get the secondary P.D.'s.

Fig. 6·26 shows a load across one pair of mains only. The load current divides between the loaded side of the mesh and the other two in series, being positive in the former and negative in both parts of the latter, or *vice versa*, so as to satisfy equations 6·09. Consequently, the loaded side of the mesh takes $\frac{2}{3}$ and the others $\frac{1}{3}$ of the load current, since the currents in both the latter are necessarily equal to one another, and opposite in phase to that in the former. After setting off these currents, V_{BA_2} can be drawn making with I_{B_2} an angle, ϕ_2 , whose cosine is the power-factor for the load. Then add the internal impedance E.M.F. corresponding to $\frac{2}{3}$ of the load current to get V_{AB_1} . The other primary P.D.'s are equal to this, and at 120° to it. By subtracting the internal impedance E.M.F.'s for the remaining third of the current from them, the other secondary P.D.'s are obtained. They are all different, one being rather greater than the normal. When compared with fig. 6·27, the construction of the transformed diagram is fairly obvious.

Three-Phase Mixed Connections.—The arrangement having a mesh-connected primary and a star-connected secondary (fig. 6·29) is a favourite one, because it gives the advantage of allowing the use of a neutral wire in the secondary system without the necessity of also having one in the primary which is present with pure-star connections. In this case, the primary quantities satisfy the relations of equations 6·06–6·09, and the secondary ones those of equations 6·03–6·05. Figs. 6·30 and 6·31 are the vector diagrams for this arrangement when symmetrically loaded. The secondary quantities are worked out in exactly the same way as in figs. 6·17 and 6·18, while the primary ones are as in figs. 6·24 and 6·25. The lines V_{AB_0} , V_{BC_0} , V_{CA_0} are what the primary line P.D.'s would be if they were also joined star fashion; they are therefore the no-load values of the secondary line P.D.'s. The difference of phase between the primary and secondary line P.D.'s should be noted.

The remaining figures give the cases when the star is loaded across one pair of mains and across one ray, and call for no special remark beyond drawing attention to the fact that only two of the primary mains are loaded in each case.

The converse arrangement, star-connected primaries and mesh-connected secondaries, can also be employed, and can be worked out in exactly the same way. A load on one side of the secondary divides between the sides of the mesh, so as to make the sum of the mesh currents zero, and this is just the condition required to make the star-connected primary work satisfactorily.

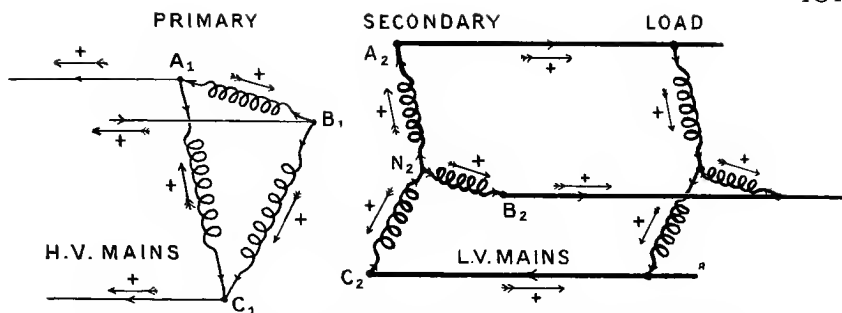


FIG. 6'29.—Connections. (The arrows alongside show the positive way; those on the windings, that of the current at one particular instant.)

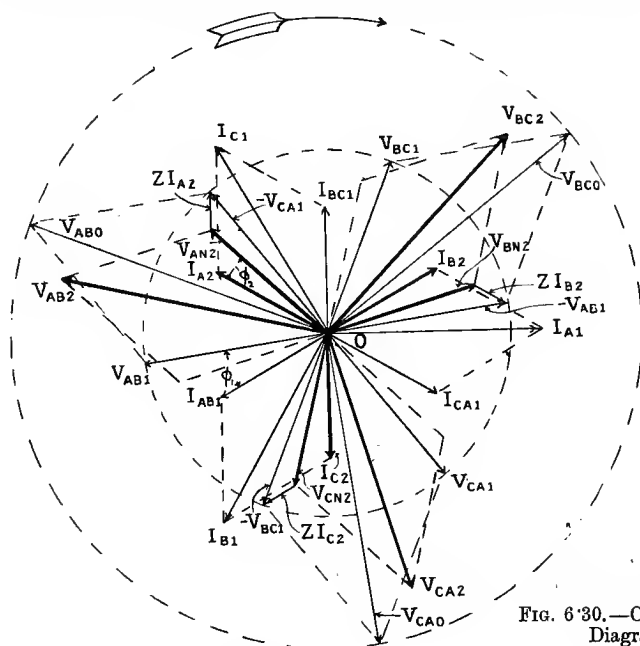


FIG. 6'30.—Clock Vector Diagram.

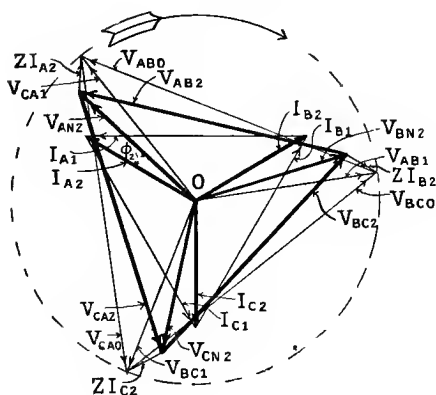


FIG. 6'31.—Transformed Vector Diagram.

Three-Phase Mixed Connections (Mesh-Star) on Balanced Load.

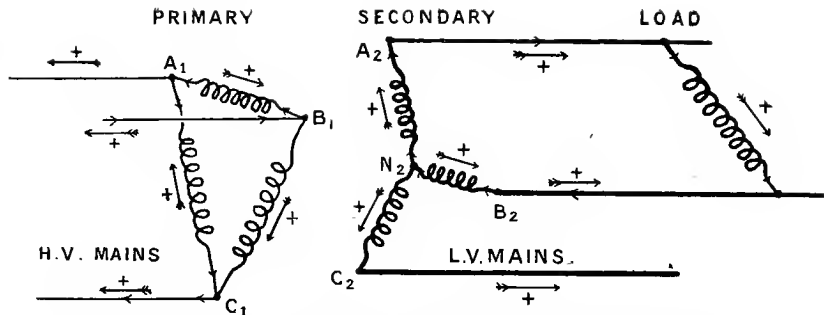


FIG. 6·32.—Connections. (The arrows alongside show the positive way ; those on the windings, that of the current at one particular instant.)

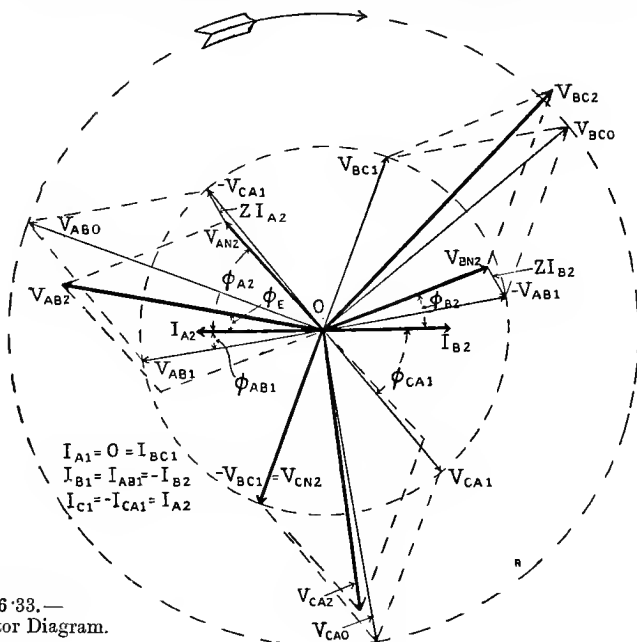


FIG. 6·33.—
Clock Vector Diagram.

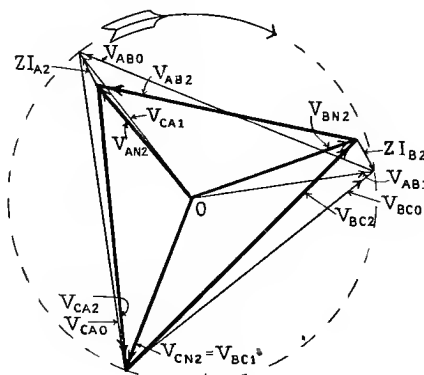


FIG. 6·34.—Transformed
Vector Diagram.

Three-Phase Mixed Connections
(Mesh-Star) loaded across One
Pair of Mains only.

$$\begin{aligned}
 & I_{A2} = I_{AC1} = I_{C1} \quad I_{B2} = I_{BA1} = I_{B1} \\
 & I_{A1} - I_{BC1} = 0
 \end{aligned}$$

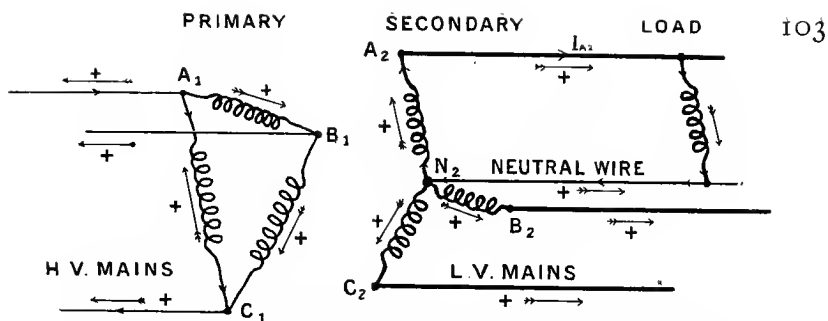


FIG. 6'35.—Connections. (The arrows alongside show the positive way; those on the windings, that of the current at one particular instant.)

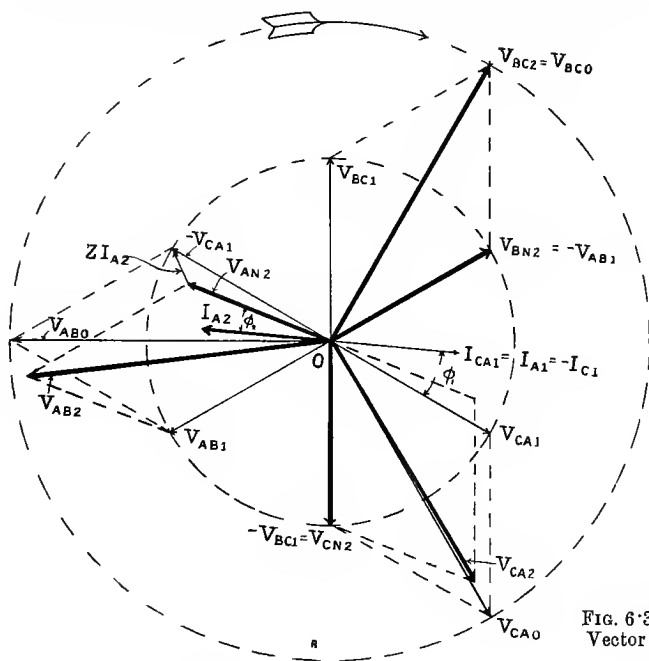


FIG. 6'36.—Clock Vector Diagram.

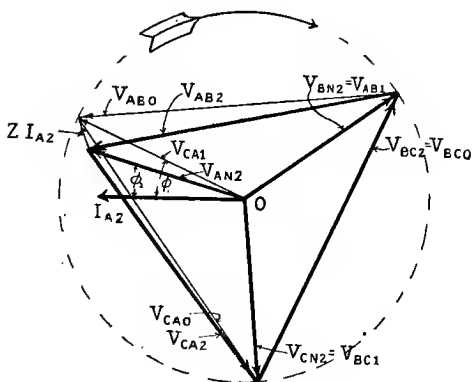


FIG. 6'37.—Transformed Vector Diagram.

Three-Phase Mixed Connections (Mesh-Star) loaded on One Ray only.

Effect of Third Harmonic in E.M.F. Wave.—It is to be noted that no triple frequency current, except a small condenser current, can flow in the mains of a three-phase system except when there is a fourth conductor. Since one-third of the fundamental period contains a whole cycle of the third harmonic, any such component in the generator E.M.F. wave will be in the same phase at any instant in all three windings. If the generator be mesh-connected, any triple frequency component in the E.M.F. wave will cause a short-circuit current to flow in the mesh; but if it be star-connected, it will merely cause a difference between the potentials of the neutral points of the generator and transformer primary, charging the latter as one coating of a condenser whose other coating is the earthed framework and the secondary windings. The secondary will consequently only receive such a triple frequency component by electrostatic induction, as the magnetic effect of the small condenser current will, in ordinary cases, be negligible.

When the neutral points of the generator and transformer primary are connected, the triple frequency current will flow in the mains and return by the fourth conductor, and can be transmitted to the secondary system if that is similarly connected. But if the secondary be mesh-connected, the mesh will be a short-circuit for it. Since the triple-frequency current is in the same (+ or -) way in all three windings at any instant, the corresponding flux has to return through the air; when three-phase transformers with only three limbs are employed, the triple-frequency magnetising current will be correspondingly large.

A third harmonic may, however, be impressed on the secondary system in another way, even when the primary P.D. wave is a sine one. As may be seen from fig. 2·01, the magnetising current of a transformer running under ordinary conditions has a prominent third harmonic. But we have just seen that star-connected transformers cannot obtain any such component from a three-wire three-phase system. The fifth harmonic has to take its place as well as it can, and so the flux wave is distorted by the absence of the triple-frequency current in such a way as to give a triple-frequency component in the E.M.F. wave, which appears in the secondary P.D. If the primary or secondary be mesh-connected, this E.M.F. causes the necessary triple-frequency component of the magnetising current to flow in the mesh, and so the effect is very small.

Clinker,¹ who has drawn attention to this effect, has found that the absence of the triple-frequency current diminishes the maximum value of the current and of the flux, and consequently also reduces the iron losses, with the result that the no-load loss is less without the fourth wire connection than with it, and less with three-wire stars than with mesh connections.

¹ "Wave Shapes in Three-Phase Transformers," *Electrician*, vol. lvi. p. 135.

CHAPTER VII.

SYSTEMATIC TESTING OF TRANSFORMERS.

Remarks on Testing.—The successful operation of an alternating current plant depends largely on the reliability of the transformers used. If their working temperature is high, the insulation is damaged and its life is shortened. Weak insulation is a source of danger to life and a frequent cause of breakdown. Poor regulation or frequent breakdowns cause annoyance to the consumers, which leads to loss of custom and injures the commercial position of the undertaking. For parallel running on the secondary side the voltage ratio must be exactly the same for all, or wasteful currents will circulate between them, and the voltage drops should also be alike if the load is to divide properly between them. Their efficiency should also be high, as the lost power means an addition to the running expenses.

It is thus extremely important to get transformers which fulfil the required conditions, and tests have to be made to see that they do in insulation, ratio, polarity, regulation, efficiency, and temperature rise. These are generally applied in the factory, but some of them should be repeated in the station after delivery for every transformer received, especially the polarity marking, ratio, insulation, and iron loss tests. The other tests are less liable to variation with different transformers of the same design, and need only be applied to a few out of each batch.

In carrying out a test it is, of course, necessary to employ leads of ample cross-section properly insulated, and to see that all the instruments are properly protected by suitable fuses. All terminals should be arranged so as to minimise risk of accident. This is especially important for the high-voltage connections, which should be protected to prevent accidental contact. Instruments of suitable type and range must be available, and these should be frequently calibrated. With many types of instrument the whole scale cannot be regarded as useful for accurate testing work, but only that part of it between 25 and 85 per cent. of the top reading, and this must be remembered when choosing them. Instruments having mirrors to avoid errors due to parallax are to be preferred, but in any case care must be taken to get the

line of vision perpendicular to the scale. When power is obtained from supply mains, the P.D. is liable to vary. It is then very important that all the instruments be read simultaneously. In many tests accuracy depends on rapidity of adjustment and observation. Errors due to a variable P.D. can be eliminated by taking the mean of several observations, four or five being generally enough. The frequency may also be out a little, and this affects the iron losses, but a correction may be applied if the exact frequency be known. All tests, except the preliminary insulation tests, should be made after the transformer has attained its working temperature.

A record sheet with all necessary rulings and headings should be prepared beforehand, and all readings ought to be entered into it immediately they are obtained. It should also contain a note of the make and serial number of the instruments employed, in case any question might arise as to their accuracy. Wherever possible, curves should be plotted as the experiment proceeds.

Insulation Tests.—Each winding must be efficiently insulated from the other, and from the core and frame of the transformer. This insulation must have a high enough resistance to reduce the leakage current to a harmless amount, or it will deteriorate rapidly, and it must also be able to withstand, without breakdown, the highest electrostatic strain it may be called upon to bear in use. It is thus subjected to tests of a twofold character. First, some sort of ohmmeter, such as the Evershed Megger, is applied to see that the insulation resistance is up to a certain standard. If not, the transformer should be rejected until the cause of the defect (*e.g.* dampness) is removed. A P.D. is then applied between the windings, and between each and the core, higher than any expected in use, and if no breakdown occurs the transformer is passed as satisfactory in this respect.

The practice of the Westinghouse Company in testing transformers designed for a 2000-volt service is to apply between the windings 10,000 volts for five minutes, the low-voltage winding being connected to the frame; then 4000 volts for the same time between the low-voltage winding and the frame; and lastly, a no-load run with twice the normal P.D. for half an hour, and of thrice the normal for five minutes. The object of the over-running test is to try the insulation between the turns, which is not done in the other tests.

The Verband Deutscher Elektrotechniker prescribes a testing E.M.F. of double the working P.D. when that does not exceed 5000 volts, 5000 volts excess from 5000 to 10,000 volts normal, and $1\frac{1}{2}$ times the working P.D. above that, this being kept on for half an hour.

It is highly important that insulation tests be made on every transformer installed. If these time tests have been made in the factory, it is sufficient to apply momentary insulation tests at the station.

For an insulation test we require a testing transformer, a regulating transformer and switch, and a suitable voltmeter, which should be connected as shown in fig. 7'01.

The regulating transformer is an auto-transformer with taps brought out so as to receive currents from the most convenient supply, say at 100 volts, and give, say, 400 volts, in steps of 40 volts.

The testing transformer should have a low impedance E.M.F., so that the ratio of transformation will be practically that of the numbers of turns, whatever current is taken from it. The high P.D. may then be determined by a measurement on the low-voltage side.

The capacity of the fuse in circuit should be slightly greater than the no-load currents of both transformers when they are operating at highest flux densities, so that, if a break occurs across the insulation of the trans-

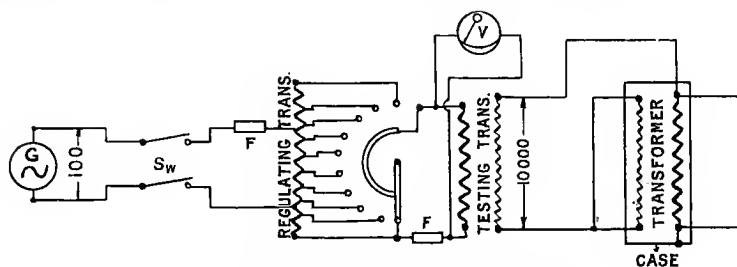


FIG. 7'01.—Connections for an Insulation Test.

former under test, the fuse will immediately blow. A fuse should also be inserted in the primary circuit of the testing transformer.

To obtain an even distribution of potential, both ends of one winding should be connected together.

Sometimes the testing P.D. is measured by means of the length of a spark gap, but this is not to be recommended. It is not very accurate, and is liable to cause a rise of P.D. which will put a great stress on both the testing transformer and the one under test.

In carrying out the test we place the regulator switch in the zero position as shown, close the main switch, and shift the regulator one step at a time until the required E.M.F. is obtained. Before the main switch is opened, the regulator switch should be turned back into its zero position.

When applying the over-running test the transformer may be excited from the low-voltage side by joining that to the secondary terminals of the regulating transformer, and adjusting the switch to give twice the rated P.D. of the transformer under test.

If a special testing transformer is not available, the high E.M.F. for

the insulation test may be obtained by connecting up a bank of stock lighting transformers with their high-voltage windings in series, and their frames insulated from earth and guarded to keep anyone from touching them. If the low-voltage windings are all joined in parallel directly to the mains, the P.D. between the secondary and primary windings of the

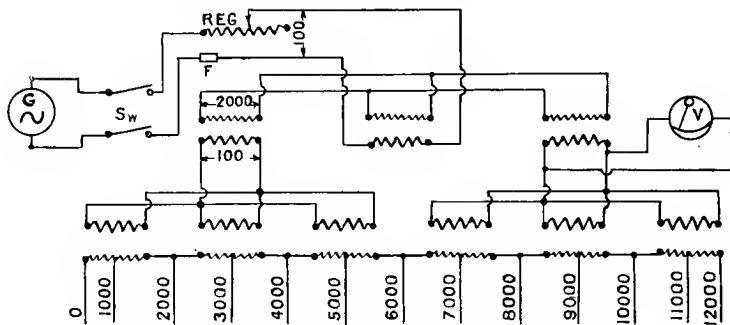


FIG. 7·02.—Insulation Test using Stock Transformers.

end transformers will be much greater than they are intended to withstand ; but by coupling them in groups, each of which is connected to the mains through transformers, as shown in fig. 7·02, this P.D. can be reduced to one-half.

In the factory it is advisable to test the individual coils as they are

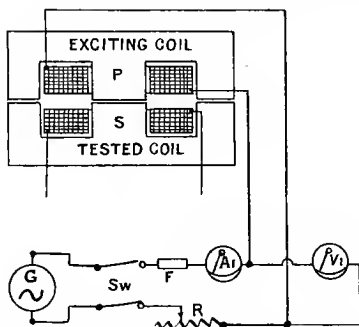


FIG. 7·03.—Transformer for Testing Single Coils.

completed for weaknesses in the insulation between the turns. This can be done by using a special transformer with a split yoke, as shown in fig. 7·03. The coil to be tested is put on one limb of this transformer, care being taken that its ends do not touch, and the core is excited by a coil on the other limb until, for one wound to give 200 volts, there is a P.D. of about 1000 volts between its ends. Any bad spot within the winding

should be ruptured by this, and would then show itself by an increase of the primary current above that taken with no coil on the test limb, and by local heating in the coil itself.

Ratio and Polarity Tests.—Before a multi-voltage transformer is tested or put in service it must be joined up correctly for the desired work. Diagrams of connections are usually sent with such transformers, giving the connections for the various voltages that may be obtained.

The ratio of transformation can be checked by connecting either winding to a suitable supply, and measuring the primary and secondary P.D.'s with suitable voltmeters. This method is not capable of sufficient accuracy when the transformer has to run in parallel with others on both sides, as the errors of two instruments come in. It is much better to keep a standard transformer which has the exact voltage ratio required, and compare all

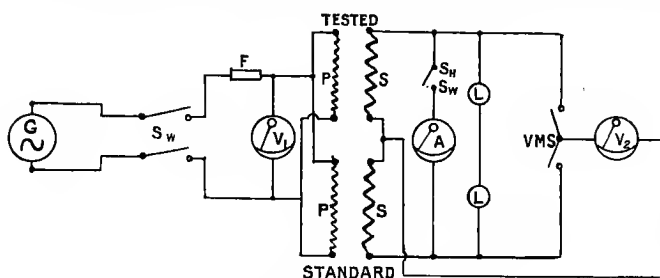


FIG. 7·04.—Ratio and Polarity Test.

the others with it. For this purpose they are connected as shown in fig. 7·04, the secondaries being put in opposition. If the ratio be far wrong, or a mistake made in the connections, or the polarity incorrectly marked, the fact will be shown by the lamps glowing. The voltmeter V_2 will show whether the ratio or the polarity is wrong, and will enable the former to be determined in terms of the standard. If the polarity is wrong, the connection should be reversed and the marking corrected. If the secondary E.M.F. is the same for both, as near as can be shown by the voltmeter, the ammeter switch can be closed. No current will flow in it if the ratio is exactly correct, and the transformer should be adjusted, if necessary, to get this right.

Short-Circuit Test—Copper Losses—Impedance—Resistance—Regulation.—In the previous chapter we saw that the total internal impedance E.M.F. referred to either winding, and for any particular current, is equal to the P.D. that must be applied to that winding to produce the desired current when the other winding is short-circuited. This P.D. seldom exceeds 3 per cent. of the normal, and consequently the iron losses on short-

allowed for by subtracting $\frac{1}{q}$ times any diminution of it from the observed change in the secondary P.D. A suitable inductive load may be obtained by employing choking coils with variable air gaps. With large transformers, however, it is seldom convenient to carry out this test, owing to the large amount of power required.

Open-Circuit Test—Magnetising Current—Iron Losses.—

The connections for this test are practically the same as before, but the winding not used for excitation is left open-circuited and other instruments are required. This time it is generally best to excite from the low-voltage side. The ammeter should have a range of say 5 per cent. of the full-load current, the voltmeter of the full P.D., and the wattmeter to correspond. Since the current is very small, the copper losses are negligible, and the power measured by the wattmeter is practically equal to the iron losses. The P.D. is adjusted to the normal value

Fig. 7'06.—Connections for Open-Circuit Test.

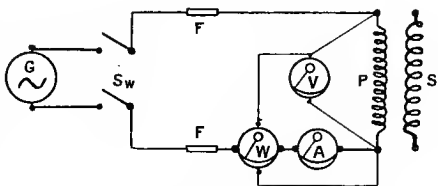


FIG. 7·06.—Connections for Open-Circuit Test.

before the instruments are read, but readings should also be taken at 1 per cent. and 2 per cent. above and below the normal if efficiency calculations are to be made.

The frequency should be measured during the test, as it affects the iron losses. With a given E.M.F. but variable frequency, the eddy current losses are constant and the hysteresis losses proportional to $f^{0.6}$ (see equations 3.13 and 3.14). Consequently, an increase of 1 per cent. in the frequency, with constant E.M.F., will diminish the hysteresis losses by 0.6 per cent., which may be taken as 0.3 per cent. of the total iron losses. If it is not far out, we must therefore add 0.3 per cent. to the observed losses for each 1 per cent. by which the frequency exceeds the normal.

The magnetising current, which may, without appreciable error, be assumed constant at all loads, has a working component in quadrature with the flux and an idle component in phase with it. These are

[illegible]

$$I_i = \sqrt{I_0^2 - I_w^2} \quad . \quad . \quad . \quad . \quad 7.05 ;$$

where I_w, I_i = working and idle components of

I_o = open-circuit current, all referred to primary,

P_o = power absorbed in open-circuit test,

V_1 = primary P.D.

Testing Transformer Iron.—In the factory it is desirable to apply similar tests to the iron before it is built up, and for that purpose the Epstein or the Searle magnetic square may be used. For the former, about 19 kilograms (42 lbs.) of the plates are taken from different bundles, cut into strips 500 mm. \times 40 mm., and then equally divided into four packets. The sheets are insulated from one another by paper, and the joints by thin sheets of micanite 0.15 mm. thick. They are then threaded through coils and clamped on a wooden base as shown in fig. 7.07. Each coil has 100 turns of insulated copper wire 2.8 mm. thick. The test is carried out

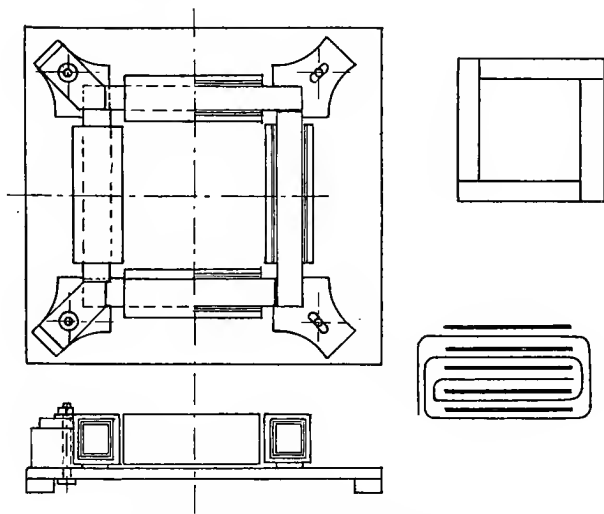


FIG. 7.07.—Epstein's Magnetic Tester.

exactly like a no-load test on a transformer, the P.D. being adjusted to give the required flux density in the iron.

Separation of Eddy and Hysteresis Losses.—If it is required to separate the eddy current and hysteresis losses in a transformer, open-circuit tests must be made at different frequencies with constant flux. This can be most easily done by running the alternator at different speeds with a constant excitation, that being adjusted to give the rated P.D. for the transformer at its normal frequency. If the power absorbed be plotted with the frequency as abscissæ, the points will lie approximately on a parabola whose axis is inclined to the axis of ordinates as in fig. 7.08, since the eddy loss is nearly proportional to the square and the hysteresis to the first power of the frequency. The energy lost per cycle by eddy currents is thus proportional to the frequency, while that by hysteresis is constant. If, therefore, the ratio of the power absorbed to the frequency

be plotted with frequency as abscissæ, the points will follow a straight line which may be produced backwards to cut the axis of ordinates, as in fig. 7·09. The intercept on this axis gives the hysteresis loss per cycle, from which that at any frequency can be calculated and the line separating the eddy and hysteresis loss in fig. 7·08 can be drawn in. The axis of the parabola is perpendicular to this line. In practice, the eddy currents often increase less rapidly than the square of the frequency with the higher frequencies, owing to the effect of the inductance of the eddy current paths. In such a case the line of fig. 7·09 is not straight, but bends downwards.

The values of the constants in equations 3·01, 3·07, and 3·10 are obtained

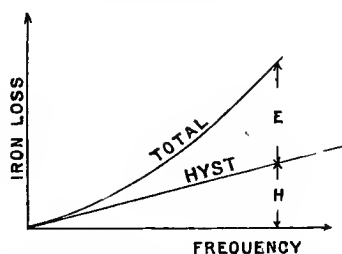


FIG. 7·08.—Variation of Iron Loss with Variable Frequency and Constant Flux.

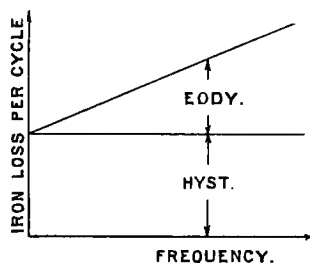


FIG. 7·09.—Separation of Iron Losses.

from experiments such as these made on actual transformers, or on special apparatus such as the Epstein tester just described.

Efficiency Calculations.—The efficiency of a machine of any kind is the ratio of the useful output of power to the total input. If, then, we can work the transformer under load and measure the input and output by suitable wattmeters we get the efficiency at once, for

$$\eta = \frac{P_2}{P_1} \quad . \quad . \quad . \quad . \quad . \quad 7·06.$$

It may not be possible in the works to put full load on a large transformer, as no suitable supply may be available, and, even if it is, since the transformer must be run on load until its steady temperature has been attained, it is rather expensive unless it can be put on a useful load. Further, it is not a very accurate way of determining the losses, as these are found as the difference of two much larger quantities, whereby the importance of errors of observation and of the instruments is enormously magnified. As, however, the iron losses are practically the same at all loads, and the copper ones are proportional to the square of the current, we may use the short-circuit and

open-circuit results, and write

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + P_o + \frac{I^2}{I_s^2} P_s} \quad \dots \quad 7.07 ;$$

where η = efficiency at output P_2 and current I ,

P_o = power absorbed in open-circuit test,

P_s = power absorbed in short-circuit test with the current I_s in the same winding as I .

With the correct frequency the iron loss depends only on the induced E.M.F. E . This is not quite constant with different loads, but varies slightly with the current and power-factor owing to the internal impedance E.M.F. If the primary P.D. is constant, E will fall as the load or lag is increased; or, if a constant secondary P.D. is to be maintained, E must be increased by raising V_1 . If an accurate calculation of the efficiency be required, the voltage drop or rise for the required load should be found by Bragstad's method (fig. 6.14). Half of this may be taken as the fall or rise of E.M.F., as Oe in fig. 6.07 will not differ much from the mean of Oa and Ob . The iron loss corresponding to that E.M.F. should be used in calculating the efficiency. If this E.M.F. is determined between the short-circuit and open-circuit tests, the primary P.D. in the latter can be set to the proper value, and the loss determined exactly. If not, the proper value can be obtained from the curve plotted from the readings taken with slightly differing P.D.'s, or, failing that, it can be estimated approximately when the difference of E.M.F. is small by subtracting 2 per cent. from the iron losses for each 1 per cent. that E is less than the P.D. used in the open-circuit test.

The copper losses depend only on the currents in the windings. That in the secondary is, of course, the load current; but that in the primary, being the resultant of this reversed and the magnetising current, differs slightly from the transferred secondary current, especially when the load is inductive. The difference of the currents may be found by the method of fig. 6.06, and the copper loss calculated from the actual load current plus half (since it is in one winding only) this difference.

If the iron and copper losses are found in this way, the total losses and the efficiency can be determined with considerable accuracy.

Since the copper losses depend on the current and the iron ones on the E.M.F., while neither is substantially altered by their relative phase, the total losses depend on the volt-ampere output irrespective of the power delivered. Consequently, the efficiency of the transformers is diminished if the load has a low power-factor.

Full-Load Tests by Opposition Method.—When two identical transformers are available, the difficulty as to a supply of sufficient power and the expense of an extended run at full load can be avoided by applying the well-known Hopkinson principle of making each act as the supply to the other on one side of the transformation involved. Then only the lost power has to be supplied by an external source. In applying this principle to two transformers they are connected in parallel and a small boosting transformer is used to cause the required current to circulate between them. This current may be adjusted either by having a booster with variable transformation ratio or by a regulating resistance in the circuit. The method can also be applied to a single transformer if both windings can be split into two equal parts. The P.D. is then, of course, only to be half the normal.

If only a heating test is to be made, for which purpose the method is

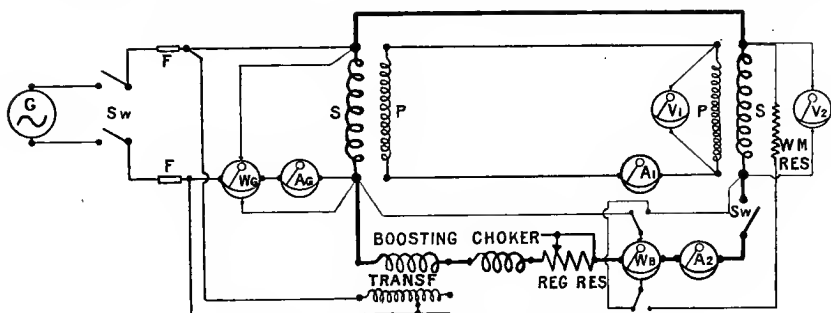


FIG. 7-10.—Load Test by Opposition Method.

especially suitable, the only instruments necessary are an ammeter and voltmeter in either the low-voltage or high-voltage local circuit, the former by preference. If the losses are to be determined, a single wattmeter may be connected to measure the total power taken from the mains, and the losses in the booster, regulators, and instruments allowed for. It is, however, better to put an ammeter in each low-voltage circuit, and to measure separately the power taken direct from the mains and that received from the booster branch by using two wattmeters connected as shown in fig. 7-10. It is then only necessary to subtract from the sum of the wattmeter readings the power used in the voltmeter, the volt coils of the wattmeters, and any instruments in the high-voltage circuit in order to get the total loss in the two transformers. The high-voltage P.D. and current may be taken as the mean of the transferred values of those for the two low-voltage windings, but a high-voltage voltmeter should be added if a regulation test is to be made. The phase of the current is of little importance when measuring the losses, but for a regulation test it is necessary to provide a means of adjust-

ing the power factor of the circulating current. For lagging currents, this can be done by suitable resistances and choking coils in the booster branch of the circuit; if leading currents are required, or a current more nearly in step than can be obtained by the expenditure of a reasonable amount of power in the regulating resistance, the booster should be excited from another phase, if a polyphase supply is available, the final adjustment being made as before. If the supply is only single phase, condensers may be used, or a suitable difference obtained by putting chokers between the supply and either the transformers under test or the booster primary.

After the connections are made, the current and P.D. should be set to their normal values and the apparatus left until a steady temperature has been attained or the run has lasted the specified time. If the temperature expected be not too high, the thermometer used should preferably be a spirit one, for eddies are induced in a mercury one and make its reading too high. The bulb should be wrapped in tinfoil and protected by cotton-wool. The instrument should be placed where it can be read without disturbing it, and with high-voltage transformers it should be observed through a telescope or only when the current is off. The room temperature near the apparatus should be noted. The thermometer is useful as a guide while the experiment is proceeding, but the mean temperature of the coils should always be obtained by measuring their resistances by the Wheatstone-bridge method before the test when they are at the room temperature, and again immediately after switching off. The temperature rise is then

$$t_r = (238^\circ \text{ C.} + t_c) \frac{R_h - R_c}{R_c} \quad . \quad . \quad . \quad 7.08 ;$$

where

R_c = resistance at the cold temperature t_c ,

R_h = resistance when hot.

The temperature rise determined in this way may be anything from 20 per cent. to 40 per cent. more than that given by the thermometer, depending on the winding depth and design of the coils.

The constants for equation 4.04 (see 4.14) are determined in this way, and should be checked every time a new design is tested.

After the working temperature has been attained, which may be hastened, if necessary, by running for some time on overload and then reducing the load to the normal value, the efficiency, regulation, short-circuit, and open-circuit tests should be made, or such of them as are required.

For the efficiency test the P.D. and current are set to the required values and the wattmeters read. As the transformer which is helped by the booster has a somewhat smaller load than the other, the one should be as

much above, both in current and P.D., as the other is below the value wanted. Half the total losses given by the wattmeters may then be taken as that of one transformer at the mean load, and the efficiency at that load calculated as before. A more accurate division of the losses can be made by testing three of them in pairs; but, owing to the difference of loading, this method is not capable of giving such good results as the simple short-circuit and open-circuit tests already explained.

The method can, however, be used for an accurate direct determination of the voltage drop or rise of the right-hand transformer. Arrange the way of the boost so that energy is transmitted from left to right in the high-voltage circuit, and from right to left in the low-voltage one, so that the primary and secondary of that transformer act in their intended functions. The

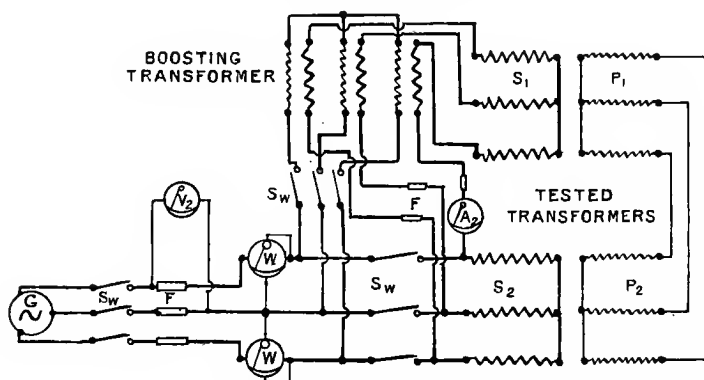


FIG. 7-11.—Load Test on Three-Phase Transformers by Opposition Method.

current in this transformer must be set exactly to the required value and phase, the voltmeter switch being changed over to test this, and either the high or low P.D., as required, set to the desired value with the current off and again with it on. The difference between the two readings on the other voltmeter gives the voltage change required. Absolute accuracy of the voltmeters themselves is not essential, but the scales should be very open at the part used, and they must be very carefully read.

This test may also be applied to polyphase transformers as shown in fig. 7-11, in which, however, the two parts of the received power are not measured separately. To make sure that the connections are right, banks of lamps should be inserted in two limbs of the high-voltage winding before switching on, and then a low voltage applied. If the connections are right, the lamps will not glow. The current can then be switched off, the lamps removed, and the test carried out as before.

Imitation Loading for Temperature Tests.—When it is not convenient to use the opposition method for a temperature test, full load conditions can be roughly imitated by exciting one winding with the normal voltage, and circulating such a direct current through the other as will give the same total copper losses as the actual full-load currents would. In applying this test to a single-phase transformer, the winding used for the direct current should be divided into two equal parts and connected as shown

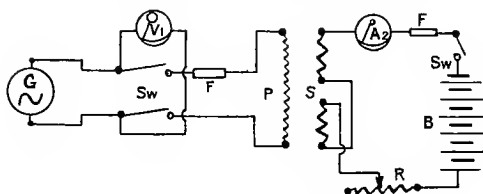


FIG. 7-12.—Imitation Loading for Temperature Test.

in fig. 7-12 so that their resultant E.M.F. is zero. If this cannot be done, a suitable choking coil would keep the A.C. within limits, but the test would not be satisfactory owing to the change in hysteresis caused by the steady magnetism produced by the direct current. The disadvantage of this test is that all the copper losses are concentrated in one winding, which consequently gets hotter than it would in actual use, while the other would heat less.

The method may be easily applied to three-phase transformers by joining

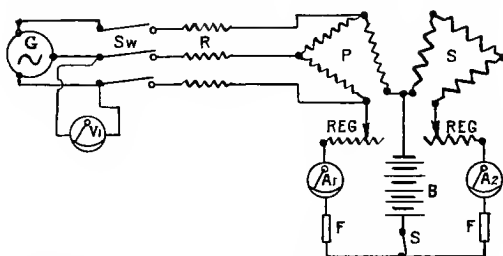


FIG. 7-13.—Imitation Loading of Three-Phase Transformer for Temperature Test.

the windings in mesh with the battery sending a current round the mesh as shown in fig. 7-13. In this case both the currents in each winding can be set to their proper values, and if they are made to circulate in opposite ways the corresponding excitation on each limb will at the same time be zero. The resistances in the main circuit prevent the battery from sending any considerable current through the generator, while they do not absorb much energy from the small magnetising current. They are not required if the

primary can be divided into equal parts connected in parallel, star fashion, as shown in fig. 7·14. Only half the normal P.D. is now to be applied, and the battery current flows between the two neutral points. The excitation due to the D.C. in the primary is zero for each limb, while that due to the A.C. is not. That of the D.C. in the secondary winding, however, does not cancel out for each limb separately, and it is therefore best to split it up in a similar way, as shown in fig. 7·15, which also shows the connections for testing two transformers at once. As the current flowing in, or out, at each neutral point is three times the

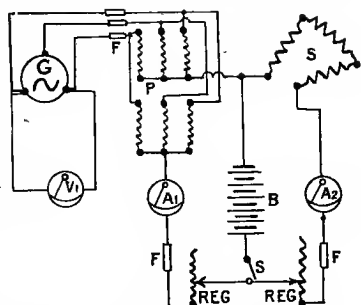


FIG. 7·14. — Imitation Loading of Divided Three-Phase Transformer.

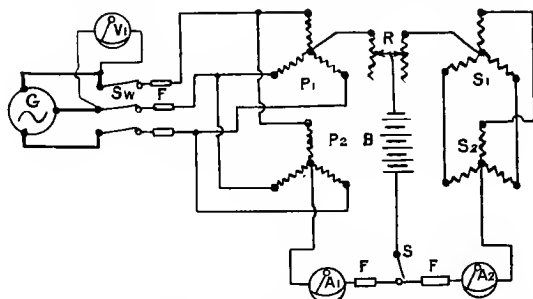


FIG. 7·15. — Imitation Loading of Divided or of Two Three-Phase Transformers.

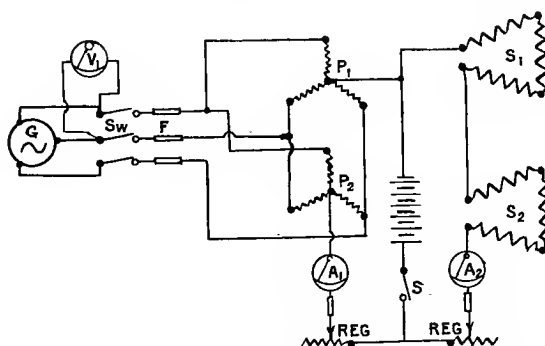


FIG. 7·16. — Imitation Loading of Two Three-Phase Transformers.

normal current in each winding, it may be preferable, in order to reduce the battery current when testing two large transformers, to adopt the connections of fig. 7·16.

CHAPTER VIII.

INSULATING MATERIALS.

Dielectric Strength—Insulation Resistance.—Insulating materials, or dielectrics, are those which do not permit electricity to flow through them; they are employed to prevent the flow of current where it is not wanted. A difference of potential between two points in a dielectric produces an electrostatic strain which may be regarded as a disturbance of the normal configuration of the electrons in the molecules or atoms, equivalent to a displacement of positive electricity from the higher towards the lower potential. The amount of this displacement is proportional to the P.D., and it takes place within the molecule only, and not from molecule to molecule as in conductors. The production of this strain while the P.D. is increasing is a “displacement current” (Maxwell) flowing in the dielectric from high to low potential, and the relief of the strain a current in the opposite way. This is the capacity current of condensers, cables, etc., and only flows while the P.D. is changing, and is proportional to the rate of change of P.D.

The electrostatic strain is elastic in the sense that the dielectric tends to return to its original condition, and that its production involves the storage of energy in the material, which is returned, in part at least, when the strain is relieved. The E.M.F. corresponding to this energy exchange is the Dielectric E.M.F., and opposes the production of the strain. It is proportional to the strain produced, *i.e.* to the total electricity which has been displaced, which is the time integral of the displacement current. It grows to such a value as will stop the current when the conditions are steady. The P.D. between any two points of the dielectric is the sum of the dielectric E.M.F.’s between them. The amount of this E.M.F. per unit length, measured along the lines of strain, or the Potential Gradient, is a measure of the corresponding electrostatic stress in the dielectric.

If the P.D. be gradually increased, a point is reached at which the material can no longer withstand the stress, and it is ruptured; if the P.D. is maintained, an arc is formed which quickly does much damage. The

potential gradient which will just produce rupture is the Dielectric Strength of the material.

No material is a perfect insulator; all allow some current to flow continuously between two points at different potentials, although it may be extremely small. This is the leakage current, and the resistance offered to it the Insulation Resistance. The leakage is probably due to the presence of some free electrons existing uncombined in the mass of the material. The resistivity of insulating materials is very high indeed, being of the order of millions of millions of millions (10^{18}) times that of the ordinary metals. A perfect dielectric may be compared with a steel wire which is stretched a definite amount by a given load, while an actual dielectric more resembles a rod of pitch which goes on yielding very slowly so long as the force is applied. In ordinary cases the leakage current is of importance only in so far as it may cause injurious heating, appreciable loss of energy, or danger of shock, and the choice of insulating material and its dimensions are settled by its dielectric strength rather than by its insulation resistance.

When several dielectrics are used in series, that of lowest dielectric constant takes more than its proper share of the total P.D., and if it also has a low dielectric strength it may break down, even although it would have been strong enough without the presence of the other materials. Thus the interposition of a plate of glass between two electrodes will cause a spark to pass through the remaining air with a lower voltage than would be required to produce a spark in the absence of the glass. This fact is of special importance in high-voltage work, for air has both a lower dielectric constant and a lower dielectric strength than solid insulating materials. Any air bubbles, however small, reduce the effective dielectric strength, and may contain small discharges with alternating P.D.'s which soon damage the solid dielectric. For this reason, air must be excluded from the insulation of coils designed for very high E.M.F.'s, either by impregnating them solid with a suitable compound, or by using them immersed in insulating oil.

Tests of Dielectric Strength.—Puncture tests of materials to determine their dielectric strength are carried out by placing a sample, large enough to prevent sparking round it, between two electrodes connected to a high-voltage testing transformer in the manner of fig. 7·01. The voltage is gradually raised and the value noted at which puncture occurs, or a specified voltage is applied and the time noted before breakdown. It is practically impossible to get thoroughly reliable tests, and the results published by different authorities differ very considerably. This is owing to the great influence of small changes in the conditions of the experiment on the results obtained.

Most of the materials used for insulating transformers are more or less hygroscopic, and their dielectric strength and insulation resistance are both much reduced by the absorption of moisture. This is a most troublesome feature to deal with, as the amount of contained moisture varies so much with the humidity of the atmosphere. Some experimenters have tested specimens previously dried in a vacuum oven, but this does not exactly represent working conditions. Others have tested undried specimens, with very variable results.

Then the temperature of the material during the test is of very great importance. Speaking broadly, the dielectric strength and insulation resistance of a material are both diminished by a rise of temperature, but with undried hygroscopic materials the first effect of heating is generally an improvement due to the expulsion of moisture. The effect of temperature rise must be borne in mind, for the materials are subject to fairly high temperatures in actual use, and tests at these temperatures are desirable rather than at the ordinary room temperature.

Alternating stresses cause an evolution of heat in the material by the expenditure of energy, partly in the leakage current, partly by discharges through the surrounding air and that contained within the mass of non-homogeneous materials, and, with some dielectrics, partly in dielectric hysteresis, which is analogous to magnetic hysteresis. The rise of temperature so produced causes a breakdown to take place with the continued application of a lower P.D. than is required to puncture it immediately (see fig. 8·01). As, however, abnormal rises of voltage in actual working do not, as a rule, last long, the application of the test voltage for one minute, or even for fifteen seconds, as specified by the Engineering Standards Committee, is considered sufficient. This effect of time is more marked with thick specimens, as they get rid of the heat less easily than thin ones, and consequently reach a higher temperature with a given P.D. The result is an apparent diminution of the dielectric strength with increase of thickness. Baur has endeavoured to show that the puncturing voltage for any material is proportional to the $\frac{2}{3}$ power of thickness, but Kinzbrunner finds it to be, as a rule, practically proportional to the square root of the thickness, instead of to the thickness as it would be with uniform strength. Some materials do not show this effect, and a few even give an increased dielectric strength with increased thickness. This is particularly the case with laminated materials, in which the probability of a fault extending right through is reduced by having a number of thicknesses.

There may also be something analogous to the fatigue of materials when

subjected to alternating mechanical stresses which has an influence on the diminution of dielectric strength with increased time of application of the testing P.D.

As the rate of generation of heat by dielectric hysteresis and internal discharges, and the corresponding rise of temperature, increase with the frequency, we would expect a diminution of the dielectric strength when the frequency is raised. Kinzbrunner¹ found that the frequency had no appreciable effect on undried materials between 20 and 75 c.p.s., but Rayner² obtained results with dried materials showing *increases* of 5 per cent. to

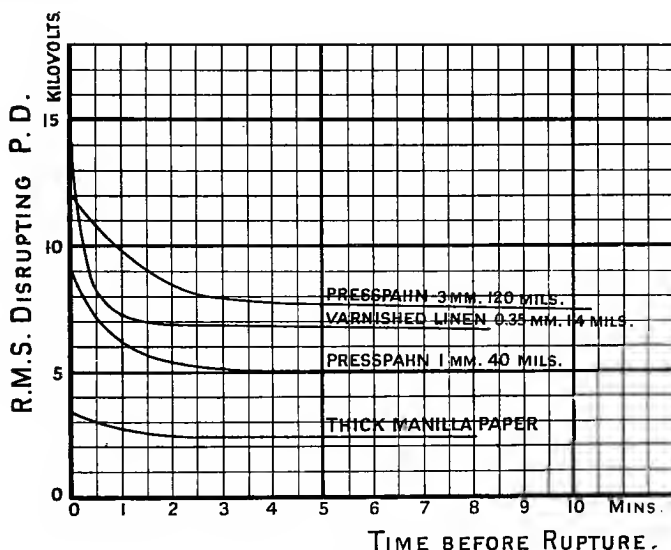


FIG. 8.01.—Effect of Time on Disrupting P.D. of Undried Materials (Kinzbrunner).

10 per cent. by increasing the frequency from 36 to 50 cycles per second. Such an increase might be due to a time lag in the production of electrostatic strain causing a diminution of the effect at higher frequencies, but Rayner's result might also be due to a change in the amplitude-factor of the E.M.F. wave of the rotary converter when the field was changed to alter the speed.

Rupture is brought about by the maximum P.D., while the voltmeter shows the R.M.S. value only. The amplitude-factor ($\text{R.M.S.} \div \text{maximum}$) of the E.M.F. wave is thus of great importance, and should be known. A sine wave is desirable (amplitude-factor $\frac{1}{\sqrt{2}} = 0.707$); a flattened wave will give

¹ "The Testing of High Tension Insulating Materials," *Electrician*, vol. lv. p. 809.

² *Journ. Inst. E.E.*, vol. xxxiv. p. 614.

a higher value and a peaky wave a lower value for the R.M.S. voltage producing rupture. It should be remembered that, although the electrostatic capacity of the testing electrodes is very small, the inductance of the high-voltage winding of the testing transformer is not, and even its capacity may be appreciable. Consequently, any high harmonics in the E.M.F. wave, such as those due to the armature teeth of a converter, or to its commutator segments, may be much magnified in the wave of the P.D. applied to the specimen. The dielectric strength with steady P.D.'s is the highest of all, being found to be from 1.5 to 2.5 times that obtained with alternating P.D.'s

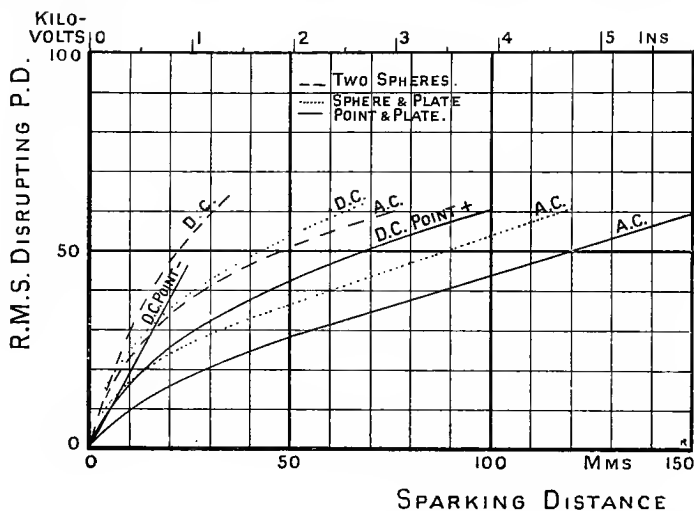


FIG. 8.02.—Comparison of Effects of Alternating and Direct E.M.F.'s on the Disrupting Voltage of Air, with Different Electrodes (Compagnie de l'Industrie Electrique and Mécanique, Geneva).¹ For alternating tests, maximum = $1.26 \times$ R.M.S. voltage.

depending on the material. This is partly due to the fact that the voltmeter now indicates the maximum value, but is also partly due to the absence of heating by hysteresis and discharges.

Fig. 8.02 shows the results of tests made on air to determine the relative value with direct and alternating stresses in connection with the Thury system of transmission with high-voltage direct currents. For these tests three direct-current machines giving one ampere at 20,000, 25,000, and 25,000 volts respectively were built, making 70,000 volts when joined in series. Comparison tests were made at a frequency of 50 cycles per second, using an alternator giving a very flat-topped wave of the high amplitude-factor of 0.8 ($1 \div 1.26$), which would be more favourable to the alternating

¹ Hirschauer, *Elektrotechnische Zeitschrift*, vol. xxv. p. 841, 22nd Sept. 1904; or Howe, *Electrician*, vol. liii. p. 996, 7th Oct. 1904.

than normal waves. Tests were made with a number of insulating materials, the most striking difference being the comparative absence of heating and side discharges with the steady P.D., and the much higher results obtained with it. Thus, 5 mm. of presspahn was punctured in $2\frac{1}{4}$ minutes by 9000 R.M.S. volts, but was only broken down by 25,000 steady volts after four minutes, following repeated applications of lower P.D.'s.

The shape of the electrodes and the smoothness of the surface of the specimen have also a very considerable influence, for these determine the form of the electrostatic field and the maximum potential gradient produced by a given total P.D. The ideal form of electrode is a pair of infinite flat

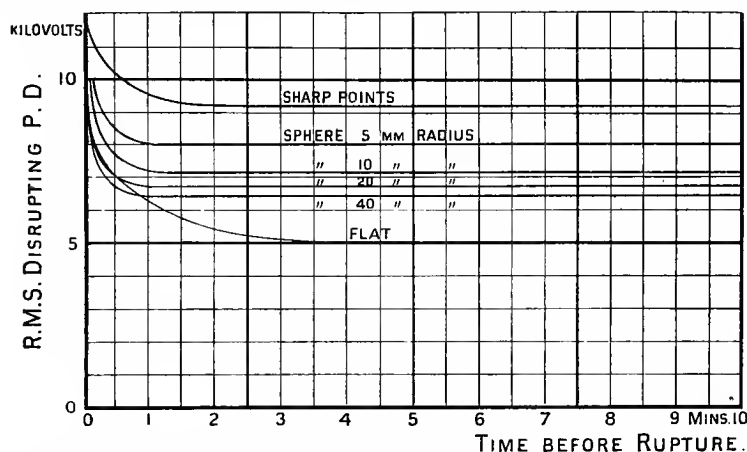


FIG. 8·03.—Time Curves with Different Shapes of Electrodes. Undried Presspahn 1 mm. thick (Kinzbrunner).

parallel plates with an infinite sheet of the material between, as this would give a uniform field. One difficulty with practical electrodes is to prevent surface heating and damage to the specimen by sparking in the air between it and the electrodes, and surrounding the electrodes. The former is prevented by pressing the electrodes into intimate contact with the test-piece so as to exclude all air, and the latter can be reduced by rounding the edges. Sharp points or edges cause very intense fields close to them, which may cause local breakdown and heating, which finally lead to rupture of the specimen with a lower voltage than would otherwise be the case (see, for example, the air curves of fig. 8·02). On the other hand, the portion of the material subject to the maximum strain is very small with sharp points, and it is able to get rid of much of the heat generated to the neighbouring unstrained portion, especially if the material is thick. This effect is well shown in fig. 8·03, which gives a summary of the results of some experiments

made by Kinzbrunner to determine the best form of electrode. It will be observed that the curves are not inconsistent with a greater P.D. being required to produce instantaneous breakdown with a larger radius of curvature, although they show very clearly that when time is allowed for the heating effect to come into play the lowest dielectric strength is obtained with flat electrodes. The smaller steepness of the curves with sharper electrodes, indicating a greater time constant, is also in agreement with the above explanation of increased sidewise cooling.

Other experiments with thinner presspahn show a smaller effect of the electrode shape (see fig. 8.04). This is just what would be expected, for the

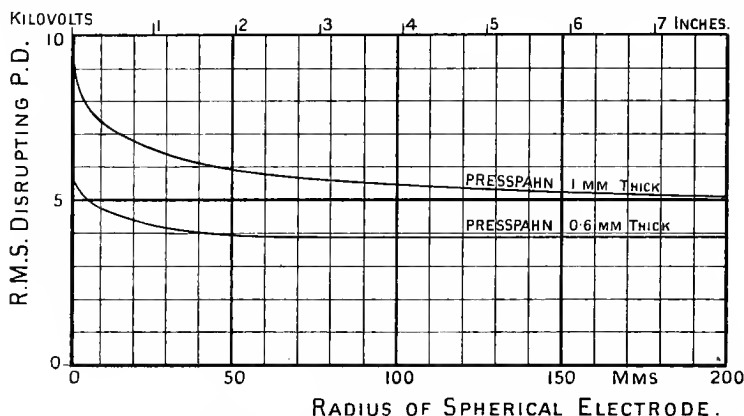


Fig. 8.04.—Effect of Curvature of Electrodes on the Minimum Disrupting P.D. of Undried Presspahn (Kinzbrunner).

thinner the specimen is, the less will be the proportion of heat carried away by the adjacent portions of the material.

Kinzbrunner tried flat electrodes of different diameters, and found that the disrupting P.D. diminished as they are made larger until a diameter of about 25 mm. (1 inch) is reached, after which little effect is produced (see fig. 8.05). This diameter is probably that at which the effect of sidewise cooling ceases to be appreciable at the centre, but there is also the greater probability of weak spots being included with large electrodes to account for the diminution of the disrupting P.D.

As may be seen from fig. 8.03, flat electrodes require a greater time than spherical to attain the steady state. The latter touch only over a very limited area, and so do not cool the specimen much, in addition to which there is a generation of heat by discharge in the small air gap surrounding the point of contact; but the flat electrodes are in intimate contact, and take heat from the specimen until they are themselves warmed up.

With flat electrodes an increase of pressure causes a diminution of the disrupting P.D. up to about $\frac{1}{4}$ kg. per square cm. ($3\frac{1}{2}$ lbs. per square in.),

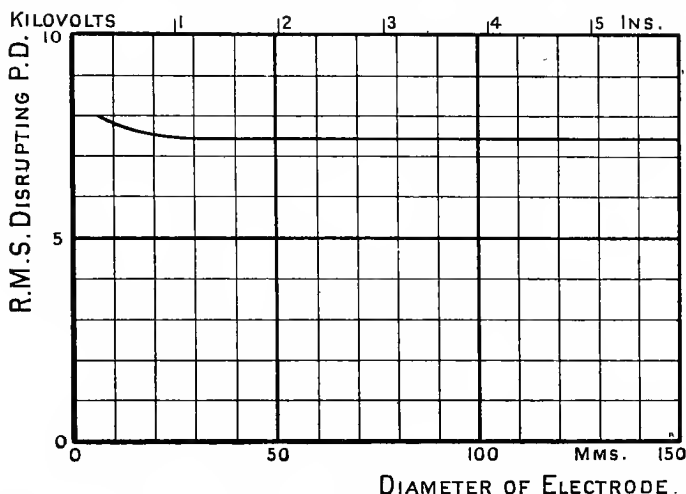


FIG. 8'05.—Effect of Area of Contact on the Minimum Disrupting P.D. of Undried Varnished Paper, 0.35 mm. thick (Kinzbrunner).

after which it makes little difference (see fig. 8'05). This initial effect of pressure is probably due to the expulsion of the films of air, which take an

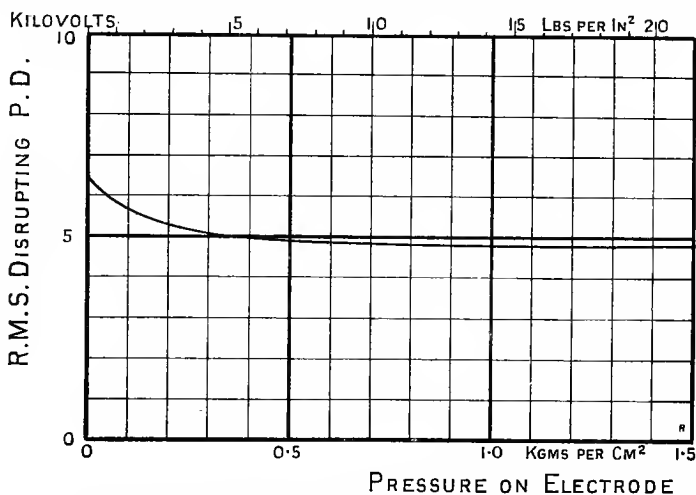


FIG. 8'06.—Effect of Contact Pressure on the Minimum Disrupting P.D. of Undried Presspahn 1 mm. thick (Kinzbrunner).

appreciable share of the total P.D., and it would probably be less with thicker specimens.

As a result of his experiments, Kinzbrunner recommends the use of flat electrodes with rounded edges, whose contact area is at least 500 square mms. (1 in. in diameter), and pressed together with at least $\frac{1}{4}$ kg. per square cm. ($3\frac{1}{2}$ lbs. per square in.). The specimens are conveniently squares of 100 mm. square for presspahn, 150 mm. for varnished paper and linen, 250 mm. for india-rubber and gutta-percha, and 450 mm. for ebonite.

By immersing the electrodes and specimen in insulating oil, or by running paraffin wax round them, the trouble with sparking can be eliminated, but

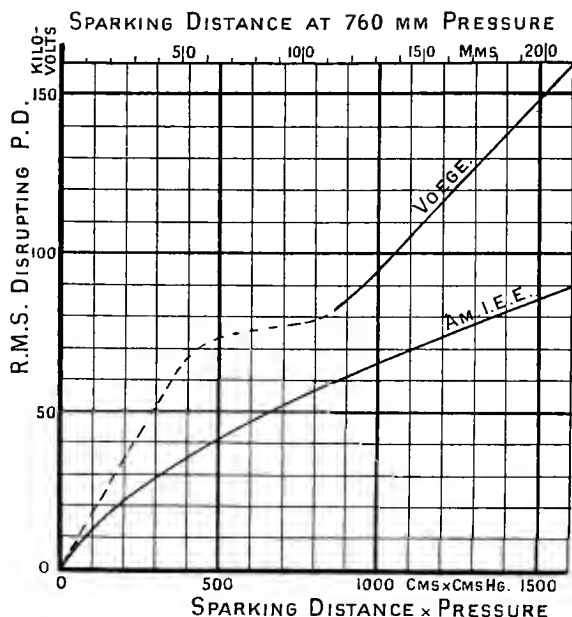


FIG. 8·07.—Disrupting Voltage of Air.

then the properties of the material may be altered by the absorption of oil or wax. If the electrodes do not actually touch the specimen, the substitution of oil for air will cause a larger proportion of the total P.D. to be taken by the specimen, and so lead to an apparent diminution of its dielectric strength, which will be more marked the greater the dielectric constant of the oil. Thus the dielectric strength of mica with point electrodes is 30 per cent. less in oil than in air, and Voegé found that glass 9 mm. thick was ruptured by 69,000 volts (R.M.S.) in linseed oil, by 78,000 volts in petroleum, and by 100,000 volts in a thick paraffin oil. When the electrodes are in perfect contact with the test-piece, the only effects that can be produced by the surrounding medium are due to its absorption by the material, its solvent action on it, and to the prevention or reduction of the external discharges,

The strength of gaseous dielectrics is much enhanced by an increase of pressure, probably owing to the greater density. Voege¹ found for air that the product of sparking distance and pressure for a given P.D. is constant when the pressure is varied. This indicates that a given mass of gas can withstand the same P.D. when contained in an extensible tube of constant cross-section reaching from one electrode to the other whatever may be the distance between them. If the air is at a pressure of about five atmospheres, its dielectric strength is about the same as that of petroleum. Voege's results (see fig. 8-07) were obtained with pointed electrodes in a glass vessel. He was unable to get constant results for the product between the values 600 to 850 cm. x cm. of mercury.

In testing liquids pointed electrodes are often employed, but with gases more consistent results are obtained with a point and a plate.

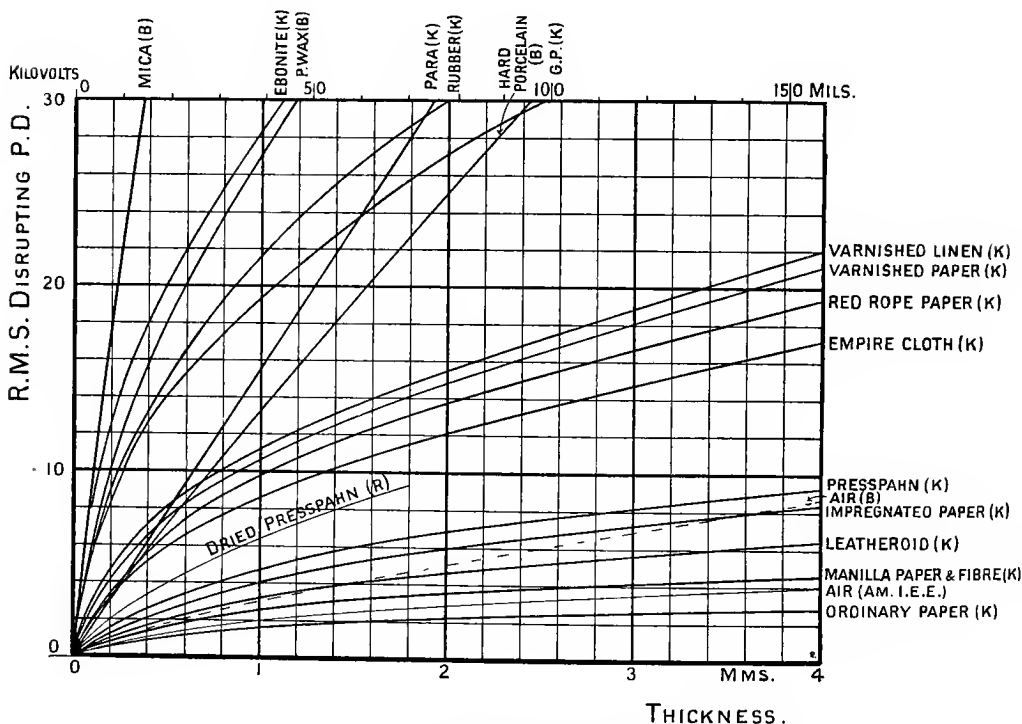
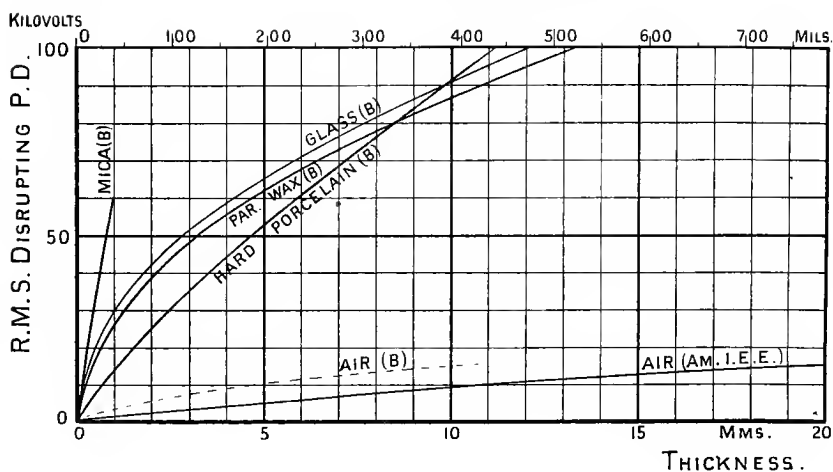
Kinzbrunner gives the following table for the dielectric strengths of various materials when tested in pieces 1 mm. thick, to be used in conjunction with his square root law. He finds that the same result is, in general, obtained whether the thickness is all one or made up of several sheets. It will be observed that his results are mostly much lower than those given by other authorities. This is largely accounted for by the fact that he applied the testing voltage for a much longer time than is customary. He contends that the usual time of application is much too short, and gives too high results.

TABLE 8-01.—AVERAGE DIELECTRIC STRENGTH WITH 1 MM. THICKNESS
(Kinzbrunner).²

Material.	Volts per mm.				
	Kinzbrunner.	Arnold.	Rayner.	Wiener.	Baur.
Presspahn .	4,600	7,400	5,000	6,000	..
Manilla paper	2,200	..	2,950
Ordinary paper .	1,450	5,500	..	8,000	5,000
Fibre .	2,250
Varnished paper	10,500	27,000
Red rope paper	9,400	19,000	10,500
Impregnated paper	4,200
Varnished linen .	10,700	21,000	..	10,000	..
Empire cloth .	8,400	12,500
Leatheroid .	3,050	5,600
Ebonite .	28,500	31,500
Rubber .	21,000	16,500	..	16,000	..
Gutta-percha	19,000	7,700
Para .	15,500

¹ *Elektrotechnische Zeitschrift*, 6th June 1907.

² *Electrician*, vol. lv. p. 980, 6th Oct. 1905.



FIGS. 8'08 and 8'09.—Disrupting Voltage for Various Materials.

K=Kinzbrunner, *Electrician*, vol. lv. pp. 938 and 977, 29th Sept. and 6th Oct. 1905.
(Undried materials.)

B=Baur, *Elektrotechnische Zeitschrift*, vol. xxv. p. 7, 7th Jan. 1904.

R=Rayner, *Journ. Inst. E.E.*, vol. xxxiv. p. 613, 9th March 1905.

His results are also plotted, along with a few others, in figs. 8·08 and 8·09.

Testing Insulation Resistance.—The insulation of a sheet may be tested by measuring, by the deflection method, the resistance between two flat electrodes placed one on each side and opposite to one another. The cross-section carrying the leakage current may be taken as the effective surface of each electrode provided they make good contact. In his tests made at the National Physical Laboratory, Rayner placed the specimen between two horizontal soft rubber discs 80 mm. (3·15 in.) in diameter and 10 mm. (0·4 in.) thick covered with thin tinfoil. A weight of 20 kgs. (44 lbs.) was placed on the upper one. As the resistance was found to vary rapidly when the weights were first put on, five minutes were allowed to elapse before making the test.

The galvanometer was of the Broca pattern, giving 5×10^9 divisions per ampere, and was provided with a set of Ayrton-Mather shunts giving ratios from 1 to 1,000,000. The battery consisted of 500 small accumulators with one pole earthed, and Price's guard wire arrangement was employed to obviate errors due to the leakage of the apparatus. Readings were taken after one minute's electrification.

Tests were made with 200 volts as well as with the full battery of 1000 volts. The higher E.M.F. always gave the lower resistance, sometimes less than half that obtained with 200 volts, and its application had a permanent effect, causing a later test with the lower E.M.F. to give a lower result than a test made before using the 1000 volts.

The influence of moisture on unvarnished fibrous materials is enormous. For example, the insulation resistance of one specimen of presspahn was raised ten million times (from 0·42 to over 4×10^6 megohms) by heating it for an hour, both measurements being made cold. Putting the specimen into an oven causes an immediate drop of the insulation to one-fifth or less, and then a gradual rise as the moisture is expelled. Tests on the hot material give a lower result than those made while it is cold but free from moisture. Varnished materials are very much less susceptible to moisture. Some of Rayner's results are given in table 8·02.¹

The following is a brief account of the properties of the chief insulating materials in ordinary use. For further information the reader is referred to the original articles by Kinzbrunner, Baur, and Voegelé, and to Turner and Hobart's *Insulation of Electrical Machines*.

¹ This table is compiled from the results given by Rayner in vol. xxxiv. p. 621 of the *Journ. Inst. E.E.* (9th March 1905). The resistivities have, however, been recalculated, as a slip has been made in the original.

TABLE 8-02.—INSULATION RESISTANCES (Rayner).

Material.	Thickness.		E.M.F. Volts.	Unheated.		Heated 75°-100° C. Tested at 75° C.	
	Mm.	Mils.		Resistance. Megohms.	Resistivity. 10 ¹² Ohm-mms.	Resistance. Megohms.	Resistivity. 10 ¹² Ohm-mms.
Presspahn . . .	0·23	9	$\begin{cases} 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} 2·8 \\ .. \\ .. \end{cases}$	$\begin{cases} 0·061 \\ .. \\ .. \end{cases}$	$\begin{cases} 22,000 \\ 9,600 \\ 18,500 \end{cases}$	$\begin{cases} 478 \\ 209 \\ 402 \end{cases}$
Presspahn . . .	0·56	22	$\begin{cases} 200 \\ 1000 \end{cases}$	$\begin{cases} 5·7 \\ .. \end{cases}$	$\begin{cases} 0·051 \\ .. \end{cases}$	$\begin{cases} 37,000 \\ 32,000 \end{cases}$	$\begin{cases} 330 \\ 288 \end{cases}$
Presspahn and stand- ard varnish }	0·34	13·5	$\begin{cases} 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} 5000 \\ 3400 \\ .. \end{cases}$	$\begin{cases} 74 \\ 50 \\ .. \end{cases}$	$\begin{cases} 10,500 \\ 9,200 \\ 10,500 \end{cases}$	$\begin{cases} 154 \\ 135 \\ 154 \end{cases}$
Manilla paper . . .	0·28	11	$\begin{cases} 80 \\ 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} 0·96 \\ .. \\ .. \\ .. \end{cases}$	$\begin{cases} 0·017 \\ .. \\ .. \\ .. \end{cases}$	$\begin{cases} .. \\ 12,500 \\ 7,600 \\ 11,000 \end{cases}$	$\begin{cases} .. \\ 223 \\ 136 \\ 196 \end{cases}$
Manilla paper and standard varnish }	0·34	13·5	$\begin{cases} 200 \\ 1000 \end{cases}$	$\begin{cases} 1950 \\ 510 \end{cases}$	$\begin{cases} 29 \\ 7·5 \end{cases}$	$\begin{cases} 16,700 \\ 11,900 \end{cases}$	$\begin{cases} 245 \\ 175 \end{cases}$
Waterproof board . .	0·44	17·5	$\begin{cases} 200 \\ 1000 \end{cases}$	$\begin{cases} 8·1 \\ .. \end{cases}$	$\begin{cases} 0·092 \\ .. \end{cases}$	$\begin{cases} > 200,000 \\ 220,000 \end{cases}$	$\begin{cases} > 2300 \\ 2500 \end{cases}$
Red oiled paper . . .	0·25	10	$\begin{cases} 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} .. \\ .. \\ .. \end{cases}$	$\begin{cases} .. \\ .. \\ .. \end{cases}$	$\begin{cases} 1,400 \\ 1,000 \\ 1,220 \end{cases}$	$\begin{cases} 28 \\ 20 \\ 24·4 \end{cases}$
Black oiled board . .	0·30	12	$\begin{cases} 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} 1300 \\ 570 \end{cases}$	$\begin{cases} 22 \\ 9·5 \\ .. \end{cases}$	$\begin{cases} 4,850 \\ 3,800 \\ 4,600 \end{cases}$	$\begin{cases} 81 \\ 63 \\ 77 \end{cases}$
Oiled linen . . .	0·23	9	$\begin{cases} 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} .. \\ .. \\ .. \end{cases}$	$\begin{cases} .. \\ .. \\ .. \end{cases}$	$\begin{cases} 220 \\ 75 \\ 185 \end{cases}$	$\begin{cases} 4·8 \\ 1·6 \\ 4·0 \end{cases}$
Excelsior paper No. 1	0·12	4·5	$\begin{cases} 200 \\ 1000 \end{cases}$	$\begin{cases} .. \\ .. \end{cases}$	$\begin{cases} .. \\ .. \end{cases}$	$\begin{cases} 3,500 \\ 1,900 \end{cases}$	$\begin{cases} 146 \\ 79 \end{cases}$
Excelsior linen No. 1	0·22	8·5	$\begin{cases} 200 \\ 1000 \\ 200 \end{cases}$	$\begin{cases} .. \\ .. \\ .. \end{cases}$	$\begin{cases} .. \\ .. \\ .. \end{cases}$	$\begin{cases} 4,200 \\ 1,750 \\ 2,600 \end{cases}$	$\begin{cases} 96 \\ 40 \\ 59 \end{cases}$
Grey fibre . . .	1·75	70	$\begin{cases} 80 \\ 200 \\ 1000 \end{cases}$	$\begin{cases} 3·0 \\ .. \\ .. \end{cases}$	$\begin{cases} 0·0086 \\ .. \\ .. \end{cases}$	$\begin{cases} .. \\ 13,700 \\ 11,000 \end{cases}$	$\begin{cases} .. \\ 39 \\ 31 \end{cases}$
Red fibre . . .	0·55	22	$\begin{cases} 200 \\ 1000 \end{cases}$	$\begin{cases} 1·2 \\ .. \end{cases}$	$\begin{cases} 0·011 \\ .. \end{cases}$	$\begin{cases} 56,000 \\ 31,500 \end{cases}$	$\begin{cases} 510 \\ 286 \end{cases}$

Glass.—Glass may be employed for insulation in a dry atmosphere for leading-in tubes, bushings, bases for terminal boards, etc., up to about 50,000 volts. Beyond this, the surface leakage becomes too great. Glass should not be used in places where it is subjected to knocks and jerks, as it easily cracks. The cracks are, however, soon detected. Glass dissolves slightly in rain-water, causing its surface to roughen and to accumulate dirt. It is therefore unsuitable for outside insulators subjected to high voltages.

Porcelain.—Good porcelain does not show the bad qualities of glass in rain-water. It does not crack, but chips. Porcelain for insulators should be thoroughly vitrified, and ought not to depend on the glazing for the exclusion

of moisture. Cheap porcelain is, however, hygroscopic, and gives very poor results as soon as the glaze is chipped. A fair idea as to its quality may be obtained by placing a drop of ink on a chipped part. If it runs through the porcelain, the material is poor. Porcelain is extensively used for outside insulators, for terminal blocks, leading-in tubes, separators for transformer coils, etc., up to the highest P.D.'s in all places where it is not subjected to knocks.

Lava.—Lava is a kind of mineral talc which can easily be machined in its natural state. It is baked at 1100°C . (2000°F .), when it becomes very hard. It is unaffected by heat, acids (except hydrochloric acid), and alkalis; it does not swell or shrink in changeable weather, and its dielectric strength is supposed to be about 40 per cent greater than that of porcelain, while its price is no higher. It is applicable in all places in which porcelain is employed.

Marble.—Marble is employed chiefly for low-voltage switchboards and terminal boards. It is hygroscopic, but possesses considerable mechanical strength. In a perfectly dry state it possesses the high dielectric strength of 6000 R.M.S. volts per millimetre (150 volts per mil). Being hygroscopic, it should not be employed for over 5000 volts.

Slate.—Slate is often employed in the place of marble. It is, however, even more hygroscopic, and very often contains metal veins which make it unsafe as an insulator. When it is used for switchboards, it is advisable to bush the holes for terminals with ebonite bushes and washers. A piece of perfect slate 25 mm. thick (1 in.) will withstand a P.D. of 10,000 R.M.S. volts.

Mica and its Compounds.—Mica is one of the best insulating materials used in electrical practice. Its dielectric strength is extremely high; it is unaffected by heat, and it is perfectly damp-proof. Its mechanical strength is, however, not very great, it is not flexible unless when heated; its uniformity is not great, and it permits of some surface leakage. It is also slightly soluble in mineral and vegetable oils, so that for oil-insulated transformers it is replaced by presspahn. The dielectric strength varies with the quality; the curve given in fig. 8.09 refers to a good quality.

Mica consists of an anhydrous silicate of potassium or sodium and aluminium, and is found in India and North America. It can be obtained in small sheets only, which have to be cemented together for many insulating purposes. The material is then called micanite. When heated, micanite may be moulded into almost any shape. It is used for the insulation of air-cooled transformers, for which it is an excellent substance; for

armature slots of high-voltage machines; and especially for commutators, since it wears down at the same rate as copper, and because it will stand any heat. The cementing together of the mica sheets is generally done with shellac varnish, using as little as possible; but, as this is slightly hygroscopic, some makers prefer other cements. The material is afterwards finished in hot presses. Mica is also combined with linen and paper, producing mica-cloth and mica-paper. The linen or paper forms the covering of the flexible mica. The dielectric strengths of these materials vary from 4000 to 12,000 R.M.S. volts per mm. (100 to 300 R.M.S. volts per mil) according to the thickness of the covered mica.

Megohmite is a similar substance to micanite, and is used for similar purposes. The adhesive matter exuding from composite micas is here withdrawn.

Wood.—Various kinds of wood, such as teak, walnut, mahogany, and maple, are often used for terminal boards, separators of transformer coils from the yoke or from one another, etc. The wood should be carefully dried in an oven and boiled in linseed oil for twelve to eighteen hours, according to the thickness. A board 25 mm. (1 in.) thick is then able to stand from 8000 to 10,000 R.M.S. volts, and has become almost damp-proof.

Fibre.—Fibre is an insulating material of great mechanical strength, elasticity, and toughness. It can be screwed almost like metal, but it is very hygroscopic. Its insulating quality is increased by heating, which, however, makes it brittle. It is insoluble in alcohols, ammonia, and mineral and vegetable oils.

Vulcanised fibre is made by treating vegetable fibre with chemicals which glutinise the exterior of each separate fibre, the whole being made uniform under high pressure. The chemicals are then extracted, and the mass is rolled, pressed, and cured in various ways. Great skill is required to prevent the absorption of moisture during the process of manufacture. Fibre should be used only in dry places. Its application is the same as that of wood. At one time it was used also for commutator insulation, but it has been discarded for the more suitable mica.

Ambroin.—Ambroin is a compound of fossil copal and silicates. It is strong, uniform, durable, and non-hygroscopic. It is employed for bushes, tubes, washers, bobbins, etc. It will stand temperatures up to 100° C. (212° F.) and more, according to the quality. Some specimens are also acid- and alkali-proof. It can be turned, drilled, cemented together with ambroin-cement and polished like wood. Its dielectric strength is about 6000 to 8000 R.M.S. volts per mm. (150 to 200 R.M.S. volts per mil.).

Isolit and Adit.—These new materials consist of papier-mâché im-

pregnated with a special insulating compound. They are very tough, unshrinkable, moisture-proof, and difficult to ignite. They are chiefly employed for bobbins of field coils, low-voltage terminal blocks, and instrument cases.

Paraffin Wax.—Paraffin wax is employed to increase the insulation of fibrous materials. It is acid- and water-proof, and, mixed with linseed oil, forms a very good impregnating compound. Its dielectric strength and insulation resistance are very high, but it is very inflammable.

Ebonite.—Ebonite in a dry state is an excellent insulator of high dielectric strength. Being hygroscopic, it is unsuitable for damp places. It becomes soft at 70° C. (158° F.), and melts at 80° C. (176° F.) It can be bent into any shape, and is used for bushes, washers, instrument bases, keys, resistance boxes, and, in fact, in all dry places in which an excellent insulation is required. A plate 1 mm. (40 mils) thick is only ruptured with 30,000 R.M.S. volts.

Presspahn.—Presspahn is a prepared paper of a smooth, homogeneous texture. It can be rendered waterproof by boiling in boiled linseed oil diluted with turpentine for five to forty hours according to the thickness (0.2 to 3 mm., or 8 to 120 mils). The presspahn is then dried in a hot chamber, but not in a vacuum, as oxygen is required for oxidation. Presspahn is also made non-hygroscopic by impregnating it with Standard varnish.

Presspahn is extensively employed for insulating the slots of electric machines, the coils of transformers, and the spaces between adjacent coils. It is especially suitable for oil-insulated transformers, as it is insoluble in transformer oil.

Manilla paper, express paper, etc., are also used for insulation; all of them should be impregnated in a similar manner to presspahn in order to make them damp-proof. Their dielectric strengths are much the same as that of presspahn.

Impregnated Cloths.—The fabrics serve to carry the film of insulating oils or varnishes; those chiefly used are muslin, cambric, and batiste, because they are free from fuzz and possess a smooth surface. Care must be taken to remove all chlorine left by the bleaching. To make them smooth, they should be singed and ironed before the varnish or oil is applied. The cloth is first dried, and then the varnish is put on with a brush; or the cloth may be passed through a vat containing linseed oil, dried, and then varnished, the varnishing and drying being repeated several times.

Impregnated cloths or tapes for insulating transformer coils must be chosen according to circumstances. If they are to be oil-cooled, do not use varnishes, as these dissolve in oil; if the machine has to withstand high temperatures, the varnish should be chosen accordingly.

The insulation of the conductors is practically always cotton in one to three layers, but silk is used for induction coils.

Insulating Varnishes.—Insulating varnishes are employed to maintain and improve the initial insulating resistance of fibrous materials. They should prevent the penetration of moisture or acid, should possess great elastic strength and a high melting-point, dry quickly, and not attack the copper.

To see whether a varnish is damp-proof, expose a treated piece of metal to the weather for a considerable time. The elastic strength may be examined by coating a piece of presspahn or sheet copper, and bending it backwards and forwards when dry. A good varnish will not crack. To find the melting-point, dry off the liquid components, and then heat the residue until it melts. To test whether a varnish attacks the copper, place some filings or turnings of copper in a vessel filled with varnish and examine them after some time.

The following are the chief varnishes in use for insulating purposes:—

Sterling Varnish.—Sterling varnish consists of concentrated linseed oil and turpentine, which possess high dielectric strengths and produce an even surface. It is perfectly damp-proof, but seems to dissolve a little in insulating oils. It can also be had of a quick-drying grade.

Empire Insulating Varnish.—This is a black oil varnish, quick-drying, flexible, and highly insulating; it is also damp- and acid-proof, and does not blister under heat.

Berrite.—This is an impregnating gum of a high dielectric strength, but rather brittle. It runs at a low temperature, but its disruptive quality is unaffected by heat. When used, care must be taken that it does not crack.

Dielectric Varnish.—This is a linseed oil gum varnish, and is supposed to have a high dielectric strength, and to be free from all acids except those of the linseed oil.

Dielectrol.—This is a kind of paraffin varnish with a flowing point high enough to permit of its use for the insulation of field and transformer coils. It is quick-drying, but is nevertheless baked after application to drive out the moisture.

Japan Varnishes.—These are compounds of bitumen dissolved in turpentine, naphtha, or carbon bisulphide. Their dielectric strengths vary greatly with their composition. Japan varnishes are used for the insulation of field and transformer coils. A coil dipped into such a liquid becomes tough and hard, and does not lose its shape when being handled. After having been dried for eight or ten hours at a temperature of about 80° C. (176° F.), the coil is dipped into another varnish, such as Sterling or Empire varnish, again

dried, and finally coated with an enamel varnish to give a smooth surface which will not readily collect dust. For coils which are subjected to high temperatures, enamel varnish alone should be employed.

Copal Varnishes.—These consist of resin acid gums, but are seldom used for insulating purposes, because they are soon reduced to powder by vibration and heat. Moreover, their damp-resisting qualities are small.

Armalac.—This is a varnish consisting of black paraffin wax dissolved in petroleum naphtha. It is used chiefly for insulating armature discs and transformer plates. It is plastic, quick-drying, and homogeneous, and retains these qualities and its insulation resistance under working conditions.

Besides these, a number of other varnishes, such as Excelsior varnish, Electro-lac, Volta-lac, Sticker, etc., are on the market. They come more or less in the classes already mentioned.

Insulating Oils.—Oil for transformers should be a pure mineral oil, obtained by fractional distillation of petroleum without any subsequent chemical treatment. Such oil gives less trouble through the formation of other carbon compounds than vegetable oils. Its specific gravity is about 0.88. It must be free from acid, alkali, or sulphur compounds, as these reduce the insulating qualities and attack the materials of the transformer. Moisture must be removed as completely as possible by keeping the oil for a considerable time near its flashing point. The effect of dissolved moisture on the dielectric strength of oil is strikingly shown in fig. 8-10.

The oil should not lose more than 0.2 per cent. of its weight when kept at 100° C. (212° F.) for eight hours. The flashing and burning points should not be less than 180° C. (356° F.) and 200° C. (392° F.) respectively. A high flash-point is accompanied by considerable viscosity at low temperatures; the oil used for transformers becomes quite fluid at 50°–60° C. (120°–140° F.).

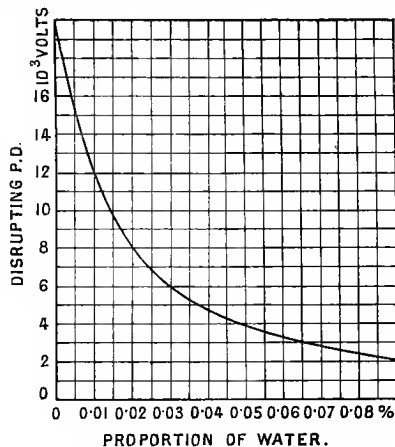


FIG. 8-10.—Influence of Moisture on the Dielectric Strength of Transil Oil (Skinner).¹

¹ *Electric Club Journal* (U.S.A.), May 1904.

CHAPTER IX.

EXAMPLES OF CONSTRUCTION.

Cores.—In small transformers the core is often placed horizontally, when the supporting brackets at each end can be identical ; but in large ones they must be vertical, for the ventilation is then better. Some makers prefer the shell type and some the core type, while others use both. In the core type the coils are generally slipped over the already completed core, and then the yoke is added, the joints being broken at every packet of sheets. In shell trans-

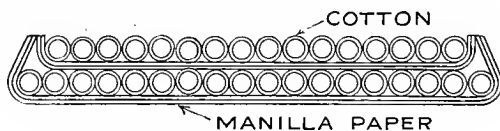


FIG. 9'01.—Insulation between Layers.

formers the magnetic circuit is built up around the coils, either from the sheets themselves or from packets of them. Single sheets are often used, especially for smaller ones, as then the joints are broken at every sheet, but the coils are then very inaccessible when repairs are required. On the whole, the core type seems to possess most advantages and to be the favourite.

The sheets are generally varnished or japanned by dipping, but sometimes only every second one is so treated, to reduce the cost and waste of space. Often a sheet of paper is inserted every few sheets. The end sheets should be thicker than the others if the transformer is not to hum, and occasionally the core is impregnated with compound with the same object.

The stampings are generally held together by bolts, but when the joints overlap they may be bound together by hempen cords wound on sheets of presspahn, as in the Ganz transformer shown in fig. 9'11. In this transformer the lower yoke is clamped by bent iron pieces insulated by presspahn. Unless the flux density is under 0.5×10^{-6} volt-secs. per sq. mm. (0.32×10^{-3} volt-secs. per sq. in. ; 5000 lines per sq. cm.), the bolts

and nuts should always be insulated, as otherwise the eddy current loss will be considerably increased. Tubes of paper or presspahn about a

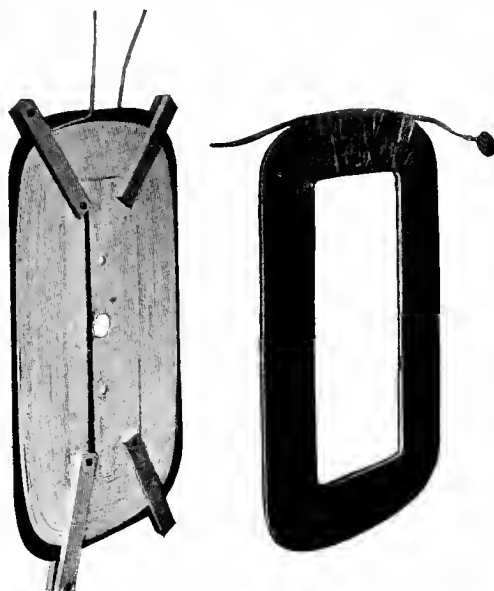


FIG. 9-02.¹—Pancake Coil on Former and Finished. (General Electric Company, Schenectady, U.S.A.)

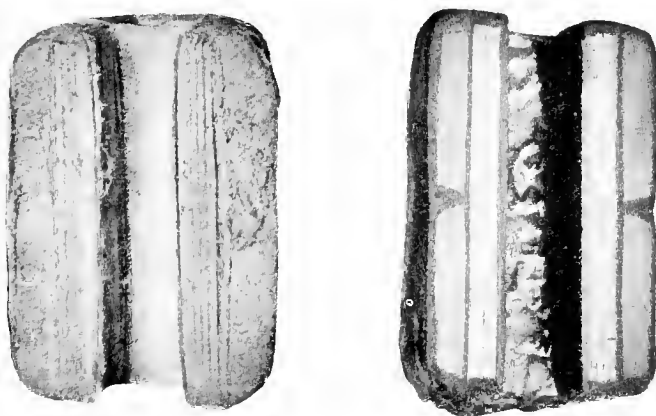


FIG. 9-03.—Compound-Filled and Untreated Coils cut by same Saw. (British Thomson-Houston Company, Rugby.)

¹ The authors are indebted to the various firms mentioned for the drawings and particulars given in this chapter.

millimetre thick are used for the bolts, and washers 3 to 4 mm. thick for the nuts.

Some makers clamp the stampings between suitable castings, as, for



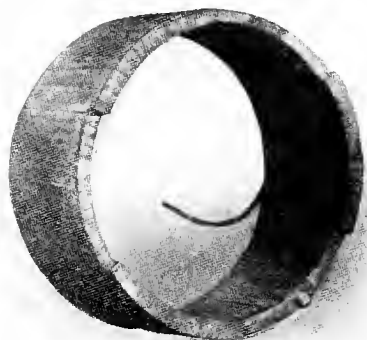
Coil just Wound.



Coil Dipped.



First Taping.

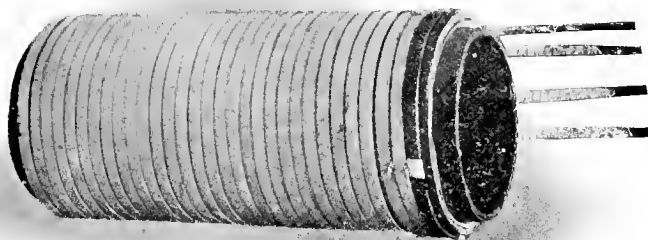


Coil Varnished after Second Taping.

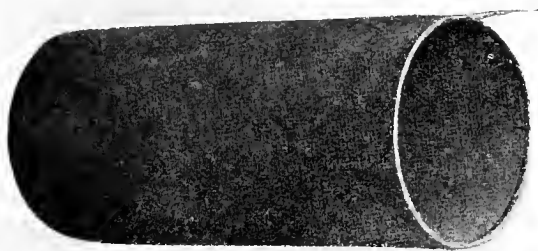
FIG. 9·04.—Construction of Circular Coils. (Brush Company, Loughborough.)

example, in figs. 9·08, 9·10, and 9·17 ; and there is generally a cast base and top, or side brackets, to hold the transformer in place and carry the terminals. All such should be light, strong, and inexpensive. The cost of the patterns is not of great importance when many castings have to be

made alike, but may be a serious item when only one or two transformers of a special type are wanted. In such cases the design should be such as to



Secondary Coil—wound direct on insulation tube.



Insulation Tube—showing earth-shield connection.



Primary Coil—slipped on insulation tube.

require as few patterns as possible, *e.g.* by making one do for top and bottom, and these should be simple.

The circular section of core has the least perimeter for a given cross-section, and therefore requires least copper, but heats most. It is seldom used, owing to the large number of different widths of sheet required.

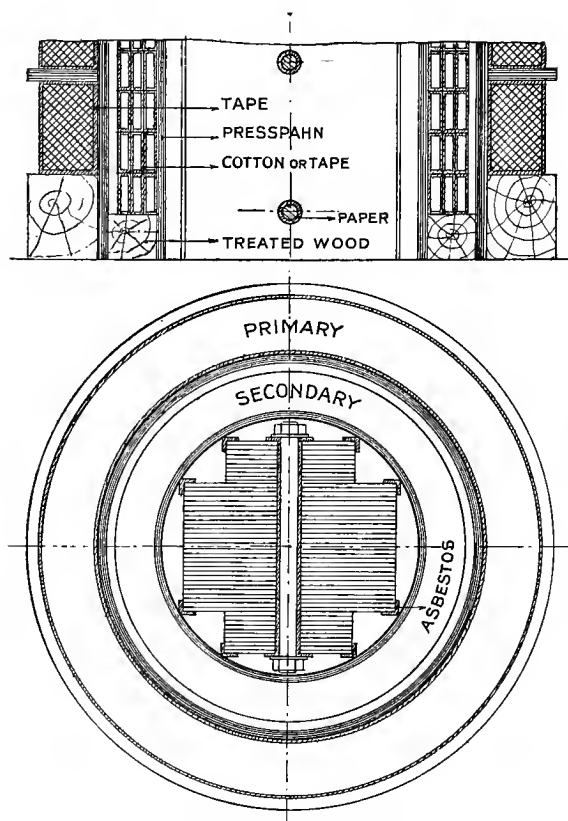


FIG. 9'06.—Insulation of Concentric Coils.

Cross-shaped and rectangular sections are best for simplicity of construction, and have the further advantages of large cooling surface and free ventilation. Ventilating ducts are formed by separating the different packets of sheets by I- or U-shaped brass bars, by small packets of sheet obtained from the stampings, or by blocks of hardwood previously soaked in oil or paraffin wax.

Examples of various methods of forming the core and ducts will be found in the plates at the end of the book. Three-phase transformers are generally made with three cores in line for simplicity of construction. Plate 11 shows

one with symmetrical magnetic circuits placed at the three corners of a triangle.

Coils.—On the whole, concentric windings are to be preferred. They give a better insulation with less material than sandwiched coils, and have less magnetic leakage, unless the subdivision is carried to extreme. In

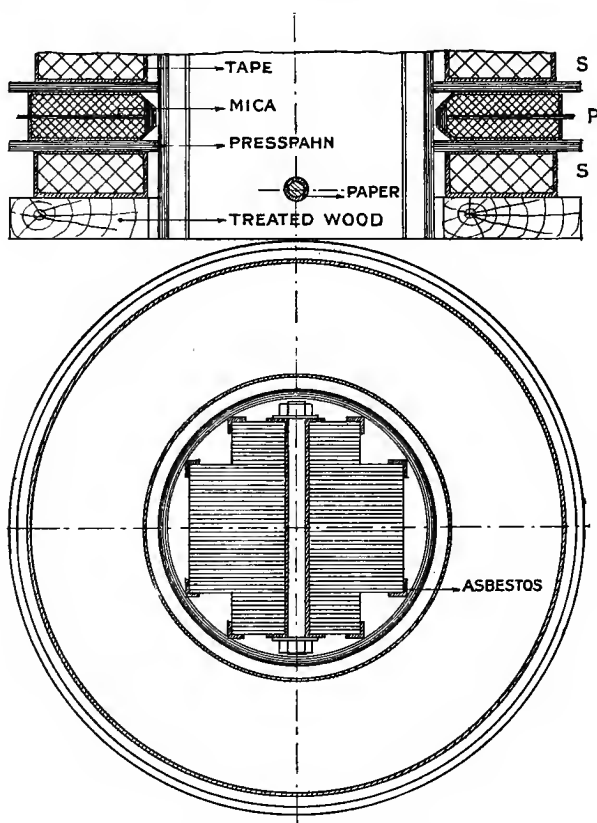


FIG. 9-07.—Insulation of Subdivided Coils.

concentric windings the low-voltage coil is generally put next the core, but sometimes it is split into two parts with the high-voltage winding between. Plate 4 shows a transformer by the Burnand Transformer Company, Ltd., in which this subdivision is carried even further, with the result that the magnetic leakage is negligible and the drop of P.D. almost independent of power factor. It also has an exceptionally short magnetic circuit.

With high voltages the concentric coils are not wound all in one, but are divided into sections, whose E.M.F. seldom exceeds 1000 volts, so as to

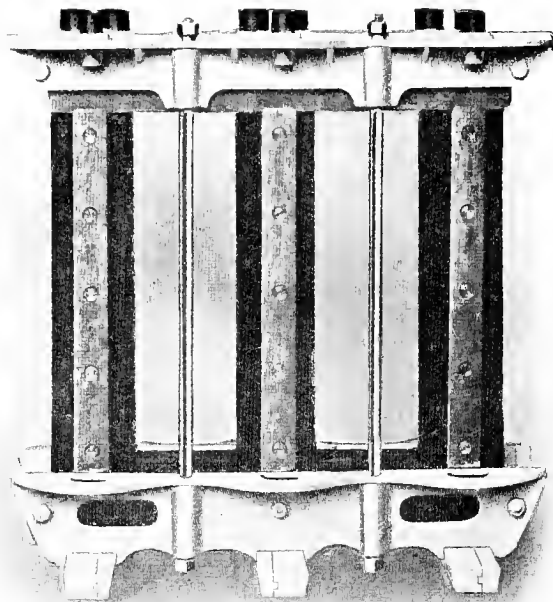


FIG. 9-08.—Core for Three-Phase Transformer. (Brush Company.)

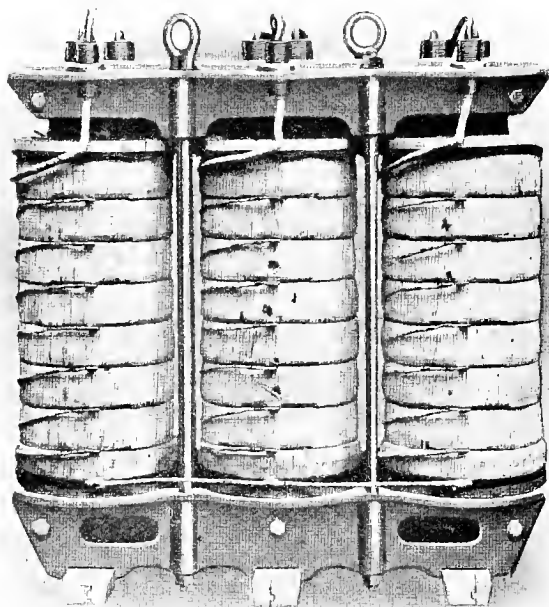


FIG. 9-09.—Three-Phase Transformer without Case. (Brush Company.)

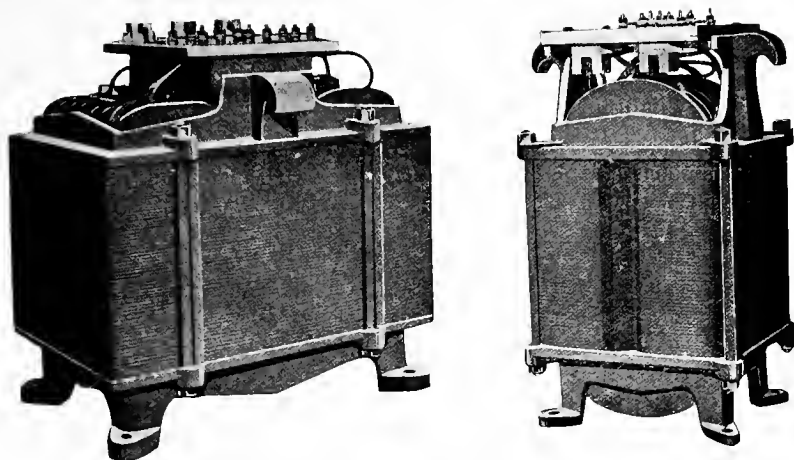


FIG. 9'10.—Completed Shell Transformers without Case. (Elektricitats Aktien Gesellschaft, Köln.)

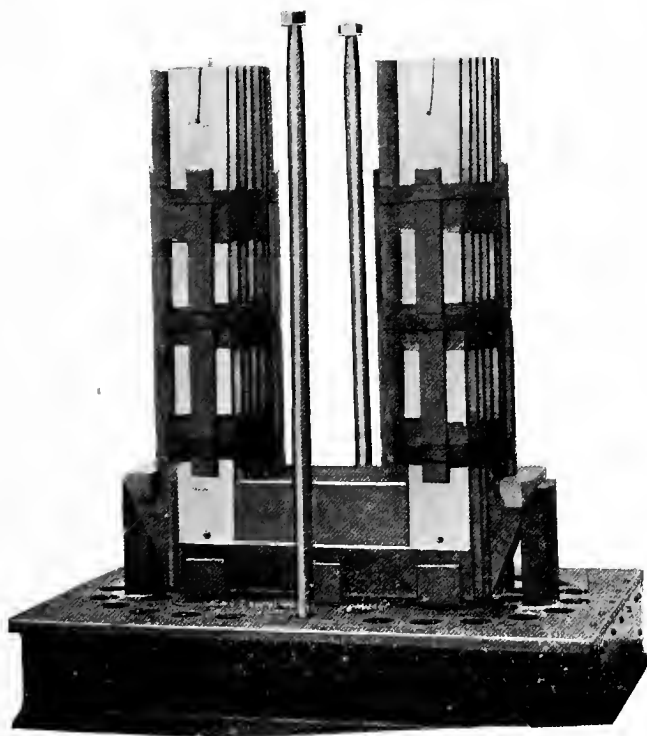


FIG. 9'11.—Cores ready for Coils. (Ganz.)

reduce the P.D. between adjacent wires in adjacent layers. This P.D. is generally under 100 volts, and should never exceed 350 volts. This is very important if the winding is liable to be subjected to sudden surges, such as are caused by lightning or the stoppage of a short-circuit. Owing to their high inductance, the propagation of a sudden disturbance through the windings is much slower than through the external mains, and consequently the P.D. due to the sudden arrival of a large charge is concentrated on the

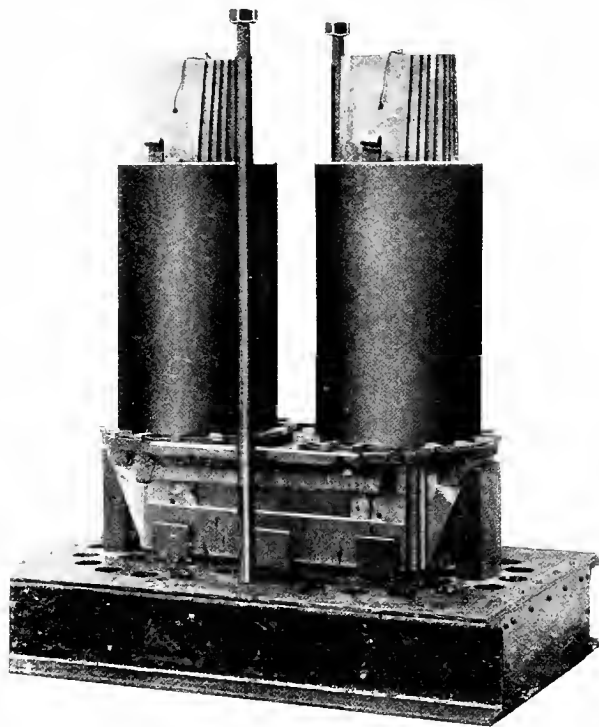


FIG. 9-12.—Transformer with Low-Voltage Coils in Place. (Ganz.)

first few turns, and may cause a breakdown of the insulation between them. The coils next the terminals of a high-voltage winding should thus be specially well insulated.

It is advisable to use circular wire up to 40 sq. mm. (0.06 sq. in.) as it is easy to get and easily wound. For larger cross-sections, flat strip or square conductors are employed, two or more being often joined in parallel. To avoid eddies in these strips, due to their being unequally acted upon by the flux, the inner and outer conductors should be interchanged half-way up the core. The wire is usually insulated by two layers of cotton laid spirally

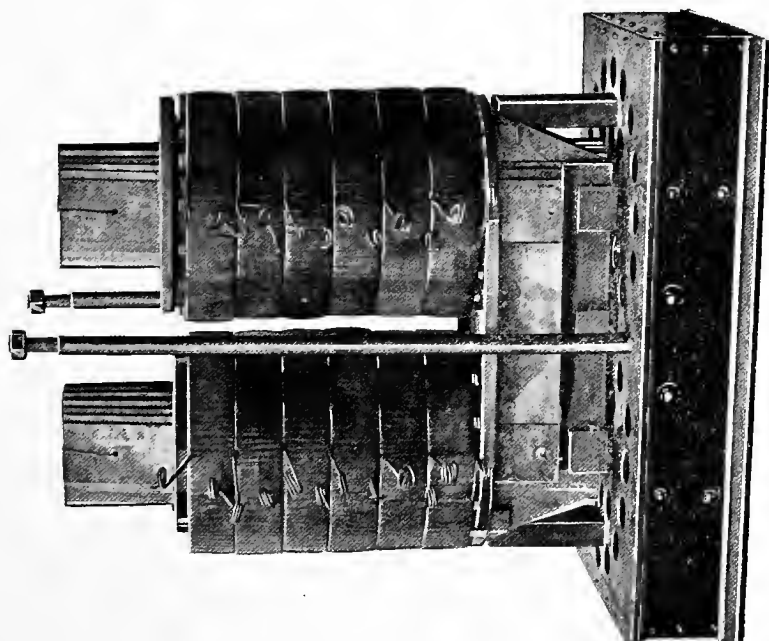


FIG. 9-13.—Transformer with High-Voltage Coils in Place. (Ganz.)

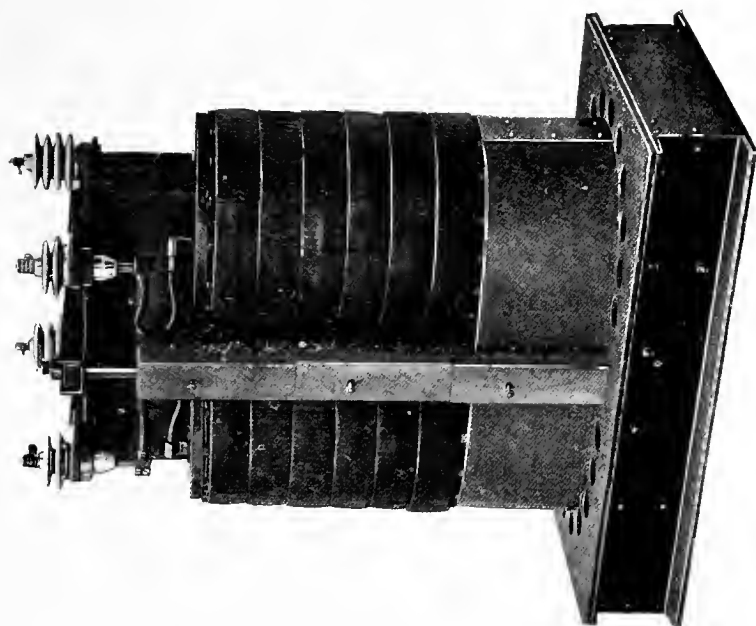


FIG. 9-14.—Transformer Complete without Case. (Ganz.)



FIG. 9.15.—Transformer Complete. (Ganz.)

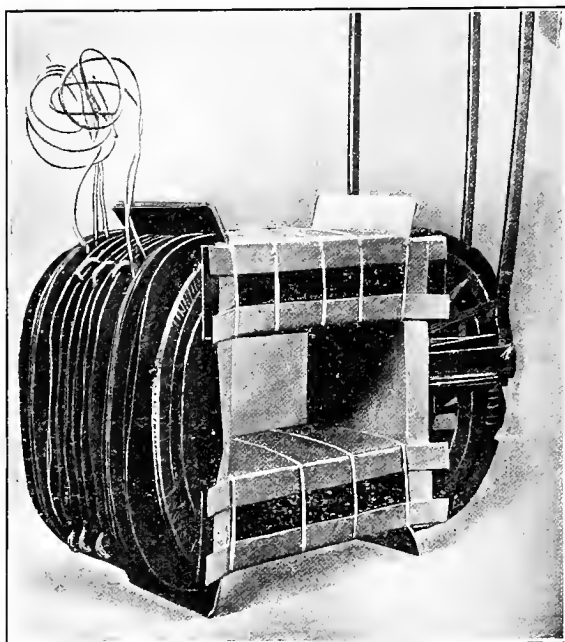


FIG. 9.16.—Sandwiched Coils ready for Core. (Westinghouse.)

round it in opposite directions ; but sometimes there is only one, and sometimes three. The larger wires and strips may be covered with braided thread, with cotton or linen tape, or, if oil insulation is to be used, with

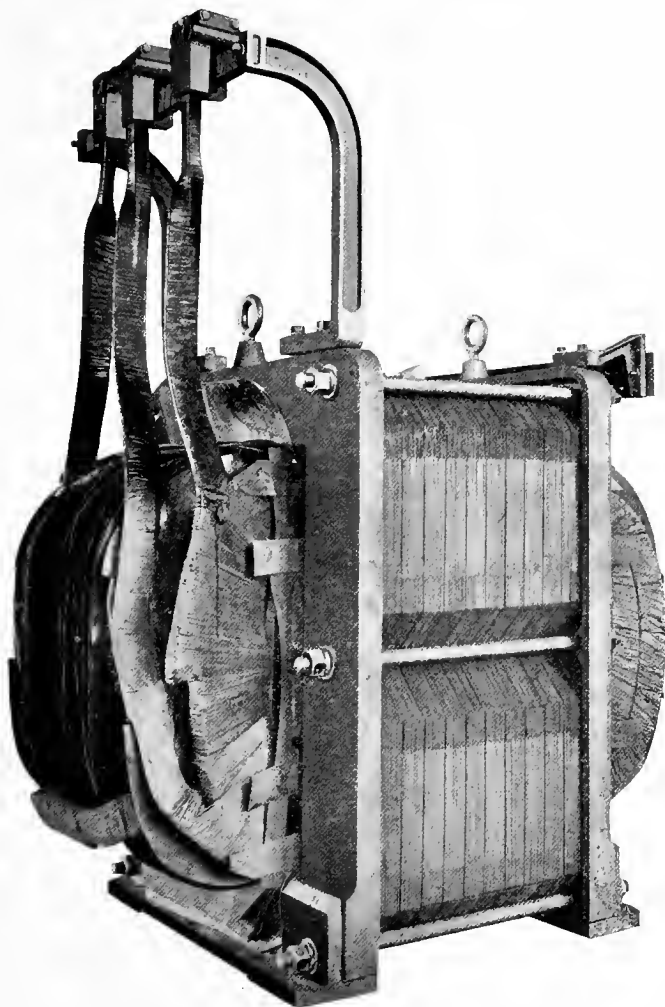


FIG. 9·17.—Single-Phase Shell Transformer without Case. (Westinghouse.)

paper strip. Extra insulation of Manilla paper or presspahn is put between the layers, and it should project beyond the wire, as shown in fig. 9·01.

In the larger sizes of transformers with subdivided windings, the coil is often made by winding a strip of copper and of insulating material together on a former ; such a coil is shown in fig. 9·02, and is generally known as the

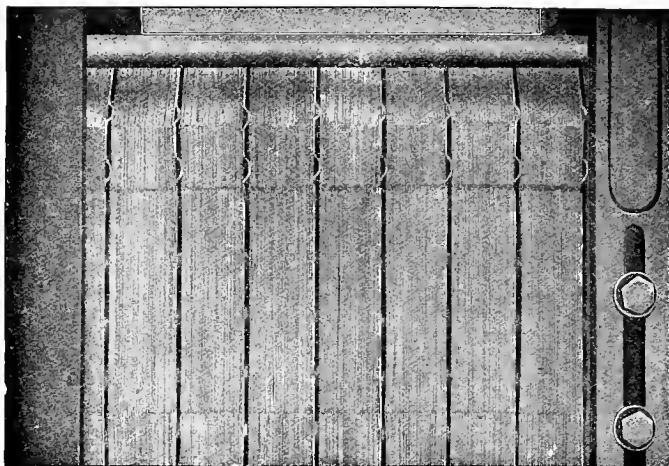


FIG. 9-18.—Cooling Ducts in Core of Westinghouse Transformer.

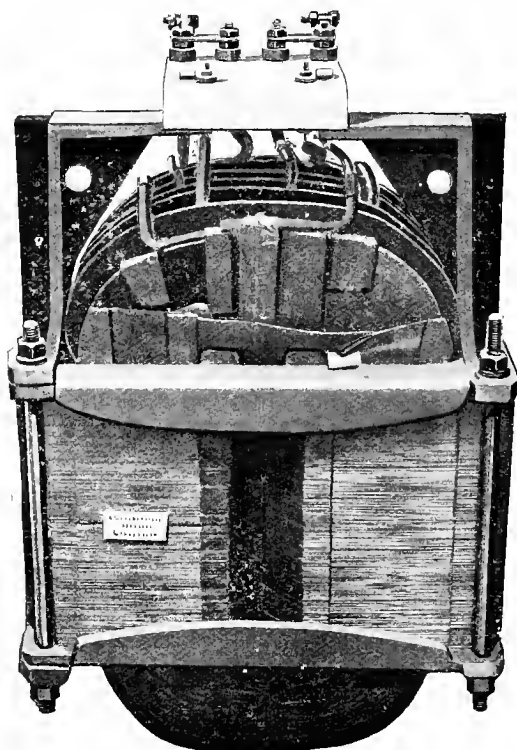


FIG. 9-19.—Small Size O. D. Transformer. (Westinghouse.)

"pancake" coil. It has the advantage of reducing the P.D. between any contiguous conductors to the E.M.F. of one turn only.

After a coil is wound, many manufacturers dry it in a vacuum oven and then impregnate it with a special compound having asphalt and coal-tar, or linseed oil, as base, depending on whether it is to be air- or oil-insulated. This improves the insulation, excludes moisture, facilitates the escape of heat, considerably increases the mechanical strength, and stops any humming of the

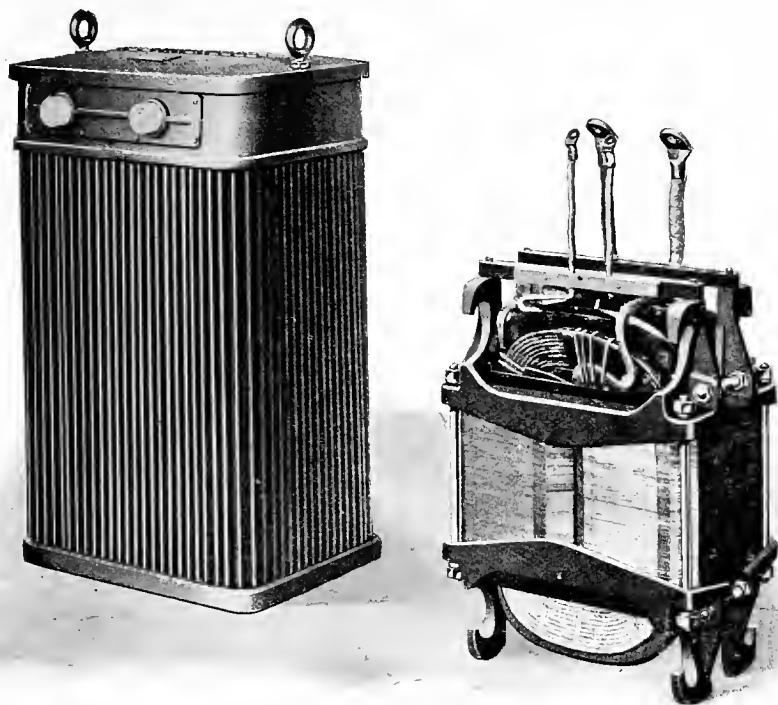


FIG. 9-20.—Large Size O.D. Transformer with Oil-Tank. (Westinghouse.)

coils. Fig. 9-03 shows sections of treated and untreated coils cut through by the same saw. The coils generally receive one or two layers of tape, which gives some mechanical protection but impedes the flow of heat, and they are varnished outside. Fig. 9-04 shows the various stages in the manufacture of a circular coil.

Insulation of Coils and Core.—Concentric coils are insulated from one another and from the core by seamless tubes of presspahn, mica-paper, or micanite, and an extra air gap is often left as well. Sometimes a metallic earthed screen is put between the primary and secondary to prevent any high-voltage current getting to the low-voltage winding by leakage there,

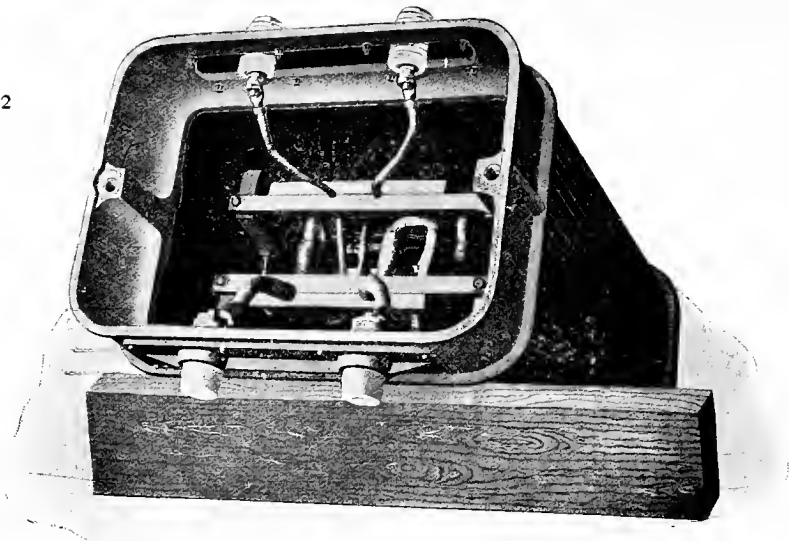


FIG. 9-21.—Top View of O.D. Transformer. (Westinghouse.)

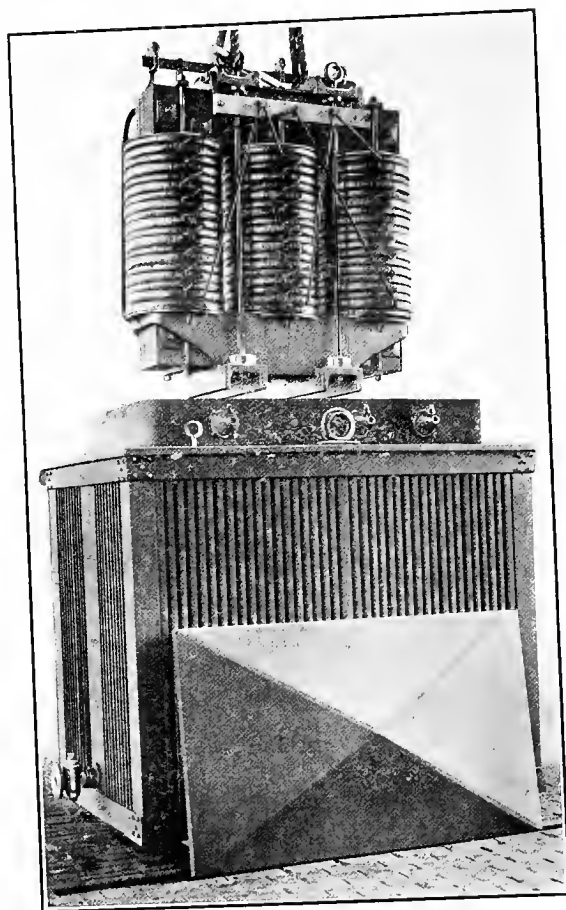


FIG. 9-22.—Three-Phase Transformer and Oil-Tank. Westinghouse.)

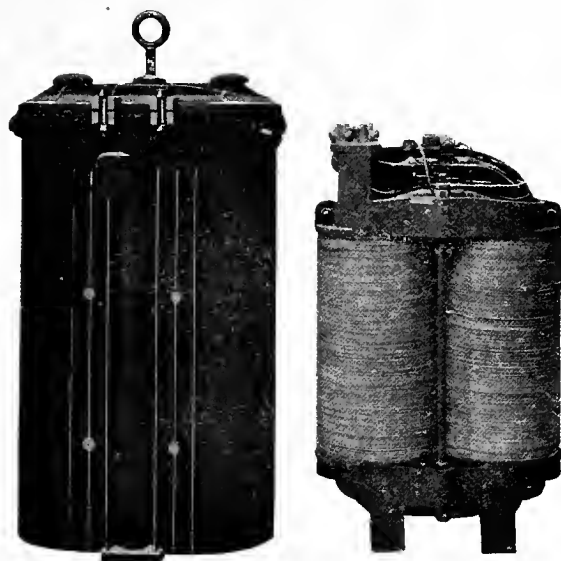


FIG. 9-23.—Small Single-Phase Transformer and Tank. (Electrical Construction Company, Wolverhampton.)

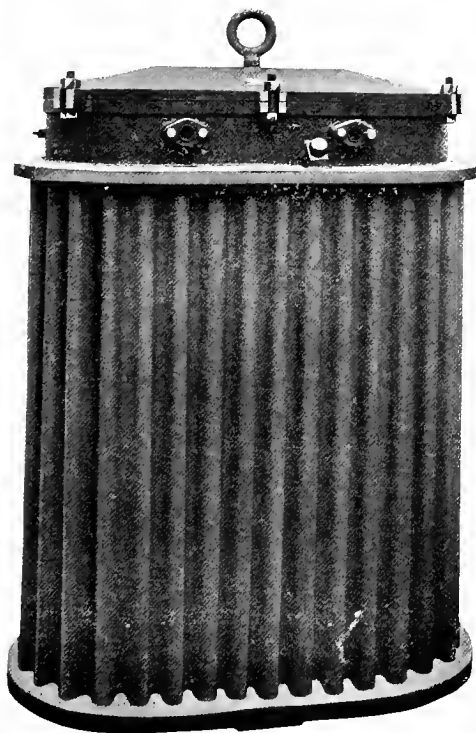


FIG. 9-24.—Corrugated Cast-Iron Oil-Tank. (Electrical Construction Company, Wolverhampton.)

but it is not of very much use, as connection is just as liable to be made outside. Such an earth shield is shown in fig. 9-05.

In subdivided windings the sections may be separated by discs of presspahn, moulded insulation, or micanite, or, better still, from the ventilation point of view, by distance pieces of these materials or of treated hardwood. See, for examples, figs. 9-06, 9-07, and 9-30.

The tube surrounding the core is sometimes replaced by oiled linen or sheets of presspahn, but this is not advisable for subdivided coils.

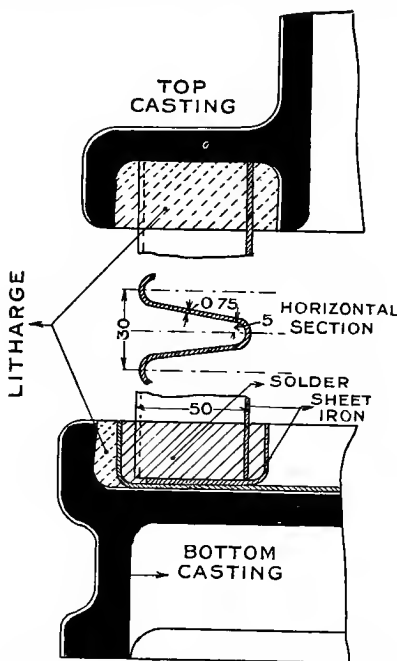


FIG. 9-25.—Joints between Corrugated Sheets and Cast-Iron Top and Bottom.
(Société Anonyme, Charleroi.)

Sufficient space should be left between the windings and the yokes, especially for the high-voltage coil. At the bottom, the coils are supported on porcelain or treated wood. The space between the coils and the yokes at the two ends of the core may be filled with presspahn or treated wood, or it may be left as an air or oil space. The length of creeping surface with presspahn may be increased by staggering the sheets at their edges. The sharp corners of the core should be well covered with pieces of asbestos or presspahn to keep them from doing damage.

The terminals are generally mounted on porcelain insulators carried on the framework, but prepared teak may be used if the voltage is low.

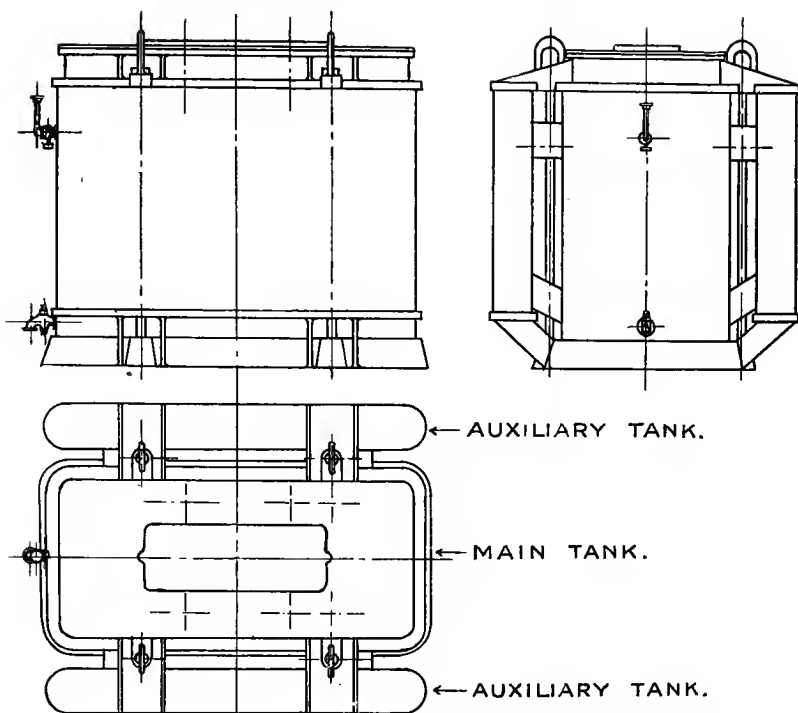


FIG. 9-26.—Oil-Tank with Auxiliary Side Vessels. (Allgemeine Electricitäts Gesellschaft, Berlin.)

Casing-Oil-Cooling.—The external case is largely determined by the conditions under which the transformer is to be run. It may be a mere protection for the windings, as the planished steel covering of fig. 9-15, or a watertight cast-iron tank like 9-20, 9-22, 9-23, or 9-24, which is generally corrugated outside to facilitate the dissipation of heat. In small sizes it may or may not be filled with oil, but in larger sizes oil is generally necessary to keep down the temperature. Oil is also necessary for small transformers with very high voltages to give sufficient insulation, as ozone and nitric acid are formed in atmospheric air by the small discharges that accompany high electrostatic stresses. Oil insulation is generally employed for E.M.F.'s over 15,000 volts, but air insulation can be used up to twice that figure.

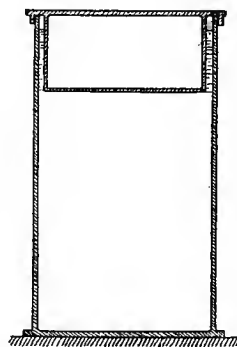


FIG. 9-27.—Transformer Case with Cooling Lid. (Lahmeyer.)

In large sizes, sheet-metal cases with cast-iron tops and bottoms may be employed instead of cast-iron cases. Great care must then be exercised in

making the joints, which are subjected to great strains by the continual expansion and contraction with temperature fluctuations. A section through such a joint is shown in fig. 9-25.

The emission of heat may be further accelerated by supplying the main vessel with special side tanks, such as shown in fig. 9-26. The auxiliary

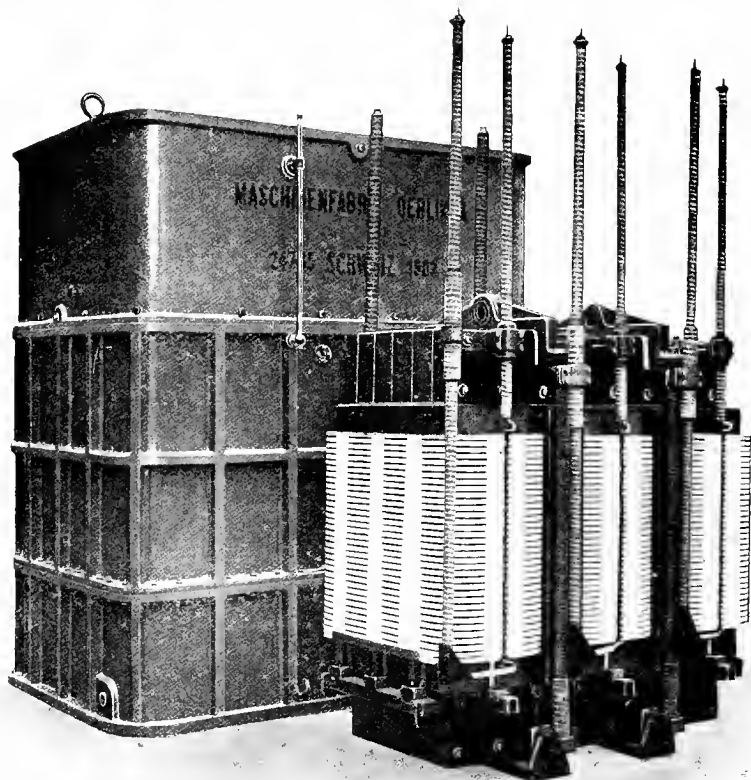


FIG. 9-28.—Three-Phase Transformer with Water-Jacketed Case for Caffaro-Brescia Transmission. (Oerlikon.) 2720 K.V.A. ; 9000 to 40,000 or 10,500 to 46,000 volts ; 42 C.P.S.

tanks are joined to the main tank by pipes near the top and the bottom and promote the circulation, since they keep cooler than the main tank. Fig. 9-27 shows another interesting method.

Water-Cooling.—A transformer case with a water-jacket is illustrated in figs. 9-28 and 9-29, its construction being such that the water is forced in serpentine fashion between the double walls of the top casting.

Very often the cooling water is passed through a serpentine pipe immersed

in the oil above the transformer. This coil is generally made of seamless

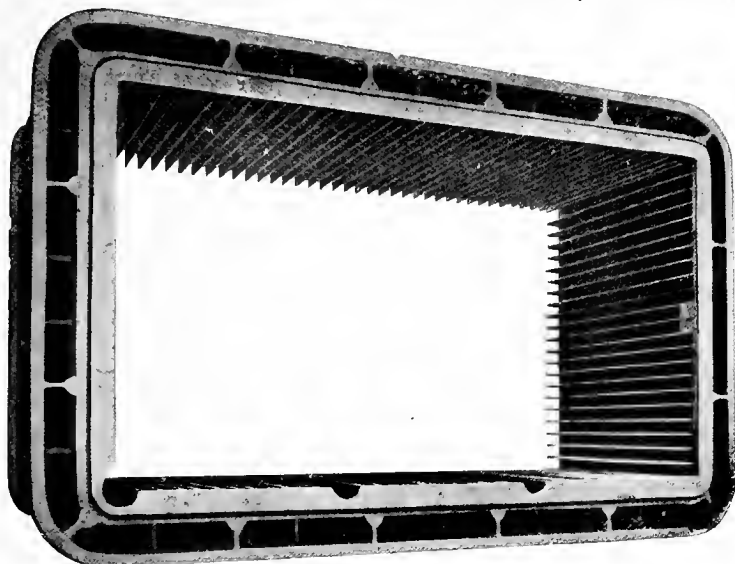


FIG. 9-29.—Water-Jacketed Upper Case of Oerlikon Transformer.

brass or copper tubing with all its joints brazed. In large transformers several coils may be joined in parallel.

Since the entering water-pipe is colder than the atmosphere above the

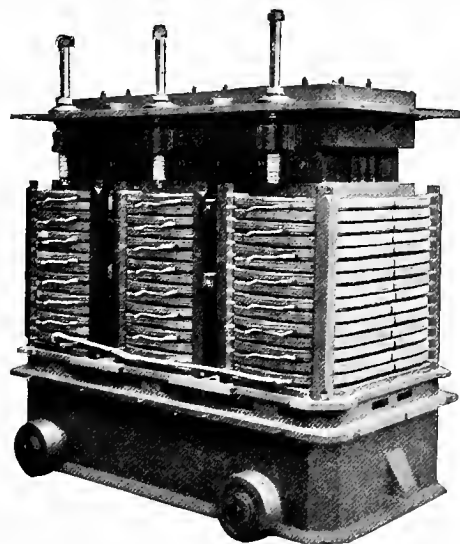


FIG. 9-30.—Oerlikon Three-Phase Transformer removed from Case.
(Drammen Transmission.)

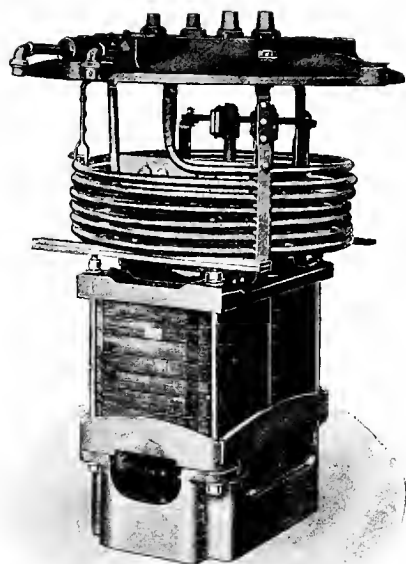
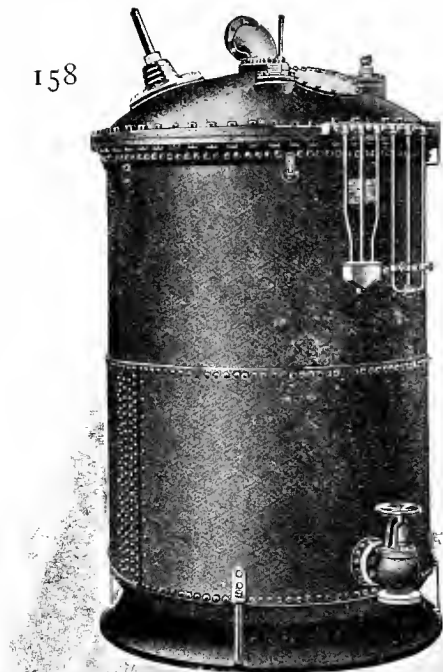


FIG. 9'31.—Oil-Cooled Transformer with Water-Cooling Coil. (Westinghouse.)

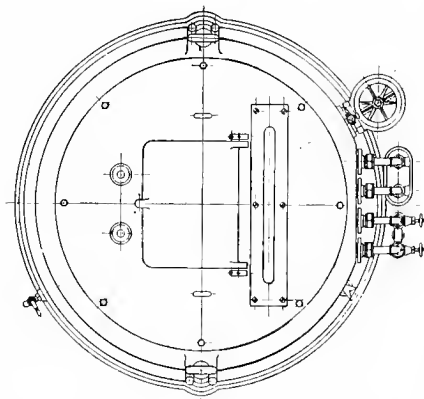
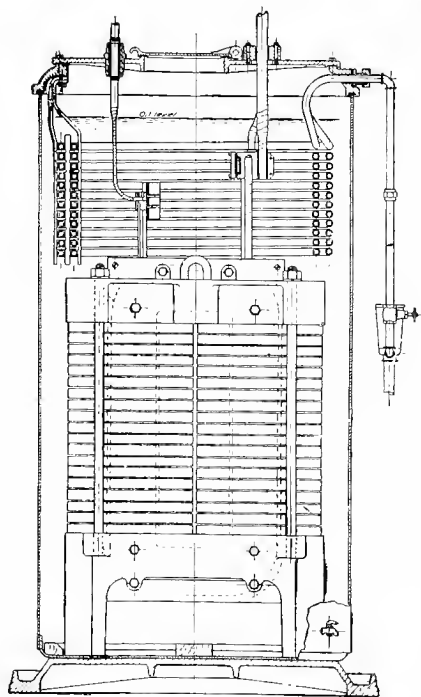


FIG. 9'32.—Oil-Cooled Transformer with Water-Cooling Coil. (Westinghouse.)
General Arrangement.

oil, water may be condensed on it and run down into the oil. To prevent this, the pipe should be heavily insulated above the oil, and care should be taken that the oil does not sink below the insulation.

The ventilating ducts of oil-cooled transformers should be made wide enough not to be choked by any deposit from the oil. The cross-shaped

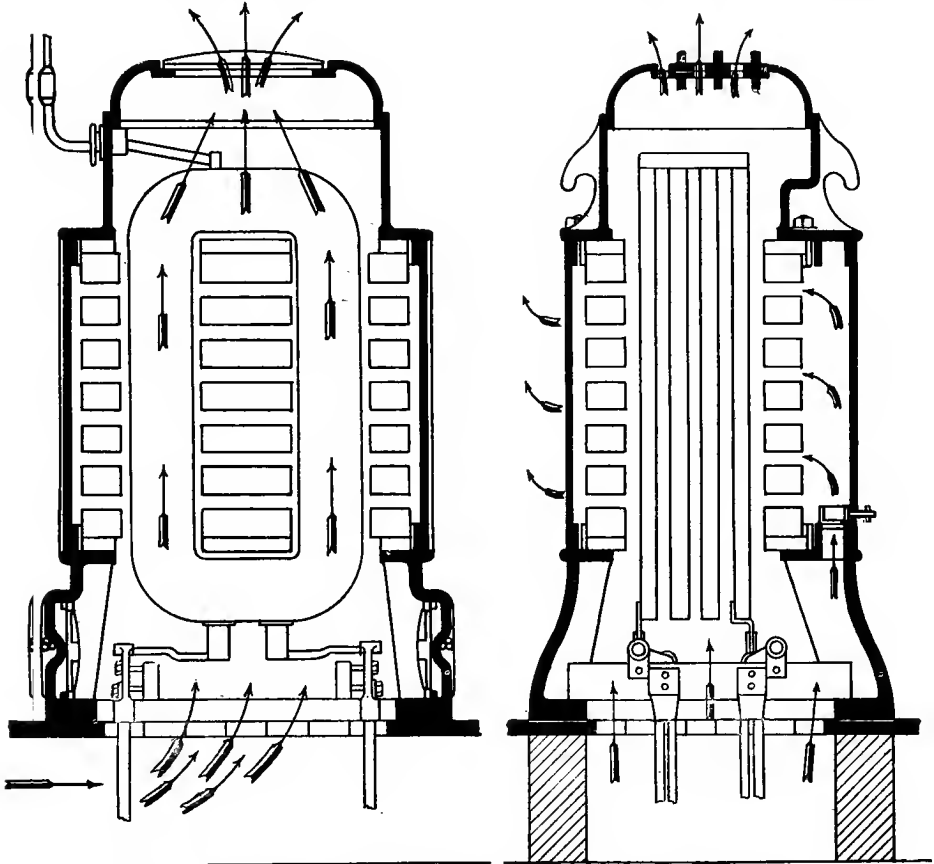


FIG. 9-33.—Transformer with Forced Draught. (General Electric Company, U.S.A.)

core is especially suitable in this case, since it provides ducts of exceptional width.

Forced Draught Cooling.—The use of forced draught originated with the General Electric Company of Schenectady, U.S.A., but it is now extensively employed all over the world. The principle of the method is illustrated in fig. 9-33, in which the course of the air through the cores and windings is clearly indicated. The iron and copper are separately cooled. The air for

ventilating the windings passes up through the transformer between the coils, which are held apart by spacing strips, and discharges through an opening at the top of the machine. This opening is provided with a damper for regulating the amount of air passing through the coils. The air for cooling the iron passes from the lower housing through a damper at one side of the transformer, and then horizontally through ventilating ducts spaced at frequent intervals in the iron. The coils are so proportioned that even in the

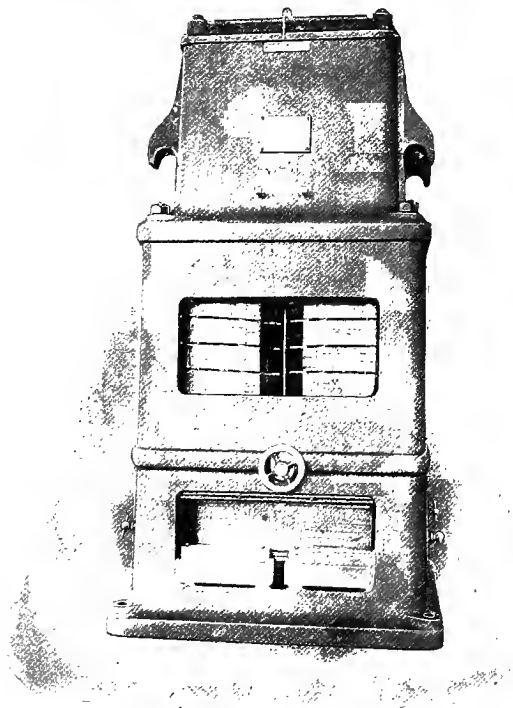


FIG. 9-34.—Transformer with Forced Draught. (Westinghouse.) Outside View.

largest sizes no part of the electric conductor is more than 15 mm. ($\frac{5}{8}$ in.) from an air duct, and the spacing strips which separate the coils from one another and from the core are on the narrow edges of the coils. Almost the entire surface is therefore exposed to a blast of cold air maintained by a ventilating fan.

In the Kolben three-phase transformers shown in figs. 9-36 and 9-37 the blower is operated by a three-phase motor with a squirrel-cage rotor fed from the low-voltage side of the transformer and rated at about $\frac{1}{2}$ b.h.p. The efficiency of the transformer is thus affected very little by the power absorbed by the blower equipment. The arrangement of the delivery pipes and the

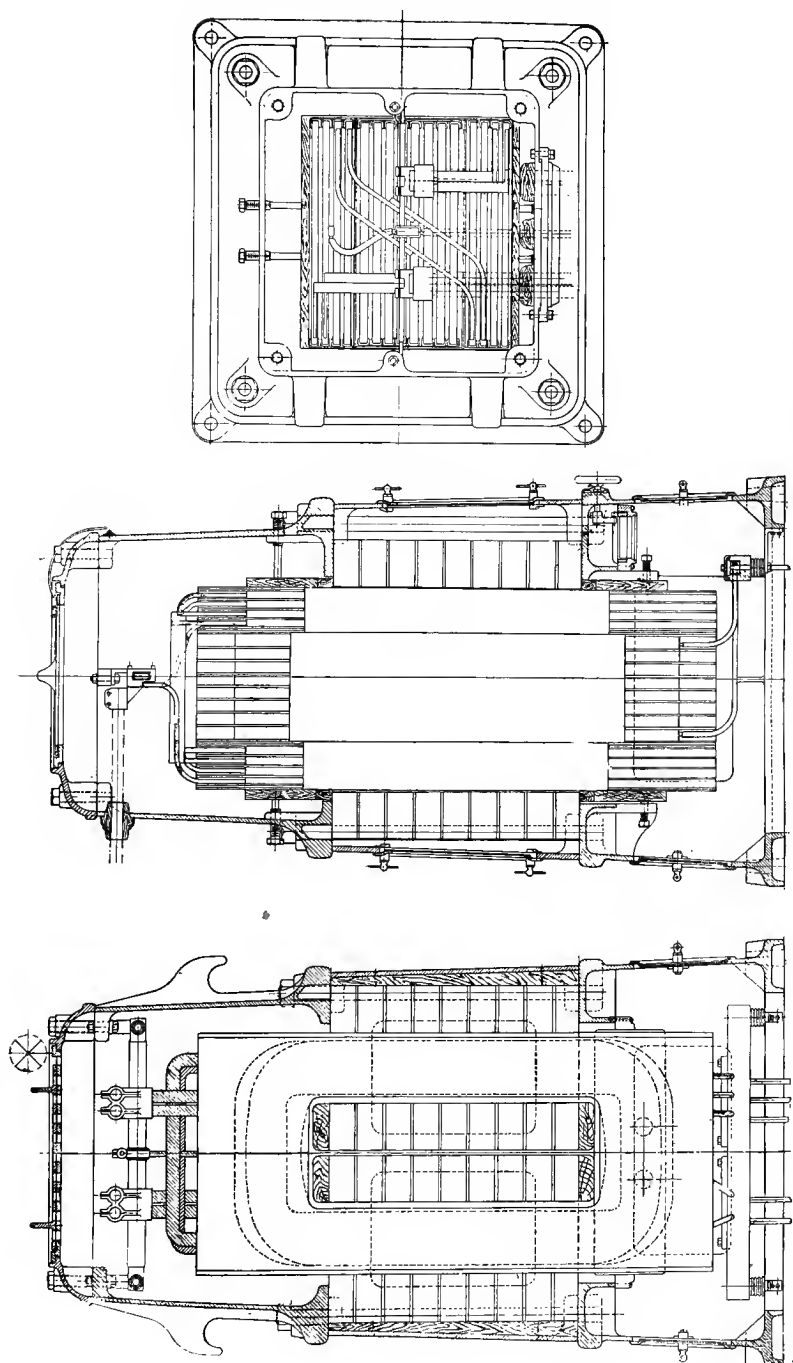


FIG. 935.—Transformer with Forced Draught. (Westinghouse.) General Arrangement.

velocity in them are so fixed that each transformer gets the same quantity of air, and therefore has practically the same temperature rise. This is of

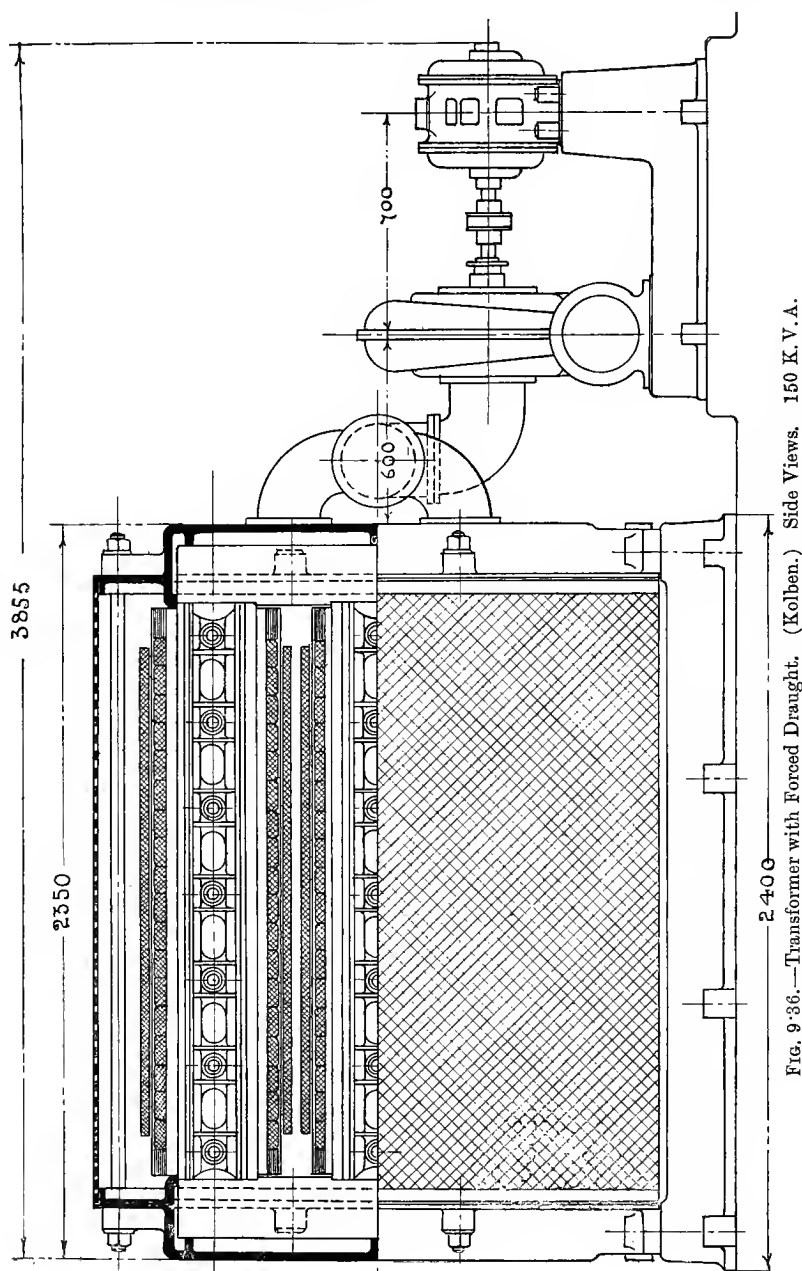


Fig. 9-36.—Transformer with Forced Draught. (Kolben.) Side Views, 150 K. V. A.

great importance whenever the transformers have to be worked in parallel in order that all may have the same voltage drop, as otherwise the load would be unequally distributed. The transformers are covered by a perforated

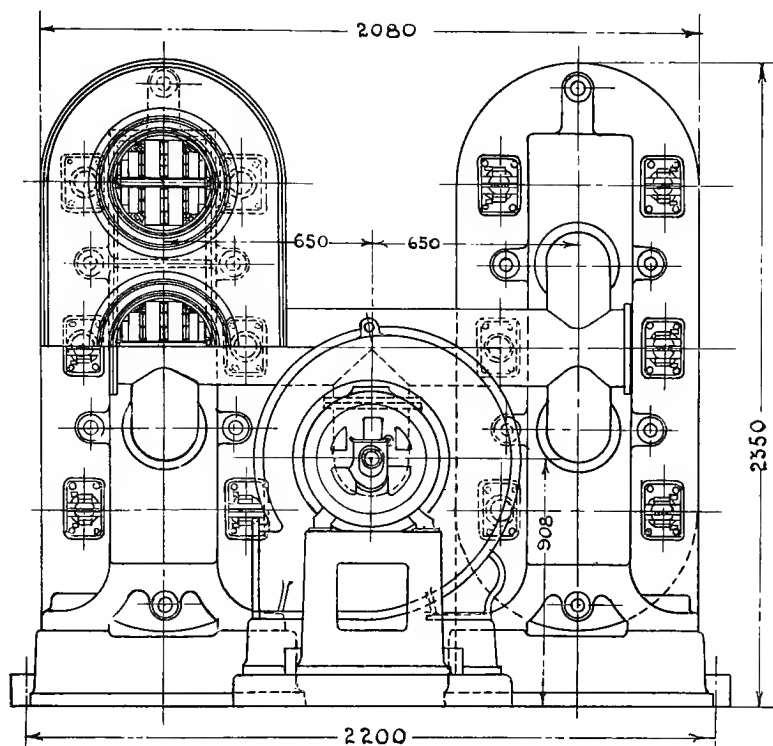


FIG. 937.—Transformer with Forced Draught. (Kolben.) End Views.

sheet-metal cover through which the hot air gets away. When more than two transformers are to be operated from a common blower, an air-valve is placed in each delivery pipe so that the air supply to each transformer can be regulated.

CHAPTER X.

DESIGN OF TRANSFORMERS.

General Considerations.—The essential things to be borne in mind when designing a transformer are (1) output ; (2) efficiency ; (3) temperature rise ; (4) voltage drop ; (5) magnetising current ; (6) reliability ; and (7) cost. The first four or five will probably have to comply with a specification, and the last has to be as small as is consistent with the others.

We have already seen in connection with equations 3·37 and 3·38 that the efficiency of a transformer remains practically constant when the current and P.D. are both increased or diminished in the same ratio. It follows from this that any output can be obtained from a given transformer working at any given efficiency not greater than its maximum, by making a suitable choice of the ratio between P.D. and current. We thus have the rather unexpected result that the dimensions of a transformer depend on its maximum efficiency rather than on its output. Of course, the losses and the rise of temperature they produce increase with the load, and a limit will be reached beyond which it is unsafe to raise the load for fear of burning out the coils. By employing oil or forced-draught cooling this limit may be raised, and by using oil-and-water cooling it can be still further increased. When our resources in the way of improving the facilities for the escape of heat are exhausted without being able to keep down the temperature to the required extent, it will be necessary to make a larger transformer having a higher maximum efficiency than that specified. It will be best to design it so as to have that efficiency at the maximum load in order to keep the losses as small as possible.

If the method of cooling be not specified, it should not be forgotten when approaching the limit of any particular way that a larger transformer of higher efficiency and consequently having less heat to get rid of, while also having more surface from which to emit that heat, may cost less than a smaller and less efficient transformer which requires a more expensive method of cooling in order to keep the temperature rise within the required amount.

Rather less active material is required by oil-cooled types than by air-cooled ones of the same rating, owing to the fact that they may be run at a lower efficiency without getting too hot. On the other hand, the casings of oil-cooled transformers are generally the more expensive. It is generally economical to employ oil-cooling above 50–80 K.V.A., and forced-draught or oil-and-water cooling above 250 K.V.A.

The temperature rise must be kept low enough to ensure the durability of the insulation (see Table 4·03), and to cause little ageing of the iron. For the latter reason the temperature of the iron should not be allowed to exceed 90°C. (190°F.). The lower the temperature is kept, the higher will be the efficiency owing to the rise of resistivity of copper with rise of temperature.

Choice of Most Efficient Load.—It is not necessarily best to design the transformer to give its maximum efficiency at its rated full load. A lighting transformer would generally only be lightly loaded during the greater part of the twenty-four hours, and fully loaded during a few hours each day in the winter time and not at all in the summer. An improvement in the light-load efficiency at the expense of that at full load would therefore cause a reduction of the total loss of energy during the year. The iron losses are approximately constant at all loads, and the copper ones are proportional to the square of the current. Consequently, the mean loss will depend on the mean square of the load, and the minimum loss of energy will be obtained if the efficiency of the transformer is greatest for the root mean square of the load. If the load-factor be very low it may not be practicable to design it for a maximum efficiency at a load as small as this, owing to the overheating of the coils which would then occur at full load.

A minimum waste of energy does not necessarily correspond with a maximum of economy, for energy lost at times of full load is worth two to ten times as much as the same amount lost at times of light load. That lost when the load is small only involves a little extra expenditure for coal and water; but that lost during the peak of the load requires as well additional capacity in the boilers, engines, generators, switchboard, mains, etc., and must therefore bear its share of the capital charges. It will be found that it is as a rule best to design so as to get the best efficiency for very nearly the maximum load it will generally have to carry. The load on the transformers can be kept up when they are grouped in sub-stations by cutting some out when not required, and this has the further advantage that they are cool when required at the time of peak load.

Since the magnetic leakage may be made as small as we please by suitably dividing and arranging the windings, the voltage drop is chiefly dependent on the resistance of the windings, especially with high power-factors. If the

percentage voltage drop desired be less than half the permissible percentage loss, it will be necessary to use more copper than would be required to give the specified efficiency. In such a case, the regulation required will determine a maximum resistance and copper loss, irrespective of the total losses, and the iron losses will be the difference between that and the total permissible losses.

The maximum efficiency will then exceed the specified efficiency, and will occur for such an overload as will make the copper losses equal to the iron ones. At this maximum efficiency,

$$P_I = RI_M^2 \quad \text{or} \quad I_M = \sqrt{P_I/R}.$$

$$\text{Hence} \quad \eta_M = \frac{VI_M \cos \phi}{VI_M \cos \phi + P_I + RI_M^2} = \frac{1}{1 + \frac{2P_I}{VI_M \cos \phi}} = \frac{1}{1 + \frac{2\sqrt{RP_I}}{V \cos \phi}} \quad 10.01.$$

Since R is limited by the specified voltage drop, the smallest value of η_M , and consequently of the cost, is that given by taking the iron loss as high as is consistent with the efficiency required at the given load; that is, by following the rule just given.

Most Economical Efficiency.—Since the cost of a transformer is largely determined by its efficiency, it is desirable to see whether extra efficiency is worth paying for or not. For instance, in order to increase the efficiency of a 50 K.V.A. transformer from 97 per cent. to 98 per cent., the cost of its active material is raised from about £25 to about £75. If this results in an annual saving of only £1 or £2 in energy saved (including the capital charges on the generating plant), the extra £50 can hardly be said to have been well spent. But if the saving be £10 or £20 per annum, then the extra efficiency would be a good investment.

In order to settle this point, it would be necessary to know the load curve which the transformer is likely to have. From that the energy lost in a year can be computed for the two transformers, and the value of this, including running costs only, estimated. If this exceeds the capital charges on the difference between the prices, minus any saving in other apparatus due to the diminished station output for the same load, then it is most economical to employ the larger transformer. If not, the smaller one would pay better. By doing this for a few efficiencies, and plotting the results, the best efficiency can be determined.

It is, however, very seldom that the load curve can be sufficiently closely estimated beforehand to make such a calculation of any value. Consequently the efficiencies which have been more or less standardised by the makers, and which are largely determined by temperature rise, are generally accepted.

Figs. 10·01 and 10·02 give typical values from the practice of first-class makers.

Assumptions and Data.—Only a few simple assumptions are made in the theory here given. These are that:—

- (1) The flux density is uniform throughout the iron.
- (2) The current density is uniform throughout the copper.
- (3) The iron losses for the given frequency are proportional to the square of the flux density.

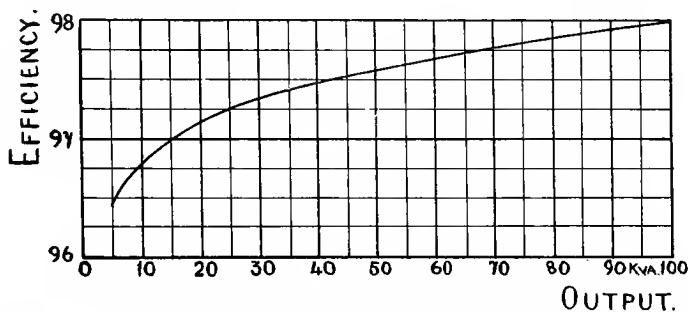


FIG. 10·01.—Usual Efficiencies for Transformers with Natural-Draught Cooling.

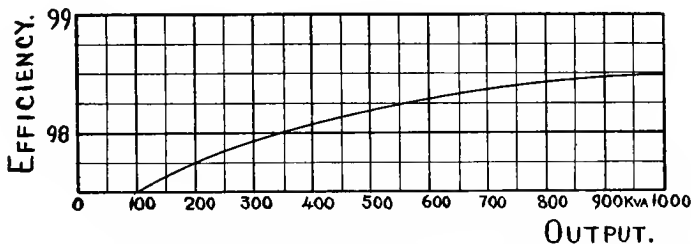


FIG. 10·02.—Usual Efficiencies for Transformers with Forced-Draught or Water Cooling.

- (4) The copper losses are proportional to the square of the current density.
- (5) The variable part of the cost can be divided into two parts, of which one is proportional to the amount of iron, and the other to the amount of copper.
- (6) The space-factors can be sufficiently accurately estimated beforehand to fix the specific cost ratio.

With regard to the first, which neglects the variations at the corners, it is fairly well known that the cost can be diminished by reducing the cross-section of the wound cores while increasing that of the unwound yokes so as

to keep the losses constant. To that extent it will be possible to make even cheaper transformers for a given efficiency than those designed by the method here discussed, but it will be found that unless the efficiency required is higher than usual for the output, such designs will have excessive magnetising currents and local heating in the wound cores. With high iron space-factors the method here explained gives quite as high a magnetising current as is desirable, and it may be found advisable for this reason to spend part of the saving by using the cheapest proportions in obtaining a higher efficiency than usual. With alloyed irons it will always be necessary to do so.

In any case, the cheapest transformer with uniform flux density will serve as a starting-point for the application of the designer's art, and give a definite standard with which the results of his modifications can be compared.

The second assumption calls for no remark, but the next is the only one which differs sensibly from the truth, although, unless the flux density be low, it is not very far out. It is true for the eddy current component of the loss, but not quite for the hysteresis loss.

The specific cost ratio is the ratio of the cost of equal *volumes* of the copper and iron *spaces*, including the labour of assembling them. It depends on the market price of the materials, the labour expended in preparing and assembling them, and on the ratio of the space-factors. Each manufacturer knows what these come to in his own works, but, in the absence of other data, the cost ratio for equal *net volumes* may be taken as 3·5 for ordinary iron and 2 for alloyed iron.

It is interesting to note that if the ratio of weights (8 : 7 for the solid metals) be taken instead of the ratio of costs, the *lightest* transformer for the given efficiency will be obtained. This may be of importance in connection with electric locomotives, portable transformers, and other cases where weight is a disadvantage.

The rated voltages, currents, and frequency are supposed to be specified, and also the copper and iron losses. The temperature rise only affects the design indirectly by setting a lower limit to the amount of ventilation required and to the efficiency permissible with a given rating.

Iron and Copper Spaces.—For the purposes of the theory, the transformer is supposed to consist of interlinked iron and copper spaces, each of which fills the entire opening in the other. It is these spaces that are shown in the illustrations, and whose dimensions are obtained from the formulæ. The cross-section of the iron space is a circle or rectangle according to the shape of the coils, while that of each portion of the copper

space is a rectangle for the ordinary types and a triangle for ring transformers.

Actually, these spaces are only partly filled with metal, and that is taken account of by the space-factors. The space-factors here dealt with are the gross ones obtained by dividing the net metal section by the gross section of the iron or copper space. At first sight it might appear that if we take the proper space-factors, the arbitrary boundary between the iron and copper spaces might be drawn anywhere in the space separating the two metals. Further examination will show that the boundary must be drawn so as to make the mean turn actually come where it is assumed to be when reckoning its length, namely, in the middle of the arbitrary winding space with cylindrical coils, and one-third up with the pyramidal coils required for ring transformers. In particular, it should be noted that the clearance

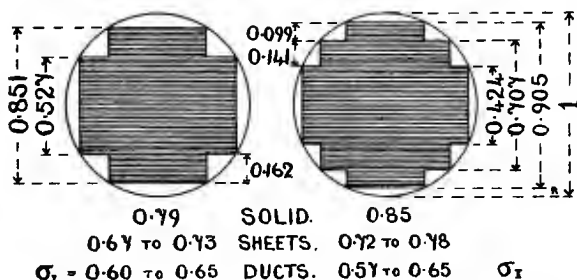


FIG. 10-03.—Proportions of Cross- and Three-Step Cores for Maximum Iron Content.

usually left between the coils and the yokes belongs to the copper space, while that at the short sides of rectangular coils belongs to the iron space.

The iron space-factor depends on the shape of the core, the thinness of the sheets, and the amount of ventilation required. With rectangular coils it ranges from 0.65 to 0.85; with circular coils and square cores from 0.45 to 0.55; with cross-shaped cores from 0.6 to 0.7; and with stepped cores from 0.55 to 0.75. Fig. 10-03 gives the proportions of the last two if the maximum amount of iron is to be got in with two and with three widths of sheet.

The copper space-factor is affected mostly by the space required for insulation, and therefore varies with the voltage. The size of the wire and the ventilation required have also to be taken into account. Fig. 10-04 gives typical values for it.

In an actual transformer, the equivalent mean turn may not come exactly at the middle of the actual winding space owing to the effects of the clearance between concentric coils, and of the different sections of the

I_0	= Exciting (or no-load) current.
I_{I0}, I_{W0}	= Idle and working components of I_0 .
\bar{I}	= R.M.S. current density (supposed uniform).
β	= Maximum flux density (supposed uniform).
ρ_C	= Copper loss coefficient (resistivity) = $P_C/\bar{I}^2 V_C$.
K_I	= Iron loss coefficient = $P_I/\beta^2 V_I$.
\mathcal{H}	= Maximum magnetising field required for iron.
μ_a	= Permeability of air.
X_I, X_J	= Excitations (R.M.S.) required to magnetise iron and joints.
X_M	= Excitation to magnetise whole circuit = $X_I + X_J$.
R_{T2}	= Total equivalent internal resistance, referred to secondary.
\mathcal{L}_{T2}	= Total equivalent leakage inductance, referred to secondary.
Z_{T2}	= Total equivalent internal impedance, referred to secondary.
L_0	= Fundamental length = $(S_{IS} S_{CS} \div L_I L_C)^{\frac{1}{2}}$.
L_L	= Loss-length = $(K_I \rho_C)^{\frac{1}{2}}/4ff$.
L_I, L_C	= Equivalent mean lengths of iron and copper such that $L_I S_I = V_I$, and $L_C S_C = V_C$.
L_F	= Length of flux.
L_{IS}	= Depth of iron space perpendicular to laminations with rectangular coils.
L_{CS}	= Distance between yokes.
l_{IS}	= Width of iron space.
l_{CS}	= Conventional winding depth.
S_0	= Fundamental surface = L_0^2 .
S_{IS}, S_{CS}	= Cross-sections of iron and copper spaces.
S_I	= Net cross-section of iron = $\sigma_I S_{IS}$.
S_C	= Net cross-section of copper = $\sigma_C S_{CS}$.
S_{CI}, S_{CC}	= Conventional cooling surfaces of iron and copper.
S_{W1}, S_{W2}	= Cross-section of primary and secondary wire.
V_0	= Fundamental volume = L_0^3 .
V_{IS}, V_{CS}	= Gross volumes of iron and copper spaces.
V_I, V_C	= Net volumes of iron and copper.
M_I, M_C	= Mass of iron and copper.
M_S	= Mass of standard iron of equal total cost.
σ_I, σ_C	= Gross space-factors of the iron and copper spaces.
x	= L_{CS}/l_{IS} .
y	= l_{CS}/l_{IS} .
z	= L_{IS}/l_{IS} .
a, b, c, d, e	= Numerical coefficients whose meaning appears in the text.
N_1, N_2	= Number of turns in primary and secondary windings.

materials, space-factors, and frequency, these equations show that the efficiency depends on the geometrical mean of the lengths obtained by dividing the cross-sections of the iron and copper spaces by the respective lengths of the iron and copper. This geometrical mean is thus a length depending on the performance required and the materials used, but independent both of the type of transformer chosen and of the proportions of that type. It is therefore convenient to express all the other dimensions in terms of this

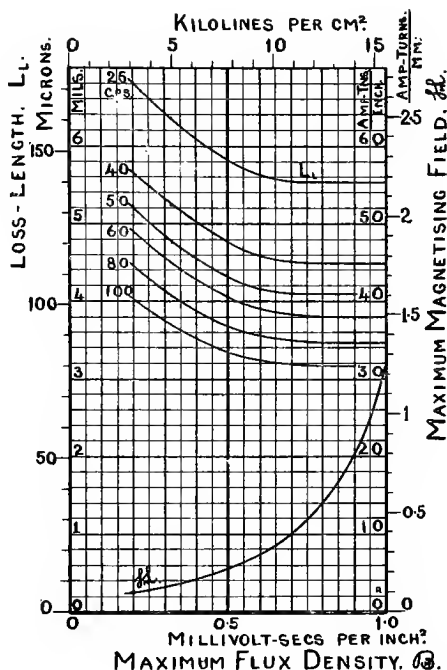


FIG. 10·05.—Loss-Length and Magnetising Field for Sankey's "Lohys" Iron 15 mils thick.

length, and to give it a name. It may be called the Fundamental Length, its square the Fundamental Surface, and its cube the Fundamental Volume.

The cost of the fundamental volume when filled with iron having the same space-factor as actually obtains in the iron space is a standard with which the cost of different transformers may be compared, and is termed the fundamental cost.

Loss-Length.—It is convenient to work out once for all the factor inside the large brackets in the expression for the fundamental length in terms of the performance. This factor takes account of the qualities of the materials, of the frequency, and of the form-factor. The frequency is included

because the iron loss coefficient varies with it; the form-factor is practically always assumed to be 1.11 as with sine waves, and arithmetic is saved by including it and the numerical factor 4. The whole factor is of the nature of a length, and since it depends chiefly on the losses in the materials, it may be termed the Loss-Length. Its value lies between two and six mils, according to the thickness and quality of the iron and to the frequency of supply.

The copper loss coefficient is practically constant; in getting out the numerical values of the loss-length it has been taken as 0.90 microhm-inches.

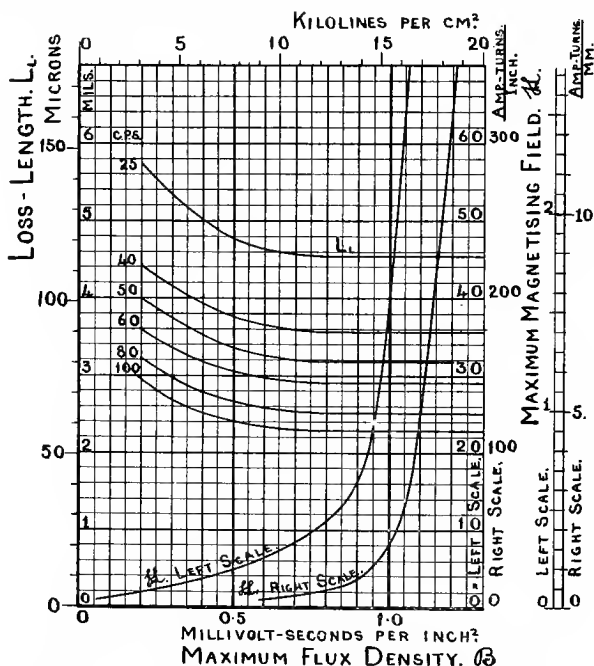


Fig. 10.06.—Loss-Length and Magnetising Field for Sankey's "Stalloy" Iron 20 mils thick.

This is the steady current value at 100° C., and may be taken as about right for alternating currents at the ordinary working temperatures. It leaves an allowance for eddies of 10 per cent. at 70° C. and of 15 per cent. at 55° C.

Figs. 10.05 and 10.06 show the variation, with flux density, at the usual frequencies, of the loss-length for two materials which represent the best quality obtainable of their respective classes. Fig. 10.06 refers to an alloyed iron having an exceedingly small eddy-current loss. In differentiating to get the cheapest proportions, it is assumed that the loss-length is constant. The curves show that this is not true until the flux density reaches about 0.65 millivolt-seconds per square inch (10,000 C.G.S.), but above that it does

not vary at all. It is rather remarkable that the degree of constancy is practically the same for both materials, although the variation with

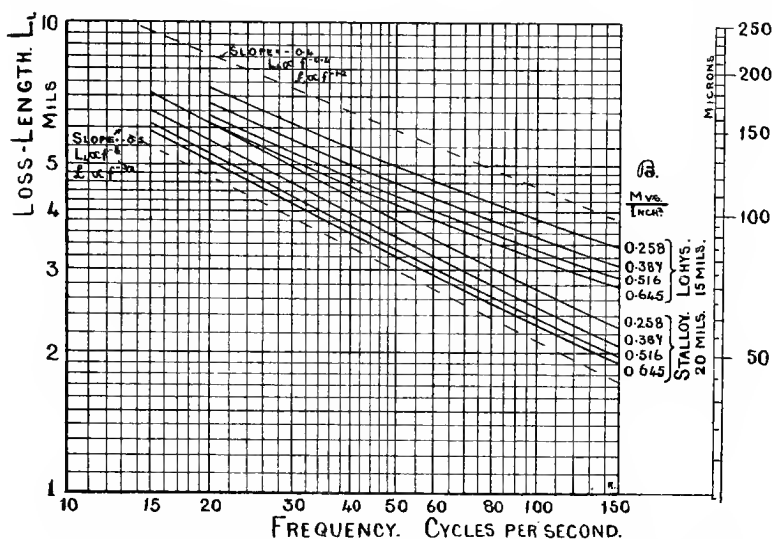


FIG. 10·07.—Logarithmic Curves of Variation of Loss-Length with Frequency.

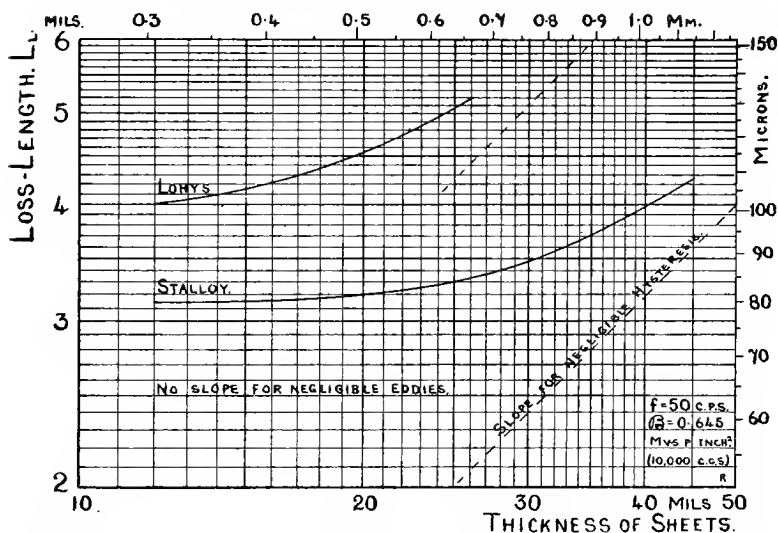


FIG 10·08.—Logarithmic Curves of Variation of Loss-Length with Thickness of Sheets.

frequency (fig. 10·07) shows that the eddies are almost negligible in the alloyed "Stalloy," as do also the curves of fig. 10·08 showing the variation with thickness of sheet. It must be concluded that the hysteresis loss in this

material follows the square law with variation of flux density more nearly than the Steinmetz law generally assumed.

Thermal Equations at Maximum Efficiency. — At maximum efficiency,

$$(I_1 V_1 + I_2 V_2) = P_{C\sqrt{\sigma_{TC}}} \cdot \frac{L_0}{L_L} \text{ (from equation 10.04)} \quad 10.11,$$

$$= (A_{C\tau_{RC}} S_{CC}) \sqrt{\sigma_{TC}} \cdot \frac{L_0}{L_L} \times \frac{L_0^2}{S_0} \quad 10.12.$$

$$\therefore V_0 = L_0^3 = \frac{(I_1 V_1 + I_2 V_2) L_L}{(A_{C\tau_{RC}})(S_{CC}/S_0) \sqrt{\sigma_{TC}}} \quad 10.13,$$

where A_C is a conventional emissivity defined so as to make this true, and $A_{C\tau_{RC}}$ is the conventional dissipation intensity or rate of dissipation of heat per unit of conventional cooling surface.

In the absence of more accurate data, $(A_{C\tau_{RC}})$ may be taken to lie between

$$\left. \begin{array}{l} 0.1 \text{ and } 0.2 \text{ watts per inch}^2. \\ 150 \text{ and } 300 \text{ watts per metre}^2. \\ 0.2 \text{ and } 0.4 \text{ watts per inch}^2. \\ 300 \text{ and } 600 \text{ watts per metre}^2. \\ 0.3 \text{ and } 0.6 \text{ watts per inch}^2. \\ 450 \text{ and } 900 \text{ watts per metre}^2. \end{array} \right\} \begin{array}{l} \text{For air-insulated natural-} \\ \text{cooled transformers} \\ \text{For oil-insulated natural-} \\ \text{cooled transformers} \\ \text{For oil-insulated water-} \\ \text{cooled transformers} \end{array} \quad 10.14.$$

By putting the value of L_0 given by equation 10.13 in equation 10.10, we get for the relation between the efficiency and rated output for a constant temperature rise:—

$$\eta_M = \frac{\{(I_1 V_1 + I_2 V_2) \sigma_{TC}\}^{\frac{1}{3}} \cos \phi - 2 \{L_L^2 (A_{C\tau_{RC}})(S_{CC}/S_0)\}^{\frac{1}{3}}}{\{(I_1 V_1 + I_2 V_2) \sigma_{TC}\}^{\frac{1}{3}} \cos \phi + 2 \{L_L^2 (A_{C\tau_{RC}})(S_{CC}/S_0)\}^{\frac{1}{3}}} \quad 10.15.$$

Cooling Functions.—If the iron and copper spaces be removed from one another, they will have certain surfaces exposed to the air, which may be taken as conventional measures of their respective cooling surfaces. The ratios of these to the fundamental surface are the Iron and Copper Cooling Functions respectively. The actual cooling surfaces will usually be much greater than these conventional surfaces, but the latter serve as a basis of comparison of different types and different proportions, as it may be assumed that the effect of ducts, etc., will be the same for all. On this assumption, the thermal equation (10.13) gives a means of settling the minimum value of the fundamental volume permissible from a consideration of heating, and shows that it is inversely proportional to the cooling function for a given rise of temperature.

The permissible heat dissipation per unit of conventional cooling surface

depends on how much the actual cooling surface exceeds the conventional one, on the method of cooling, and on the temperature rise allowed. It will be greater for large sizes than for small ones, owing to the greater subdivision possible.

The curves (figs. 10·09 and 10·10) give the cooling functions for the cheapest proportions, and are rather remarkable, especially those for the copper, which are almost perfectly constant for a given type for all practical values of the specific cost ratio above 1 or 2, according to the type. The iron cooling functions for the shell and polyphase types with circular coils are

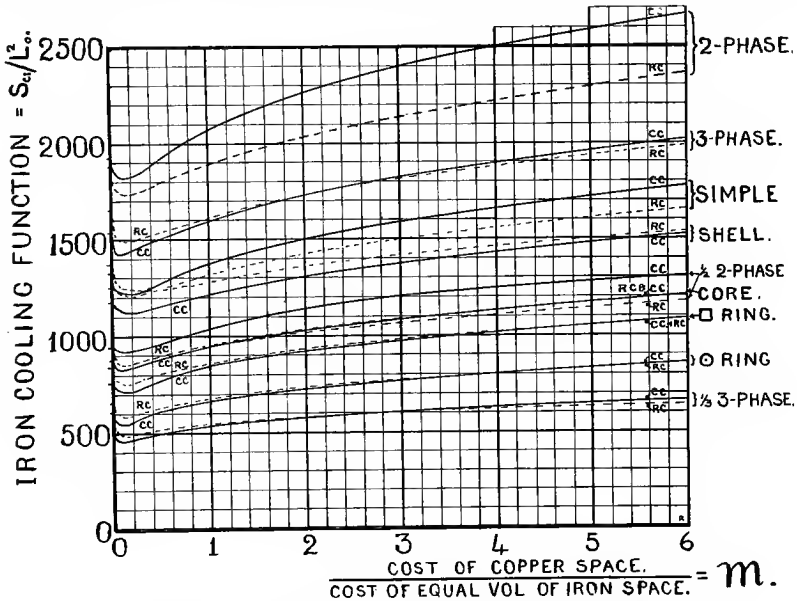


FIG. 10·09.—Iron Cooling Functions of Cheapest Transformers.

(The full lines refer to transformers with Circular Coils, and the dotted ones to those with Rectangular Coils.)

only approximate, as no account has been taken of the loss of surface at the intersection of the cylinders, nor of the difference between the circumference of an ellipse and a circle having the same mean diameter.

Cost Equations for a given Efficiency:—

$$\mathcal{L} = c_I \sigma_I V_{IS} + c_C \sigma_C V_{CS} \quad . \quad . \quad . \quad 10\cdot16,$$

$$= c_I \sigma_I V_O \times \frac{V_{IS}}{L_O^3} \left\{ 1 + \frac{c_C \sigma_C}{c_I \sigma_I} \cdot \frac{V_{CS}}{V_{IS}} \right\} \quad . \quad . \quad . \quad 10\cdot17.$$

Hence, using equation 10·05, the cost function is

$$\frac{\mathcal{L}}{\mathcal{L}_O} = \frac{L_I^{\frac{2}{3}} L_C^{\frac{1}{3}}}{S_{IS}^{\frac{2}{3}} S_{CS}^{\frac{1}{3}}} \left\{ 1 + m \frac{L_C S_{CS}}{L_I S_{IS}} \right\} \quad . \quad . \quad . \quad 10\cdot18,$$

where $\mathcal{E}_0 = c_{I\sigma I} V_0 = \text{fundamental cost.}$

If we assume $\frac{\mathcal{E}}{\mathcal{E}_0} \propto m^n$

i.e., $\propto \left(\frac{C_C \sigma_C}{C_I \sigma_I} \right)^n,$

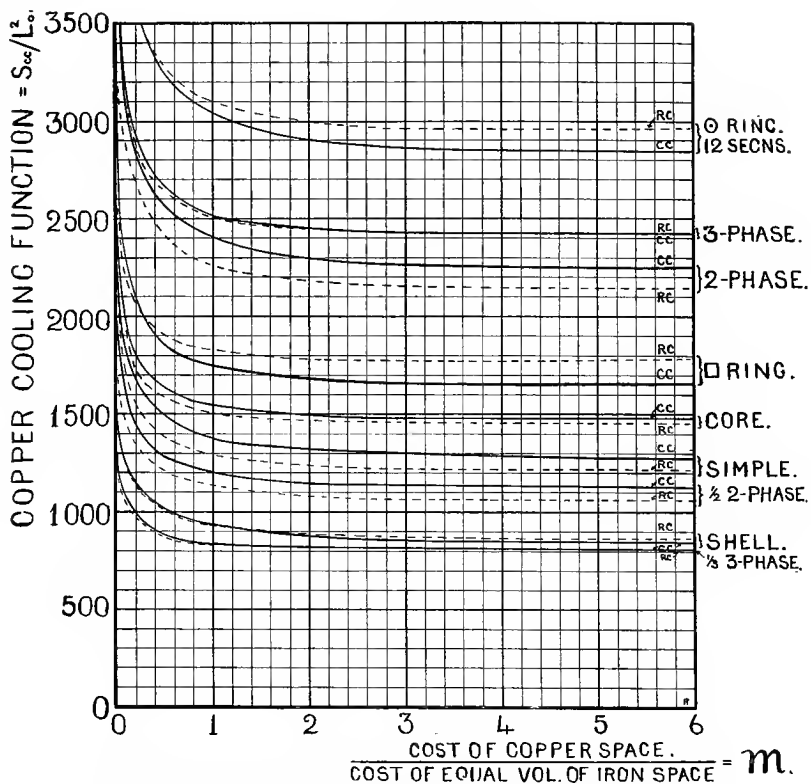


FIG. 10·10.—Copper Cooling Functions of Cheapest Transformers.

(The full lines refer to transformers with Circular Coils, and the dotted ones to those with Rectangular Coils.)

then

$$\frac{\mathcal{E}}{\mathcal{E}_0} \cdot \mathcal{E}_0 = \frac{\mathcal{E}}{\mathcal{E}_0} \times c_{I\sigma I} V_0 \quad . \quad . \quad 10·19,$$

$$\propto \left(\frac{C_C \sigma_C}{C_I \sigma_I} \right)^n \times c_{I\sigma I} \times \frac{L_L^3}{(\sigma_I \sigma_C)^{\frac{3}{2}}}$$

$$\propto \frac{C_I^{1-n} C_C^n}{\sigma_I^{\frac{1}{2}+n} \sigma_C^{\frac{3}{2}-n}} L_L^3 \quad . \quad . \quad 10·20,$$

and, if n be taken as $\frac{1}{2}$,

$$\mathcal{E} \propto \frac{C_I^{\frac{1}{2}} C_C^{\frac{1}{2}}}{\sigma_I \sigma_C} L_L^3 \quad . \quad . \quad 10·21.$$

Cost Function.—Only that part of the cost which varies with the amount of active material is taken into account in this theory, it being assumed that the remainder is the same for all types. The ratio of this portion of the cost to the fundamental cost is the Cost Function, which, for

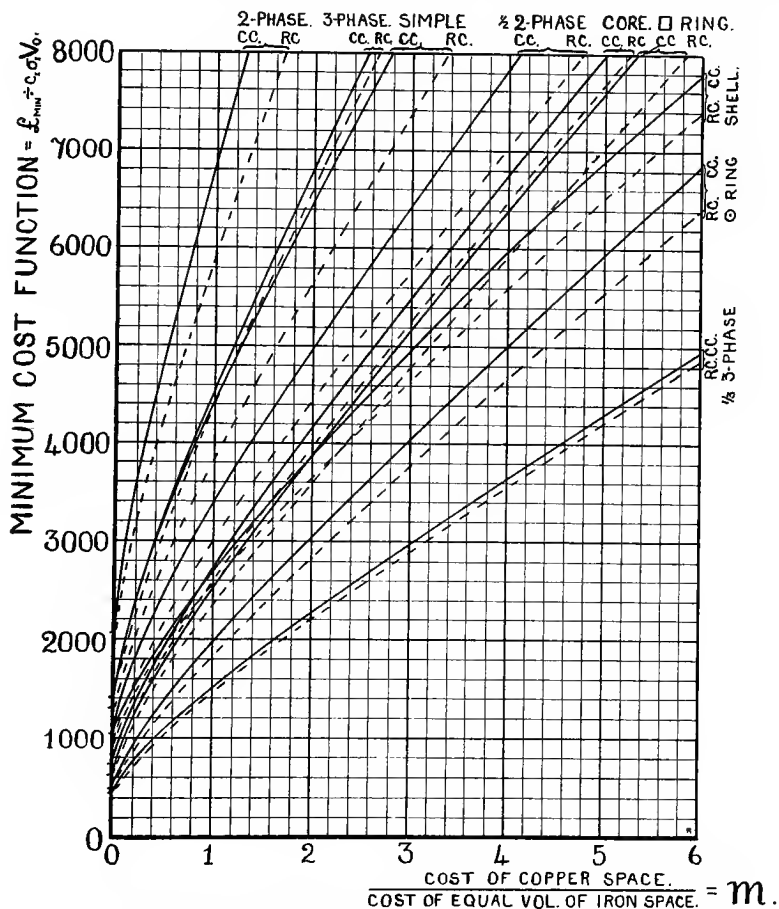


FIG. 10·11.—Minimum Values of the Cost Function.

(The full lines refer to transformers with Circular Coils, and the dotted ones to those with Rectangular Coils.)

the ordinary types of transformers, varies from two to five thousand within the usual limits of specific cost ratio. The actual dimensions cancel out in this cost function, leaving only the ratios of the others to one of them, conveniently the width of the iron space, and thus reducing the number of variables by one. This shows that the cost function is independent of the fundamental length, and hence also of the performance required and of the

quality of the materials, although not of their price. It therefore gives a means of studying the effect of the proportions and type apart from con-

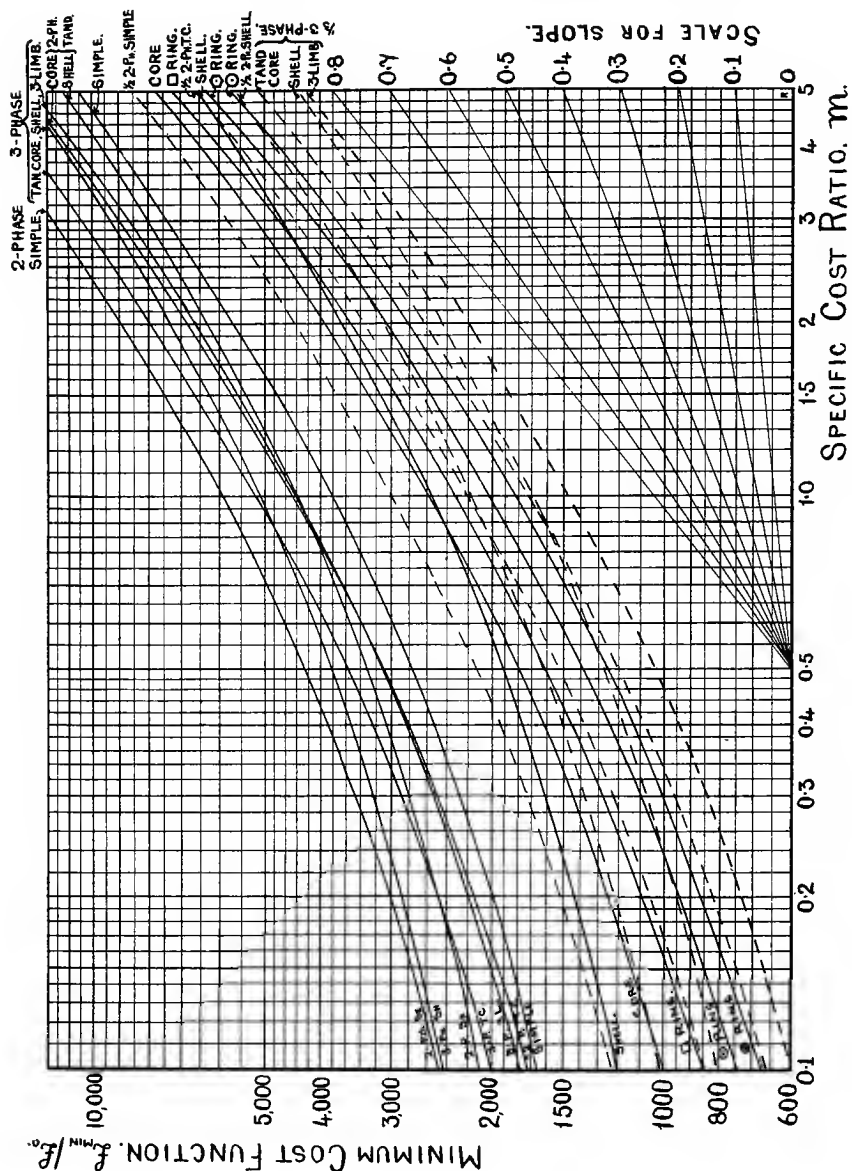


FIG. 10 12.—Logarithmic Curves for Minimum Cost Function. (Rectangular Coils only.)

siderations of size. The cost is proportional to the fundamental cost for different performances or qualities of materials, or for different prices or

space-factors of the iron; and to the cost function for different types, proportions, or relative prices and space-factors of the copper.

The cost function gives the necessary starting-point for the differentiation required to determine the cheapest proportions of any type, and a means of comparing other proportions with these. It will be found that it varies very slowly indeed near the minimum, and that a considerable latitude may be allowed in the proportions without greatly affecting the total cost of the material required.

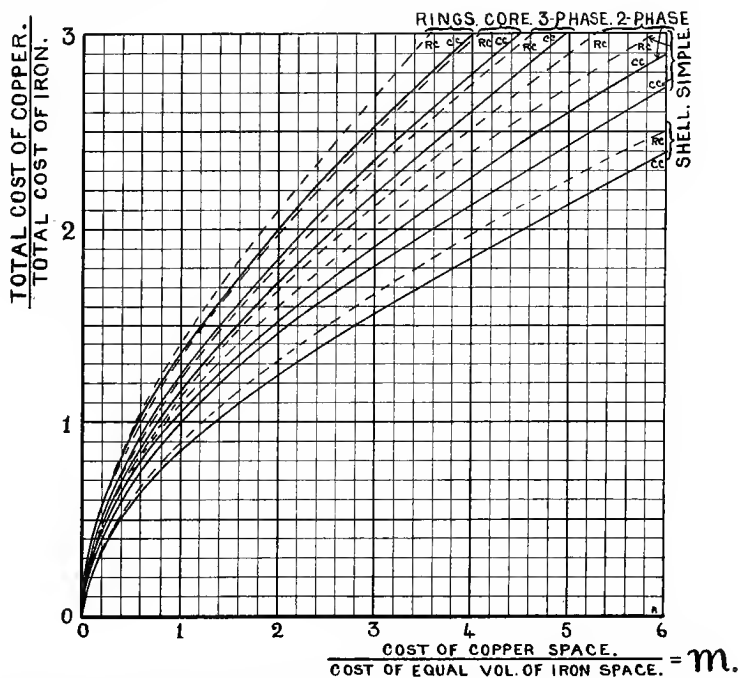


FIG. 10·13.—Division of Total Cost between Copper and Iron with Cheapest Transformers.
(The full lines refer to transformers with Circular Coils, and the dotted ones to those with Rectangular Coils.)

Figs. 10·11 and 10·12 show how the minimum values of the cost function change with the specific cost ratio. The latter, in which the logarithms are plotted, is most interesting, for on it the distance apart of two curves gives a measure of their ratio. It will be seen that, leaving out the shell types, there is a nearly constant ratio between the different types within the usual range of the specific cost ratio, and also that within these limits the logarithmic curves are fairly nearly straight lines with a slope of $\frac{1}{2}$, showing that the cost function is approximately proportional to the square root of the specific cost ratio.

Fig. 10.13 gives the division of the total cost between the copper and iron when the cheapest proportions are adopted. Some writers have assumed that this ratio is unity with the cheapest transformer, whereas the curves show that although that might happen to be about right in a particular case it may be very far wrong in another.

Effect of Variables on the Cost for a given Efficiency.—The cost equations, together with the logarithmic curves for the cost function and loss-length, show that for a specified efficiency the cost varies in proportion to :—

- (1) The inverse cube of the geometrical mean of the iron and copper losses, from which it follows that the cost is least with given total losses when they are divided equally between the iron and copper.
- (2) The inverse cube of the power-factor.
- (3) The inverse cube of the form-factor of the E.M.F. wave.
- (4) The cube of the loss-length.
- (5) The inverse of some power of the frequency lying between 1.5 (no eddies) and zero (no hysteresis). For the 20 mils "Stalloy" this index is 1.5, and for 15 mils "Lohys" it is 1.0 to 1.2.
- (6) The inverse of some power of either space-factor not far from unity, and lying between 0.8 and 1.2 for extreme practical values of the specific cost ratio. The lower figure refers to the iron and the upper one to the copper at very small values of this ratio, while their positions are reversed at high values, and their sum is always 2. Within the usual range the index for the iron space-factor may be taken as 1.0 for the shell types, and 1.1 for all the others ; the corresponding values for the copper are 1.0 and 0.9.
- (7) Some power of the cost of either metal not far from $\frac{1}{2}$ and lying between 0.3 and 0.7 for extreme practical values of the specific cost ratio. The lower figure refers to the copper and the upper one to the iron at low values of this ratio, while their positions are reversed at high values, and their sum is always 1. Within the usual range, the index for the iron may be taken as 0.5 for the shell types and 0.4 for the others, the corresponding values for the copper being 0.5 and 0.6.

Cost Equations for a given Temperature Rise.—If we use the same proportions, when the size is limited by temperature rise, as give the cheapest transformer having a specified efficiency, we get the best efficiency for the money expended. In that case, using equation 10.13,

$$\begin{aligned}
 \mathcal{L} &= \left(\frac{\mathcal{L}}{\mathcal{L}_0} \right) \mathcal{L}_0 \\
 &= \left(\frac{\mathcal{L}}{\mathcal{L}_0} \right) c_I \sigma_I V_0 \\
 &= \left(\frac{\mathcal{L}}{\mathcal{L}_0} \right) \frac{c_I \sigma_I (I_1 V_1 + I_2 V_2) L_L}{(A_{CTRC}) (\mathbf{S}_{CC}/\mathbf{S}_0) \sqrt{\sigma_I \sigma_C}} \quad . \quad . \quad . \quad 10 \cdot 22.
 \end{aligned}$$

And if, as before, we assume that the cost function is proportional to m^n , we have

$$\begin{aligned}
 \mathcal{L} &\propto \left(\frac{c_C \sigma_C}{c_I \sigma_I} \right)^n \times c_I \sigma_I \times \frac{L_L}{\sqrt{\sigma_I \sigma_C} (\mathbf{S}_{CC} \mathbf{S}_0)} \\
 &\propto \frac{c_I^{1-n} c_C^n}{\sigma_I^{n-\frac{1}{2}} \sigma_C^{\frac{1}{2}-n}} \frac{L_L}{\mathbf{S}_{CC}/\mathbf{S}_0} \quad . \quad . \quad . \quad 10 \cdot 23.
 \end{aligned}$$

And if, in addition, n be taken as $\frac{1}{2}$,

$$\mathcal{L} \propto \frac{c_I^{\frac{1}{2}} c_C^{\frac{1}{2}} L_L}{(\mathbf{S}_{CC}/\mathbf{S}_0)} \quad . \quad . \quad . \quad 10 \cdot 24.$$

Effect of Variables on the Cost for a given Temperature Rise.—If the efficiency be increased with the rated load so as to keep the temperature rise the same, the cost varies in proportion to :—

- (1) The output.
- (2) The inverse of the power-factor.
- (3) The inverse of the form-factor of the E.M.F. wave.
- (4) The loss-length.
- (5) The inverse of some power of the frequency lying between 0·5 (no eddies) and zero (no hysteresis). For 20 mils “Stalloy” this index is 0·5, and for 15 mils “Lohys” it is 0·35 to 0·40.
- (6) Some power of either space-factor not far from zero, and lying between $\pm 0\cdot2$ for extreme practical values of the specific cost ratio. The negative figure refers to the iron and the positive one to the copper at very small values of this ratio, while their positions are reversed at high values and their sum is always zero. Within the usual range the index for the iron space-factor may be taken as zero for the shell types and 0·1 for the others; the corresponding values for the copper are 0 and $-0\cdot1$.
- (7) Some power of the cost of either metal not far from 0·5, and lying between 0·3 and 0·7 for extreme practical values of the specific cost ratio. The lower figure refers to the copper and the upper one to the iron at low values of this ratio, while their positions are reversed at high values and their sum is always 1. Within the usual range, the index for the iron may be taken as 0·5 for the

shell types and 0.4 for the others, the corresponding values for the copper being 0.5 and 0.6.

- (8) The inverse of the cooling function.
- (9) The inverse of the permissible rise of temperature.
- (10) The inverse of the effective emissivity.

Relative Value of Different Materials and Thicknesses of Sheet.—The logarithmic curves show that the cost function is fairly nearly proportional to the square root of the specific cost ratio for the ordinary values of that ratio. If we assume that this is quite true, the formulæ enable tolerably accurate conclusions, applicable to all types and sizes of transformers, to be drawn as to the relative values of different qualities and thicknesses of iron. More accurate comparisons can of course be made when the actual space-factors and prices are known, but these would only apply to one particular case and would involve considerably more trouble.

Assuming the square-root law, the cost equations show that for a given total cost and a specified efficiency the cost per unit of gross volume of iron space may be increased in proportion to the inverse sixth power of the loss-length for the material at the given frequency (equation 10.21). If different thicknesses are employed, the price might increase proportionally to the square of the space-factor divided by the sixth power of the loss-length. The dearer iron will, however, give a smaller transformer, which will get hotter than the other, and so it will have to be cheaper than given by this rule to a sufficient extent to allow the efficiency to be increased enough to keep the temperature rise down to the permissible amount. The transformer using the better material will also require a greater magnetising current if made to give the same efficiency.

Neglecting the very small change in the cooling functions with change of specific cost ratio, equation 10.24 shows that if the total cost and the temperature rise are to remain constant, the price of the iron may only be increased in proportion to the inverse square of the loss-length. The space-factor cancels out this time, and so does not affect the comparison. The transformer with the dearer iron will still be smaller and lighter than the other, but it will also be more efficient.

That relative cost of two materials which would make the total cost the same whichever is employed, may be termed the Relative Value of these materials for equal efficiency or for equal temperature rise as the case may be. That is the best material and thickness which, at the given frequency, gives the greatest ratio between the relative value and the relative cost. If the relative values and relative costs be plotted to a thickness base on logarithmic paper, the best thickness is that at which there is the greatest

distance between the two curves. Table 10·01 gives the constants of two brands of iron and their relative values with different thicknesss at 50 cycles

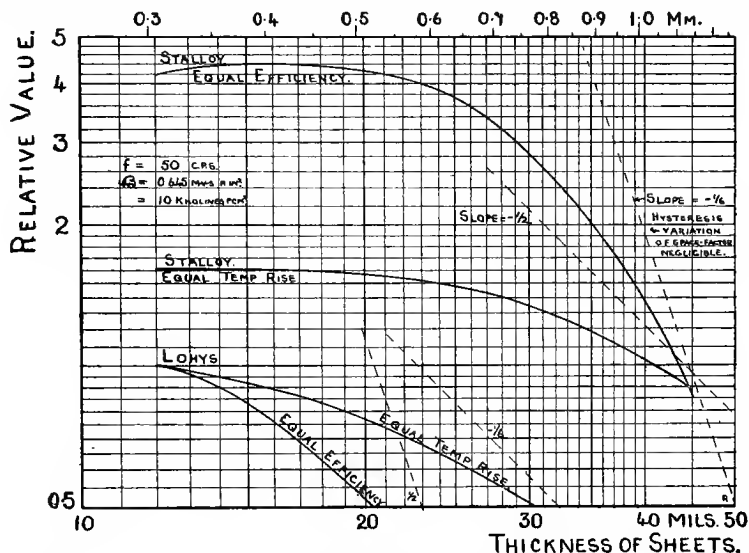


FIG. 10·14.—Logarithmic Curves of Variation of Relative Value with Thickness for "Stalloy" and "Lohys."

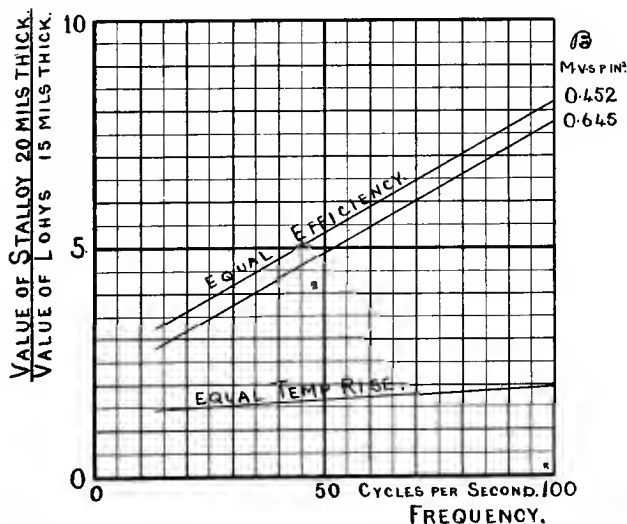


FIG. 10·15.—Relative Value of 20 mils "Stalloy" and 15 mils "Lohys" at Different Frequencies.

per second worked out in this way, as compared with a material giving a loss-length of 4 mils at this frequency, and also their relative value com-

pared with one another at different frequencies. Fig. 10-14 shows the variation with thickness. It will be noticed that "Stalloy" has the greatest relative value at a thickness of about 16 mils (0.4 mm.), and that it decreases very slowly with increased thickness. Allowing for the slight decrease of the cost of the material and labour of assembling it as the thickness of sheet is made greater, the cheapest thickness will come about 20 mils (0.5 mm.) at this frequency. At higher frequencies the best thickness would be less, and at lower ones greater, than this. Owing to the much larger eddy current loss in "Lohys," its greatest relative value lies below the practical range of thinness (about 12 mils, or 0.3 mm.) at this or higher frequencies.

Fig. 10-15 shows how the relative value of 20 mils "Stalloy" and 15 mils "Lohys," these being about the cheapest thicknesses of the two materials, vary with the frequency. As nearly as may be, this follows a straight-line law.

TABLE 10-01.—COMPARISON OF SANKEY'S LOHYS (ORDINARY) AND STALLOY (ALLOYED) IRONS.

Frequency. Cycles per sec.	Flux Density. 10 ⁻³ volt. sec. inch ²	Thickness of Sheet, Mils.	Sheet Space. Factor (2 mils lost per sheet).	Iron Loss.* Watts per lb.		Iron Loss Coefficient. Watts per inch ³ {10 ⁻³ V.S. per inch ² } ²		Loss-Length. Mils.		Rel. Value for Equal Temp. Rise.		Rel. Value for Equal Efficiency.	
				Lohys.	Stalloy.	Lohys.	Stalloy.	Lohys.	Stalloy.	Lohys.	Stalloy.	Lohys.	Stalloy.
50	0.645 (10,000 C.G.S.)	12	0.857	1.30	0.80	0.88	0.541	4.00	3.14	1.00	1.61	1.00	4.17
		14	0.875	1.36	0.80	0.92	0.541	4.10	3.14	0.95	1.61	0.90	4.35
		16	0.889	1.45	0.81	0.98	0.548	4.23	3.16	0.89	1.60	0.75	4.40
		18	0.900	1.55	0.82	1.05	0.554	4.37	3.18	0.84	1.58	0.65	4.35
		20	0.909	1.68	0.83	1.14	0.561	4.55	3.20	0.77	1.56	0.52	4.28
		22	0.917	1.82	0.85	1.23	0.575	4.74	3.24	0.71	1.53	0.41	4.10
		24	0.924	1.99	0.87	1.35	0.588	4.95	3.27	0.65	1.50	0.31	3.93
		26	0.929	2.19	0.90	1.48	0.609	5.20	3.33	0.59	1.45	0.25	3.58
		28	0.934	...	0.94	...	0.635	...	3.40	...	1.39	...	3.19
		30	0.938	...	0.98	...	0.662	...	3.47	...	1.33	...	2.82
		35	0.946	...	1.11	...	0.716	...	3.62	...	1.22	...	2.22
		40	0.953	...	1.28	...	0.865	...	3.97	...	1.02	...	1.32
		45	0.958	...	1.48	...	1.000	...	4.27	...	0.88	...	0.85
25	0.645	<div> <div>Lohys</div> <div>15</div> <div>Stalloy</div> <div>20</div> </div>	0.883	0.625	0.42	0.423	0.284	5.56	4.55	1.00	1.49	1.00	3.51
40	(10,000			1.060	0.67	0.717	0.453	4.52	3.59	1.00	1.59	1.00	4.26
50	C.G.S.)			1.375	0.82	0.930	0.554	4.11	3.18	1.00	1.67	1.00	4.95
60	...			1.72	1.00	1.164	0.676	3.84	2.93	1.00	1.72	1.00	5.40
80	...			2.50	1.34	1.69	0.905	3.47	2.54	1.00	1.87	1.00	6.95
100	3.30	1.70	2.23	1.149	3.19	2.29	1.00	1.94	1.00	7.76
25	0.452	Ditto.	Ditto.	0.350	0.23	0.484	0.318	5.94	4.81	1.00	1.53	1.00	3.80
40	(7000			0.600	0.37	0.829	0.511	4.85	3.82	1.00	1.61	1.00	4.42
50	C.G.S.)			0.775	0.46	1.071	0.635	4.42	3.40	1.00	1.69	1.00	5.12
60	...			0.982	0.55	1.358	0.760	4.15	3.10	1.00	1.79	1.00	6.10
80	...			1.41	0.74	1.95	1.022	3.73	2.70	1.00	1.91	1.00	7.40
100	1.86	0.95	2.57	1.310	3.42	2.44	1.00	1.97	1.00	8.10

* From curves supplied by the makers. See also figs. 10-64 and 10-65.

Dimension Coefficients : Proportions.—The dimension coefficients are the ratios of the chief dimensions of the transformer to the fundamental

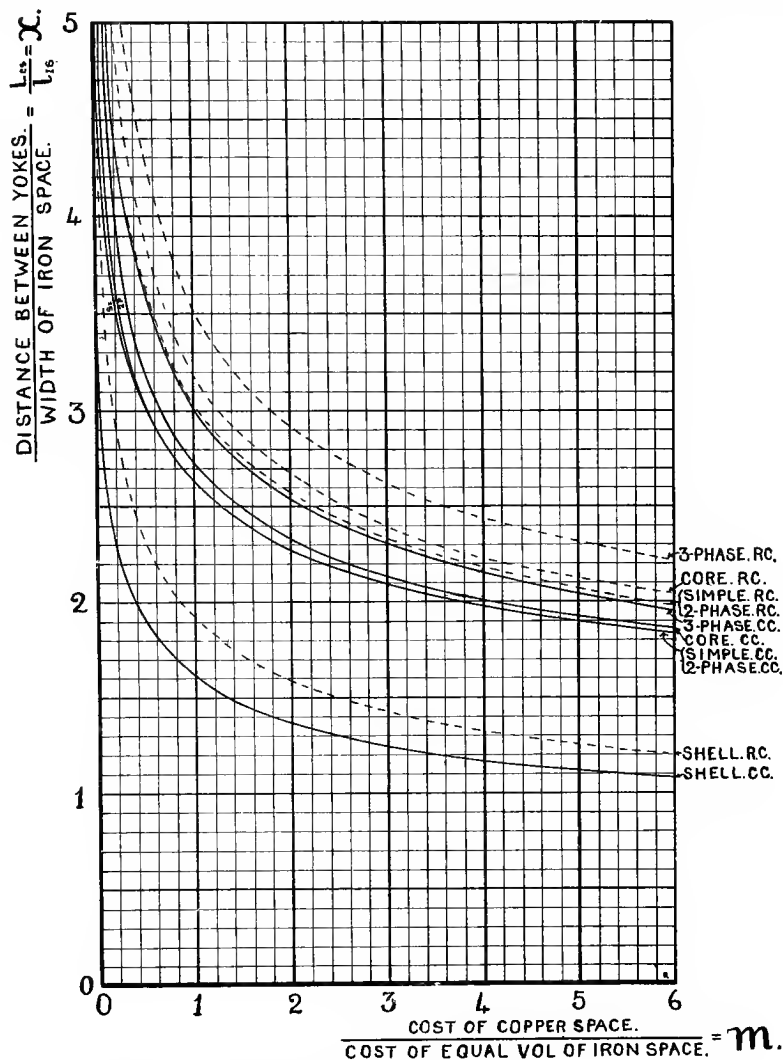


FIG. 10'16.—Ratio of Distance between Yokes to Width of Iron Space of Cheapest Transformers.

(The full lines refer to transformers with Circular Coils, and the dotted ones to those with Rectangular Coils.)

length; in the formulæ they have all been expressed in terms of the ratio of the other dimensions to the width of the iron space. Those for the cheapest proportions have been calculated once for all for each type, and

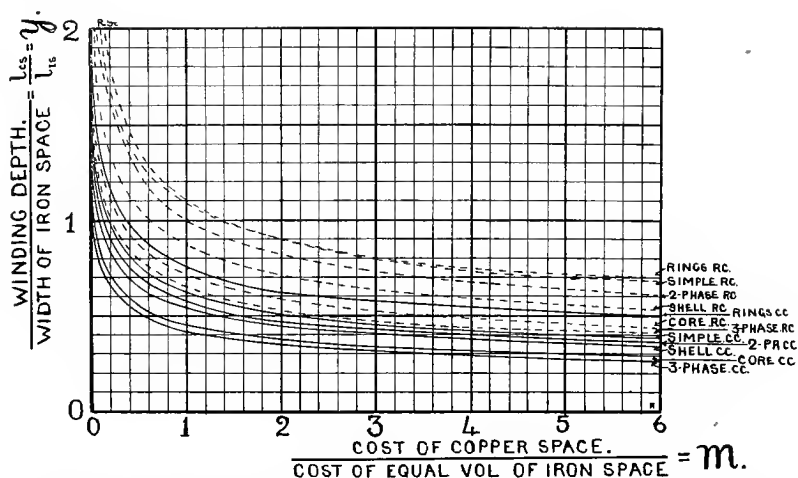


FIG. 10-17.—Ratio of Winding Depth to Width of Iron Space of Cheapest Transformers.

(The full lines refer to transformers with Circular Coils, and the dotted ones to those with Rectangular Coils.)

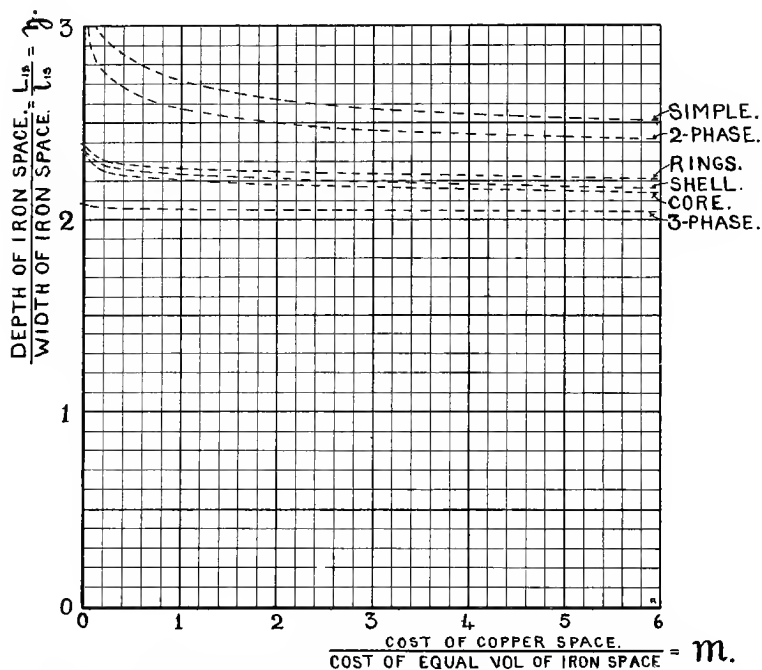


FIG. 10-18.—Ratio of Depth to Width of Iron Space of Cheapest Transformers with Rectangular Coils.

their values are given in the tables and curves. Curves are also given showing the proportions of the cheapest transformers. It will be seen that the best proportional length of core (fig. 10·16) and of winding depth (fig. 10·17) diminish with increase of the specific cost ratio. The ratio of the two dimensions of the iron space for rectangular coils (fig. 10·18) is remarkably constant for all values of the specific cost ratio for any one type, and varies exceedingly little from one type to another. For the shell, core, and ring

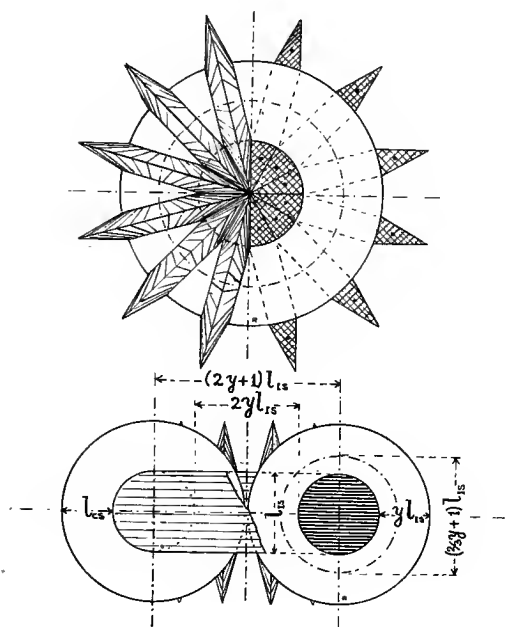


FIG. 10·19.—Circular Ring Transformer with Circular Coils. (Modified Faraday Type.)

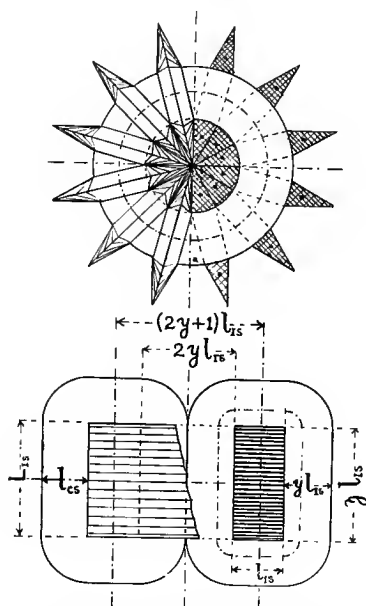


FIG. 10·20.—Circular Ring Transformer with Rectangular Coils.

transformers it is practically always 2·2 for all values of the other variables, and 2·05 for the three-limb three-phase type.

Comparison of Types: Single-Phase.—Six distinct types of single-phase transformers are illustrated in figs. 10·19–10·38, and each of these may either have circular or rectangular coils. All similar curves are drawn to the same scale, and all the transformers are shown in the correct proportions, and to the same scale, for a specific cost ratio of 2. In all cases the rectangular coils are cheaper than circular for the same efficiency, have practically the same cooling functions, and a slightly greater magnetising current. They are capable of higher iron space-factors, but probably not

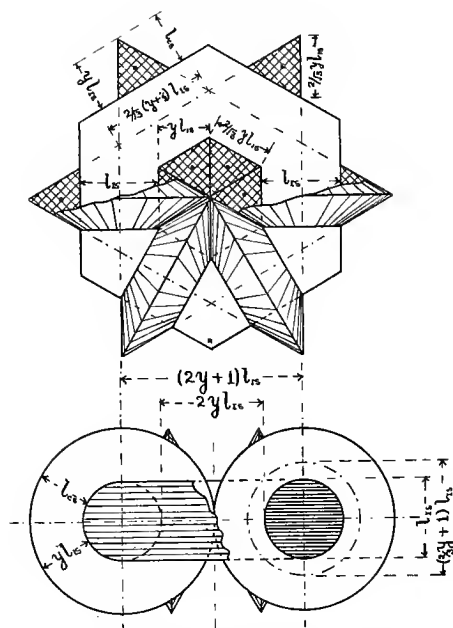


FIG. 10-21.—Hexagonal Ring Transformer with Circular Coils.

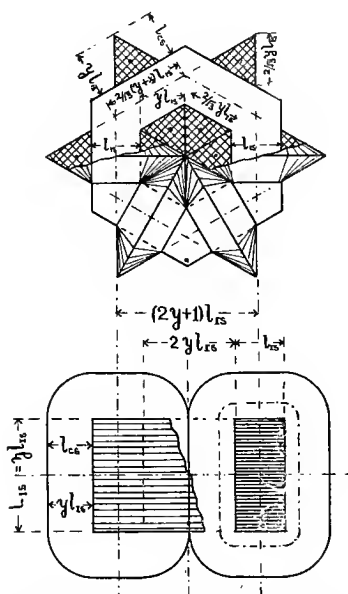


FIG. 10-22.—Hexagonal Ring Transformer with Rectangular Coils.

quite such high copper space-factors with small wires owing to the want of pressure between the wires in the straight parts.

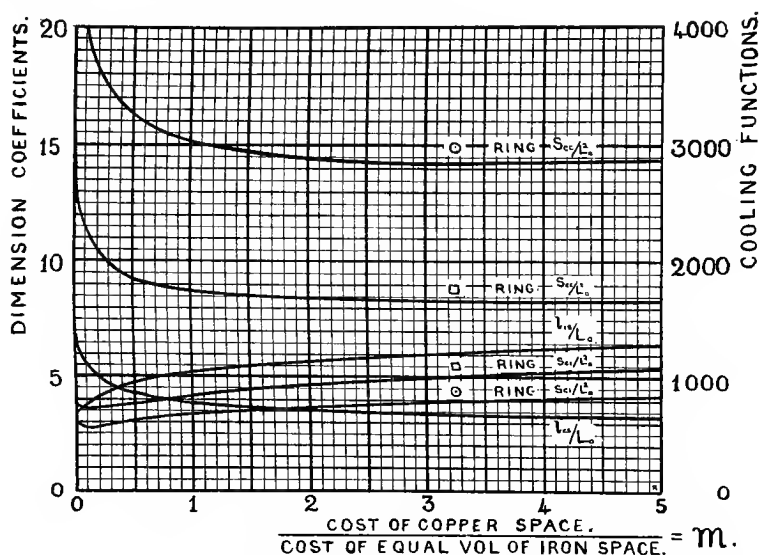


FIG. 10-23.—Dimension Coefficients for Ring Transformers with Circular Coils.

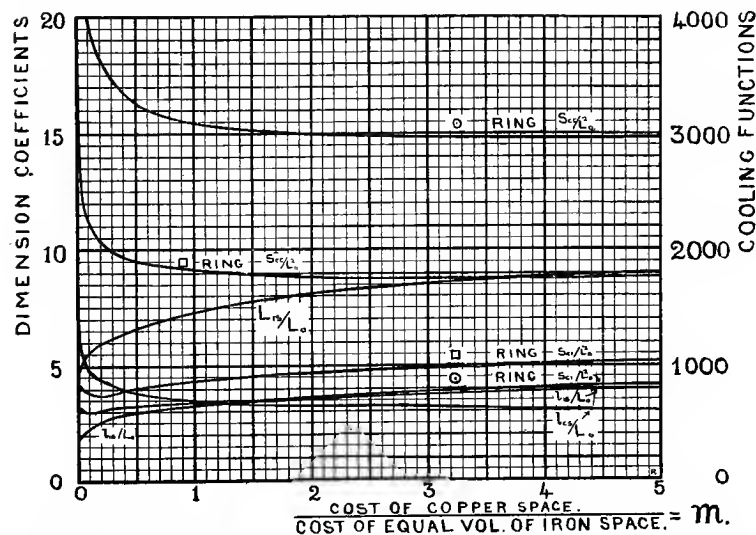


FIG. 10-24.—Dimension Coefficients for Ring Transformers with Rectangular Coils.

The fact that rectangular coils are cheaper than circular is quite unexpected, and may be explained thus. The proportions for the circular

coils also apply to the square coils, 4 being substituted for π in getting the cost function. Although the change from square to rectangular increases the perimeter for a given cross-section, it also reduces the mean length of the iron, and consequently permits a higher flux density and smaller cross-section to be employed. The smaller cross-section leads to a further diminution of volume and consequent increase of permissible flux density, and so on until not only the disadvantage as compared with the square but even the initial handicap of the square as compared with the circle is wiped out,

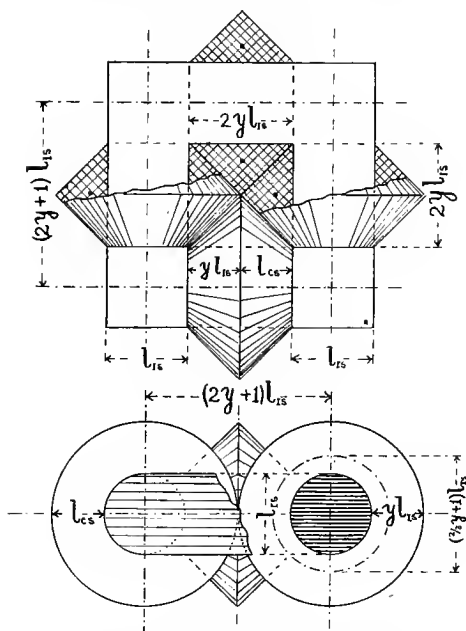


FIG. 10-25.—Square Ring Transformer with Circular Coils.

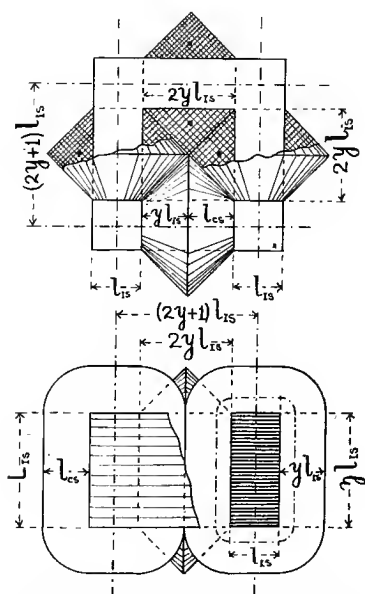


FIG. 10-26.—Square Ring Transformer with Rectangular Coils. (Burnand Type).

and in the end the rectangular proves cheapest of all. The difference amounts to 3 per cent. for the ordinary three-limb three-phase type, 5 per cent. for the core and shell types, 9 per cent. for ring transformers, 12 per cent. for the ordinary three-limb type of two-phase transformer, and 15 per cent. for simple transformers.

The first type is the ring type as originally made by Faraday. When made in the correct proportions (see figs. 10-19 and 10-20), this type requires less material than any other for the same efficiency, and it also has a large copper cooling surface. The circular ring is hardly a practical design, but special attention is directed to the hexagonal (figs. 10-21 and 10-22) and

octagonal rings, which retain a large part of the advantages of the circular ring; and although they have not, as far as the authors are aware, been proposed before, they do not appear to present any great structural difficulties. The correct proportions are the same for all the ring types, and are given in figs. 10·23 and 10·24. With the same efficiency, these will have a rather greater magnetising current than the shell type, and it will probably be found advisable to spend part of the saving in getting an increased efficiency. The square ring (figs. 10·25 and 10·26) has been patented by Burnand.¹

We may give the name of Simple Transformer to the type with one simple iron space interlinked with one simple copper space (see figs. 10·27–10·30). It is of no practical importance, as it is more expensive than the other types. But it is interesting as the basis from which the core and shell types are derived. It can be improved in either of two ways. If the iron be divided so that half goes on one side of the copper and half on the other, the mean length of the iron is reduced. This division of the iron space is the characteristic of the Shell Transformer (figs. 10·31–10·34). If, without altering the iron, the copper be divided so that half is wound on each limb, the mean length of one turn of copper is reduced. This division of the copper is the characteristic of a Core Transformer (figs. 10·35–10·38). The value of the material saved is only a small percentage in either case, but owing to the reduction of volume without reduction of effectiveness the efficiency is at the same time increased, and a still greater reduction ensues when the dimensions are all reduced to bring the efficiency to what it was before. The result is that what appears at first to be only a slight saving actually leads to the cost being little more than half what it was.

One would naturally expect that the one in which the initial saving was in the more expensive material would be the cheaper of the two types, but the reverse is true. The shell and core types cost the same when the specific cost ratio is about unity; the shell is the cheaper for greater values of this ratio, and the core for smaller values than unity. The reason for this is probably similar to the one for the cheapness of rectangular coils. The difference amounts to 10 per cent. in favour of the shell when the specific cost ratio is about 2·5, and in favour of the core when it is about 0·1.

We have already seen, in connection with fig. 10·12, that, except for the shell type, there is a nearly constant ratio between the costs of the different types for equal efficiencies. This ratio for rectangular coils is given in Table 10·02:—

¹ British Patent No. 21,410 of 1897, and No. 20,423 of 1901.

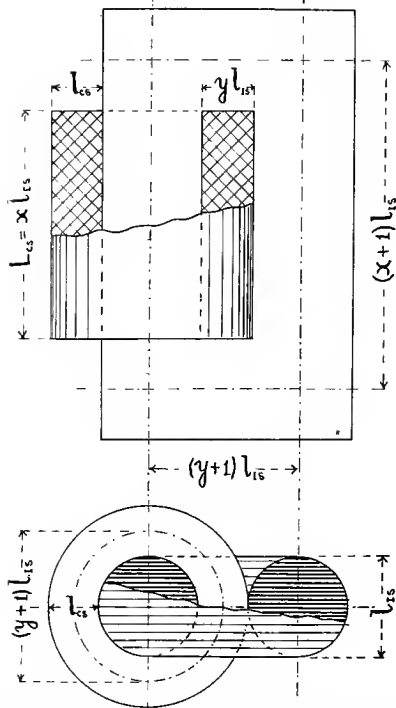


FIG. 10-27.—Simple Transformer with Circular Coils.

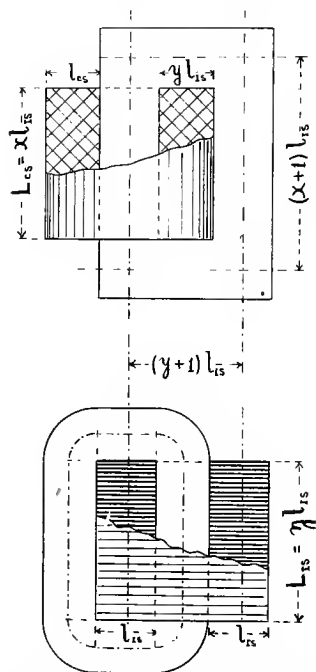


FIG. 10-29.—Simple Transformer with Rectangular Coils.

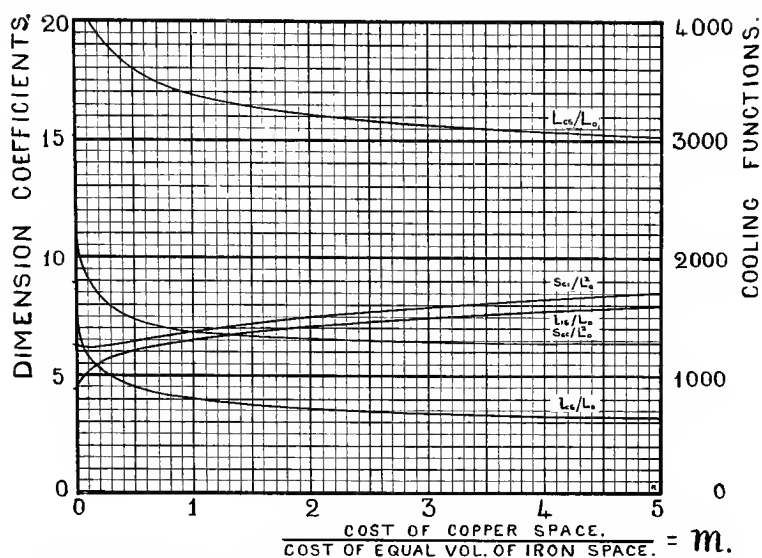


FIG. 10-28.—Dimension Coefficients for Simple Transformer with Circular Coils.

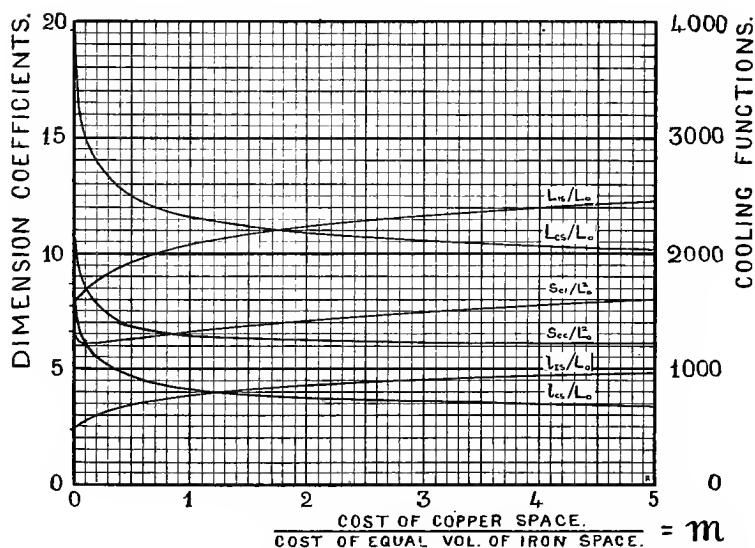


FIG. 10-30.—Dimension Coefficients for Simple Transformer with Rectangular Coils.

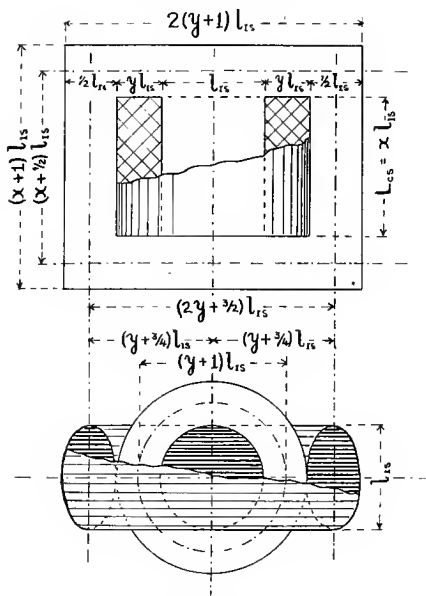


FIG. 10·31.—Shell Transformer with Circular Coils.

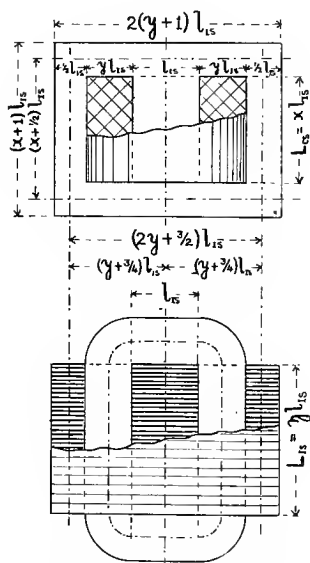


FIG. 10·33.—Shell Transformer with Rectangular Coils.

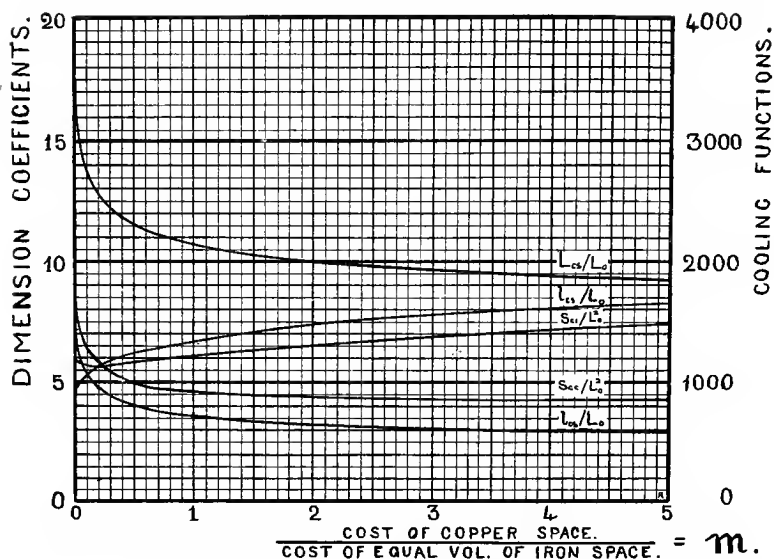


FIG. 10'32. —Dimension Coefficients for Shell Transformer with Circular Coils.

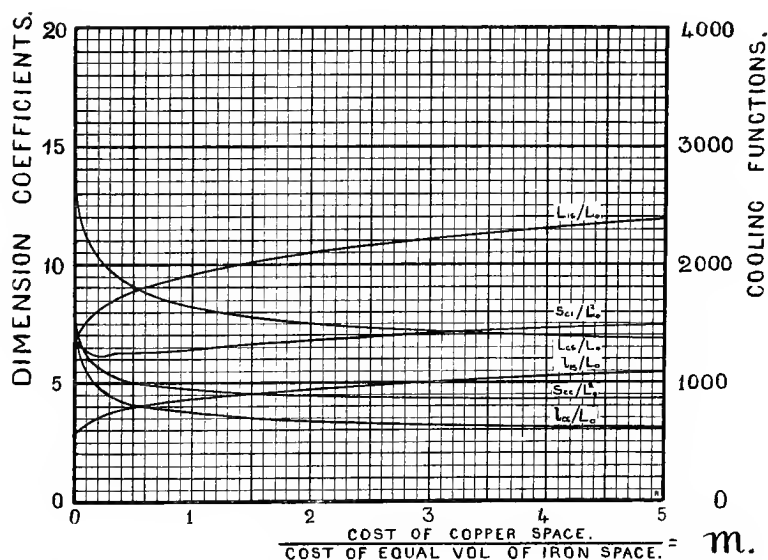


FIG. 10'34. —Dimension Coefficients for Shell Transformer with Rectangular Coils.

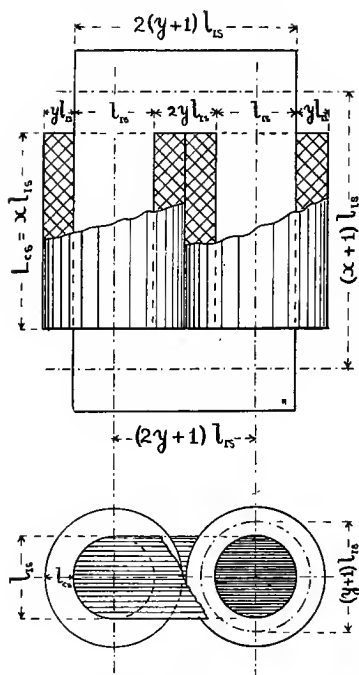


FIG. 10.35.—Core Transformer with Circular Coils.

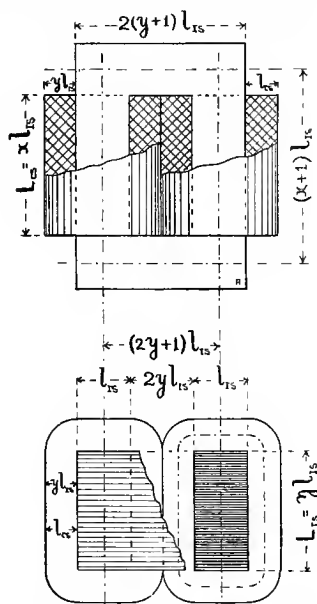


FIG. 10.37.—Core Transformer with Rectangular Coils.

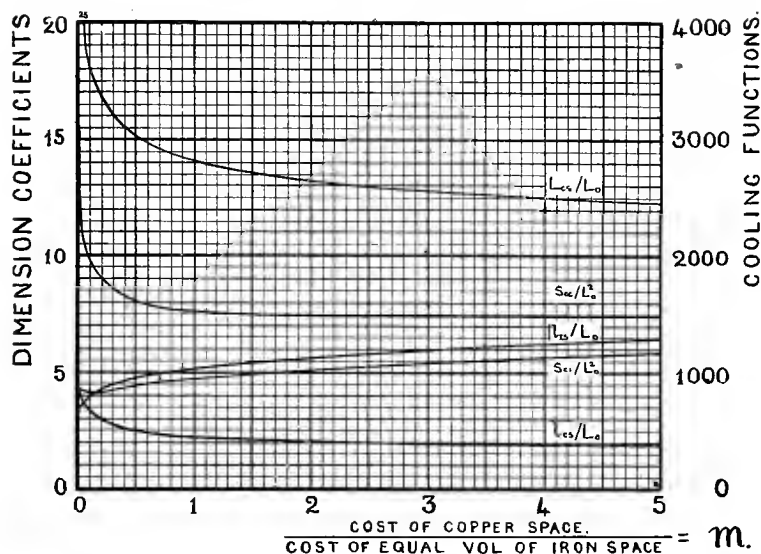


FIG. 10-36.—Dimension Coefficients for Core Transformer with Circular Coils.

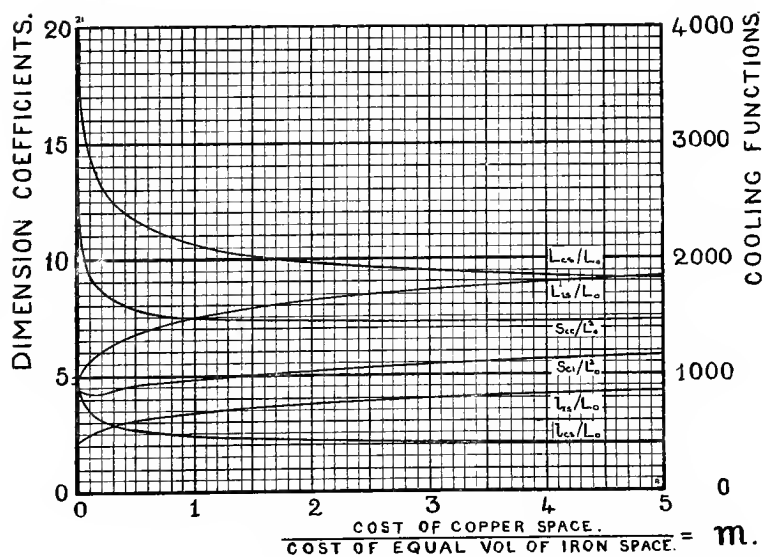


FIG. 10-38.—Dimension Coefficients for Core Transformer with Rectangular Coils.

TABLE 10·02.—APPROXIMATE RELATIVE COSTS FOR EQUAL EFFICIENCY OF RING, CORE, AND SIMPLE TRANSFORMERS WITH RECTANGULAR COILS.

Circular Ring.	Octagonal Ring.	Hexagonal Ring.	Square Ring.	Core.	Simple.
1·00	1·06	1·10	1·27	1·40	2·00

The curve for the shell type is similar, but displaced sidewise, and so the ratio of its cost to that of the others varies with the cost ratio, as shown in Table 10·03 :—

TABLE 10·03.—APPROXIMATE RELATIVE COSTS FOR EQUAL EFFICIENCY OF CORE AND SHELL TRANSFORMERS WITH RECTANGULAR COILS.

Specific Cost Ratio.	0	0·5	0·7	1·0	1·5	2·0	3·0	4·0
Relative cost of Shell compared with Circular Ring	2·14	1·51	1·46	1·40	1·34	1·29	1·25	1·21
Relative cost of Core compared with Shell	0·71	0·95	0·97	1·00	1·05	1·09	1·12	1·15

It will, in fact, be noticed that for the shell transformer the values of y and z are the same, of x half, of the distance between the yokes $1/\sqrt{2}$ times, of the other dimension coefficients and of the cost function $\sqrt{2}$ times that for a core transformer designed for half the specific cost ratio.

The core has a larger total cooling surface than the shell, and most of that is on the copper, where it is most necessary, whereas most of that on the shell type is on the iron. The coils of the core type are both longer and thinner, making the leakage inductance less with the same amount of subdivision. The shell works at a rather lower flux density and takes a somewhat smaller magnetising current, and also simplifies matters when several voltages are required from one transformer. As a rule, the core type is to be preferred, for with modern alloyed iron the specific cost ratio is brought down to a figure at which its relative cost is either less, or very slightly over, that of the shell.

The formulæ for the circular coil shell type apply whether the iron is arranged in one plane as usual, or radially as in the Berry type. The latter must be expensive for its efficiency, owing to the fact that it is based on the

circular coil, and to the small iron space-factor which it gives because of the space that must be wasted in the centre. It has an exceptionally large iron surface, but the ordinary shell transformer has already much more cooling surface on the iron than on the copper.

Comparison of Types: Two-Phase Transformers.—The ordinary two-phase transformer (figs. 10·39–10·42) may be obtained by combining two simple transformers with their unwound limbs together. The section of the common return need then only be $\sqrt{2}$ times that of the wound limbs, and this reduction of the iron leads to further reductions all round. The two-phase is thus considerably cheaper than the two transformers from which it was derived, but the saving is not enough to balance the initial disadvantage of the simple type, and so it is still dearer than two separate transformers of the ordinary types. Compared with the circular ring, its price is approximately $3\cdot10$ or $2 \times 1\cdot55$ times as much. There therefore seems to be no sufficient reason why this type should be made.

Two shell or core transformers may be combined tandem fashion with their cores in line (figs. 10·43–10·50), leading in each case to a saving in iron and consequent remodelling of the design. When the cheapest proportions of this arrangement are obtained it is found in both cases that the distance between the yokes is reduced in the ratio of $(2 + \sqrt{2}) : 4$, or $0\cdot854 : 1$, from that of the single-phase type, while the other dimensions remain the same. The cost has been increased in the ratio $(2 + \sqrt{2}) : 2$, and so it is cheaper than two single-phase transformers of the same efficiency in the ratio $(2 + \sqrt{2}) : 4$, and the relative cost of the core type becomes $2 \times 1\cdot19$ when compared with the circular ring. The relative merits of the tandem core and shell types are the same as for single-phase transformers.

Comparison of Types: Three - Phase Transformers.—When three simple transformers are united to form a single three-phase unit the return core is not required, and so a very large saving is possible. The three cores are usually placed in one plane for convenience of manufacture (figs. 10·51–10·54), but occasionally symmetrical types are made. The saving has been so great that the usual three-phase transformer is much cheaper than three separate ones, its relative figure being approximately $2\cdot34$ or $3 \times 0\cdot78$, which is less than that of two core or shell transformers. Two complete three-phase transformers are thus rather cheaper than a set of four single-phase core or shell transformers of the same efficiency, so that, even allowing for spares, the three-phase is to be preferred.

When core or shell transformers are combined in tandem to make a three-phase one (figs. 10·55–10·62), the distance between the yokes is reduced to $\frac{2}{3}$ and the cost is doubled. The other dimensions remain the

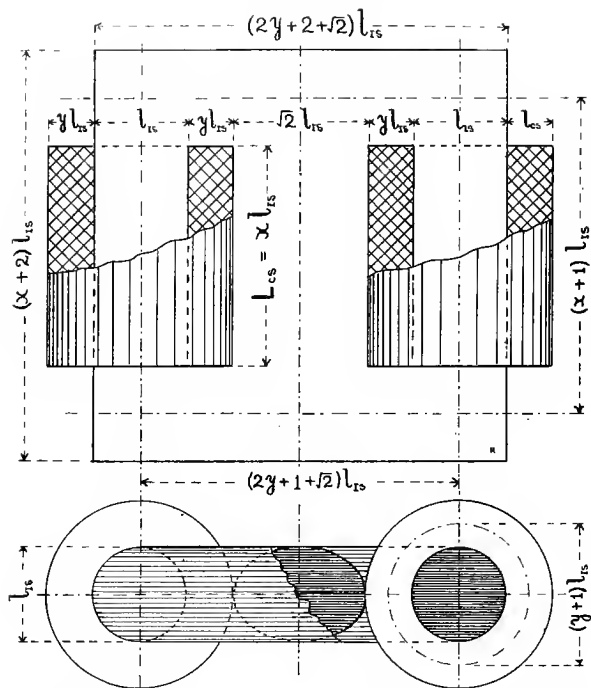


FIG. 10'39.—Two-Phase Simple Transformer with Circular Coils.

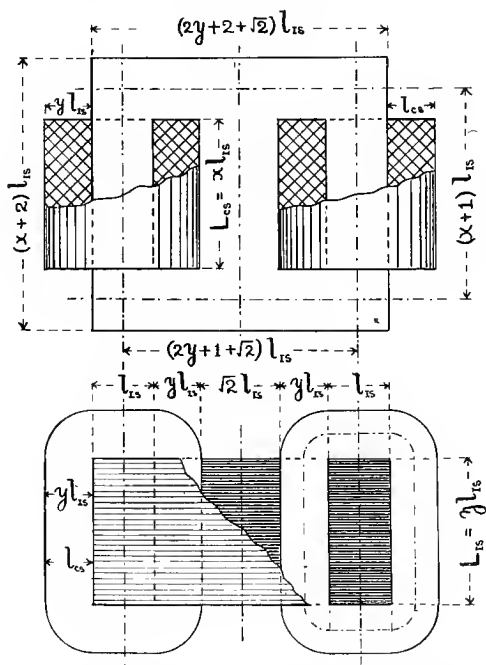


FIG. 10'41.—Two-Phase Simple Transformer with Rectangular Coils.

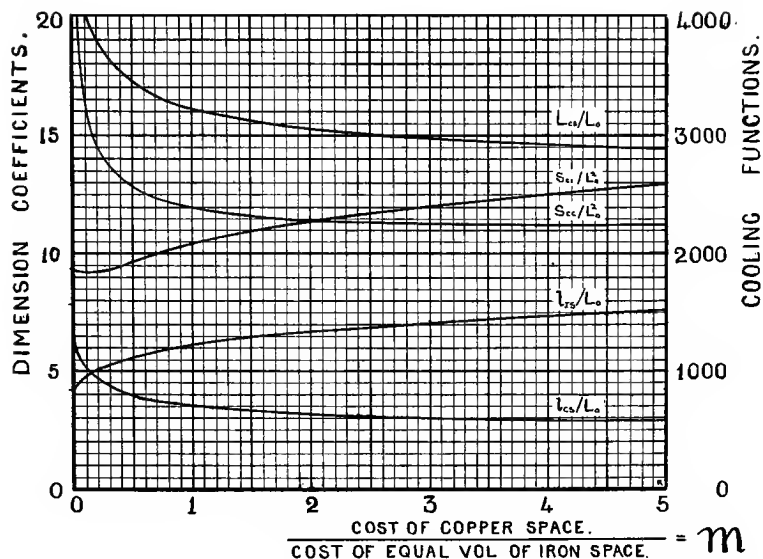


FIG. 10.40.—Dimension Coefficients for Two-Phase Simple Transformer with Circular Coils.

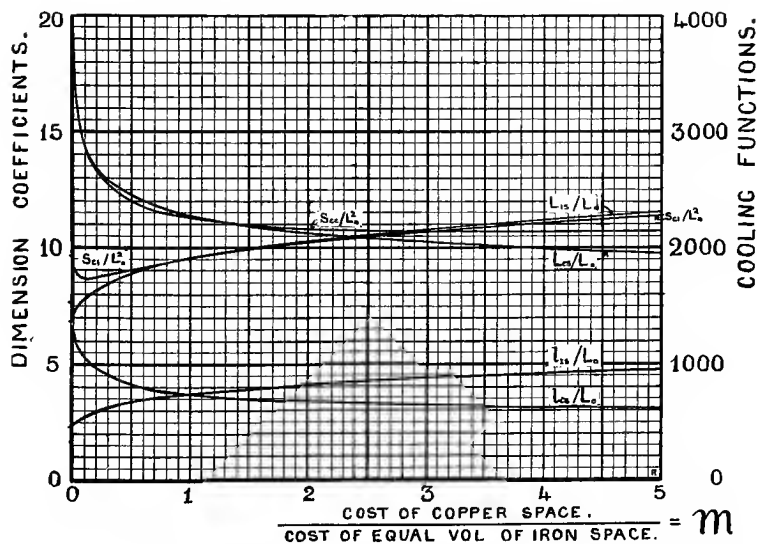


FIG. 10.42.—Dimension Coefficients for Two-Phase Simple Transformer with Rectangular Coils.

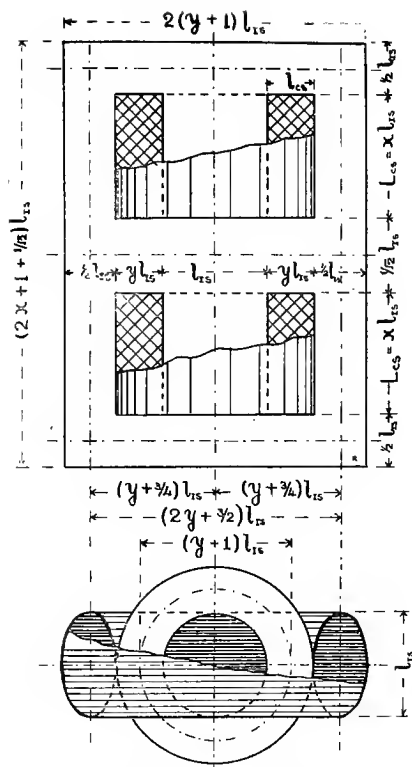


FIG. 10·43.—Two-Phase Shell Transformer with Circular Coils.

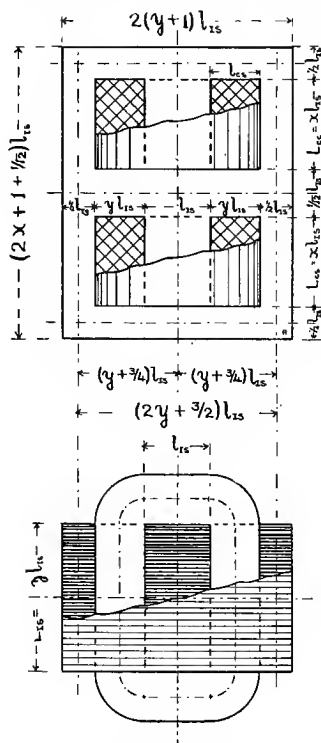


FIG. 10·45.—Two-Phase Shell Transformer with Rectangular Coils.

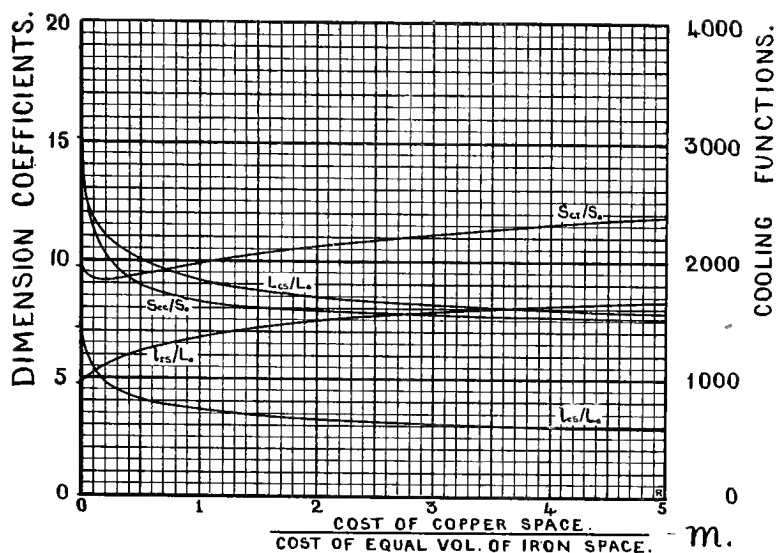


Fig. 10'44.—Dimension Coefficients for Two-Phase Shell Transformer with Circular Coils.

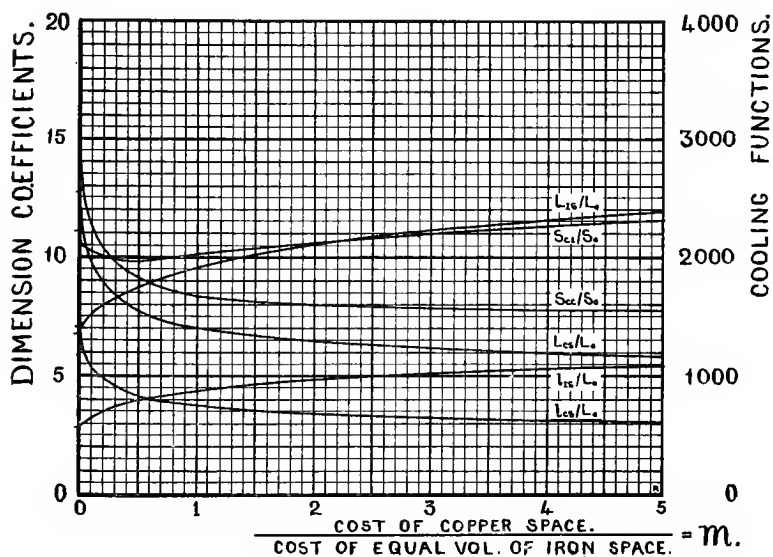


Fig. 10'46.—Dimension Coefficients for Two-Phase Shell Transformer with Rectangular Coils.

same. One of these arrangements thus costs $\frac{2}{3}$ of a set of three single-phase transformers of the same type to give the same efficiency, and the relative figure is 3×0.93 for the core type. The comparison between the two types is the same as for single-phase.

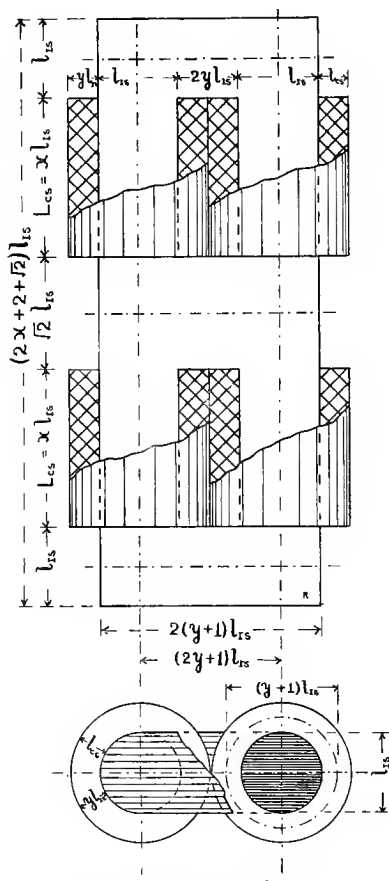


FIG. 10.47.—Two-Phase Tandem Core Transformer with Circular Coils.

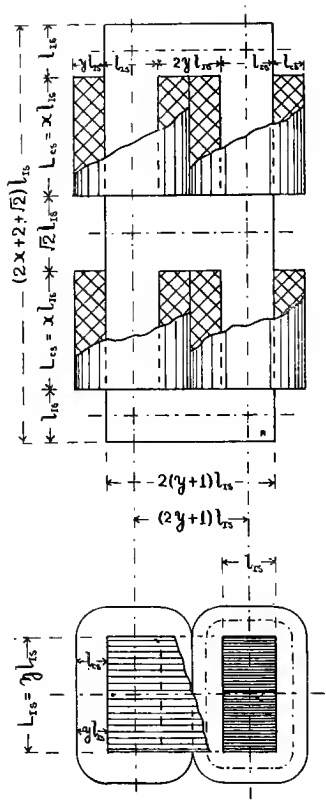


FIG. 10.49.—Two-Phase Tandem Core Transformer with Rectangular Coils.

It will be found that the prices of polyphase transformers listed by the leading makers do not show such a large saving as is here indicated, because part of the saving has to be spent in obtaining a larger efficiency than is customary with single-phase transformers of the corresponding output. Otherwise the polyphase one would get too hot.

Figures have not been worked out for the symmetrical three-phase type,

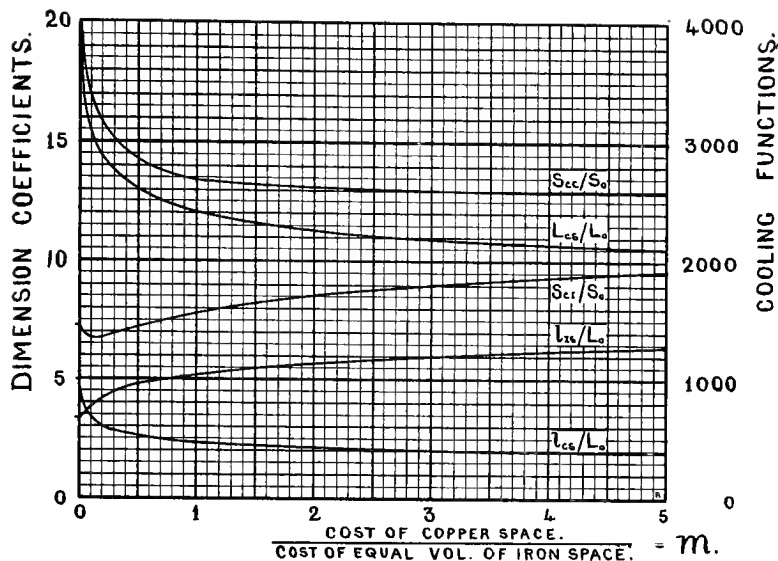


FIG. 10-48.—Dimension Coefficients for Two-Phase Tandem Core Transformer with Circular Coils.

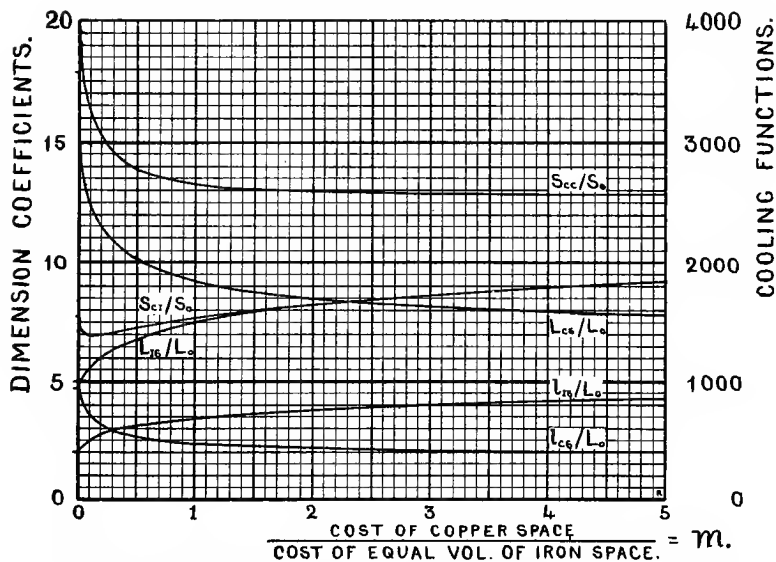


FIG. 10-50.—Dimension Coefficients for Two-Phase Tandem Core Transformer with Rectangular Coils.

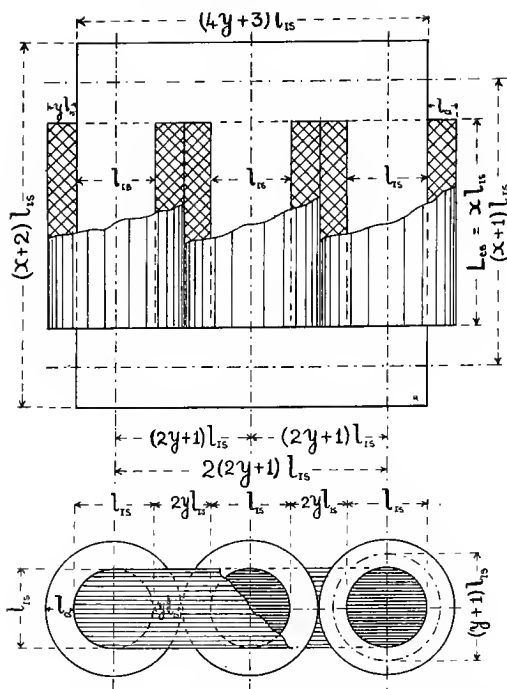


FIG. 10·51.—Three-Phase Three-Limb Transformer with Circular Coils.

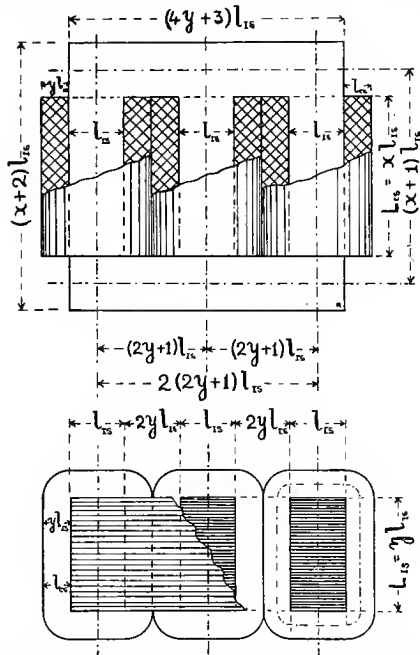


FIG. 10·53.—Three-Phase Three-Limb Transformer with Rectangular Coils.

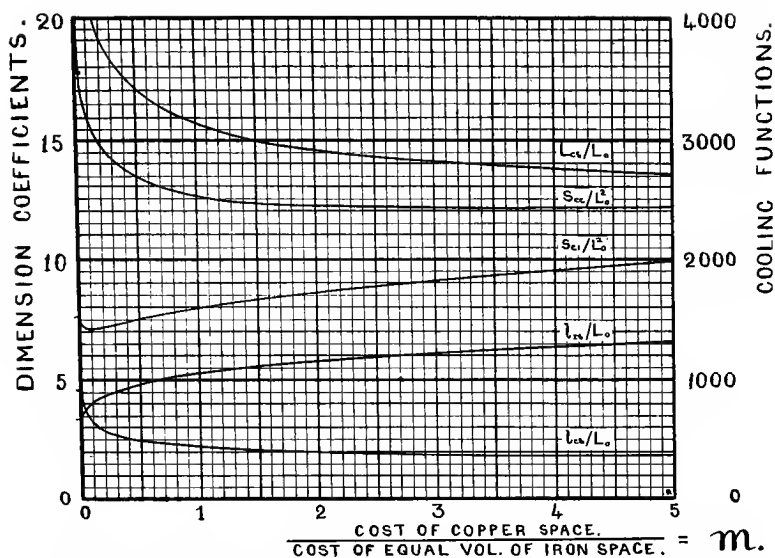


FIG. 10.52.—Dimension Coefficients for Three-Phase Three-Limb Transformer with Circular Coils.

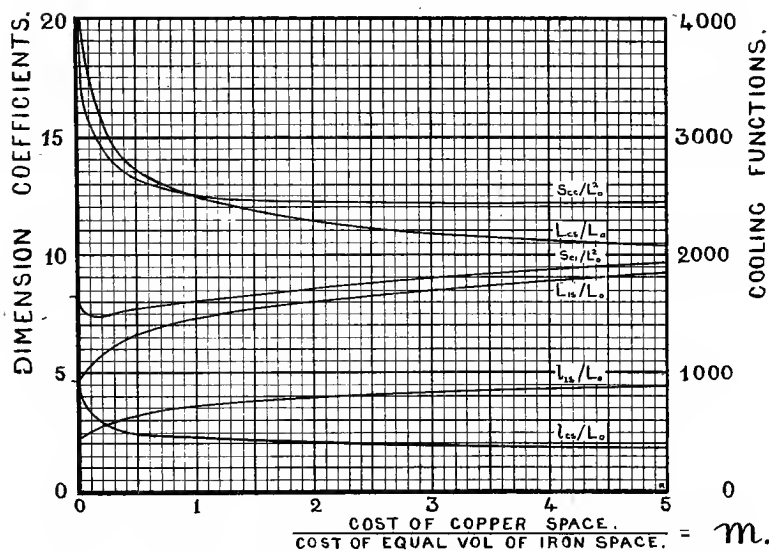


FIG. 10.54.—Dimension Coefficients for Three-Phase Three-Limb Transformer with Rectangular Coils.

as it is much more expensive to make. There is no difficulty, however, in applying the formulæ to find its best proportions. They could not, however, be used for the type with a common magnetic return, as that does no useful work. If such a type were required, it would be best

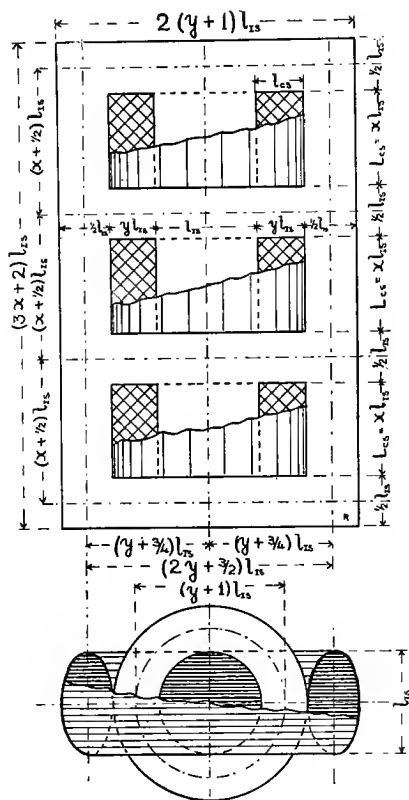


FIG. 10-55. —Three-Phase Shell Transformer with Circular Coils.

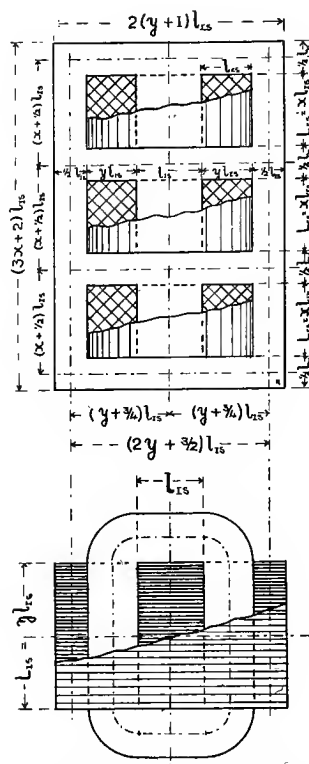


FIG. 10-57. —Three-Phase Shell Transformer with Rectangular Coils.

to employ one of the tandem types which have independent magnetic circuits.

Ring Transformers with Conical Coils.—The ring-shaped transformer, as originally made by Faraday, is the simplest to deal with mathematically, for with it there is only one variable when the coils are circular, and two when they are rectangular. Taking the general case in which the ring is a regular polygon having d sides, each wound with

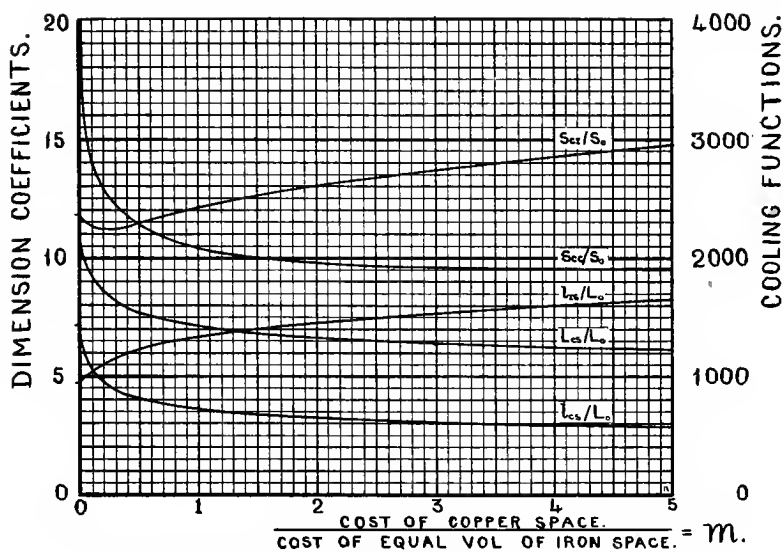


FIG. 10·56.—Dimension Coefficients for Three-Phase Shell Transformer with Circular Coils.

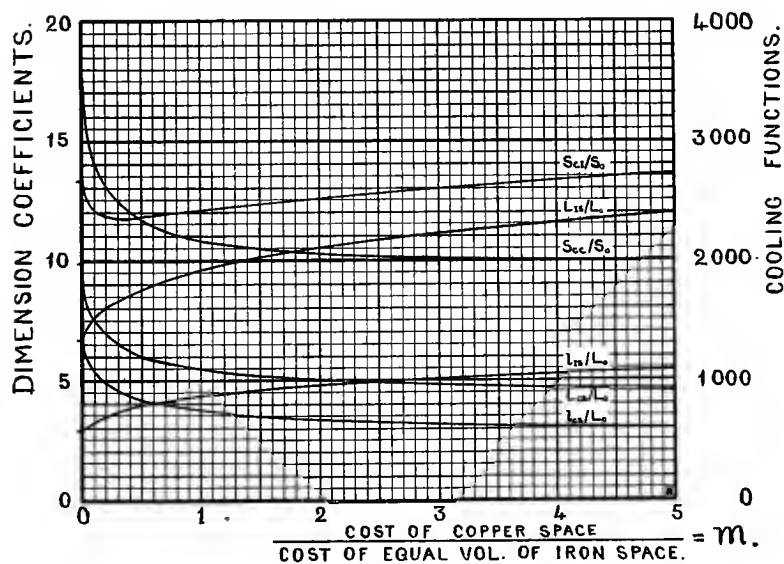


FIG. 10·58.—Dimension Coefficients for Three-Phase Shell Transformer with Rectangular Coils.

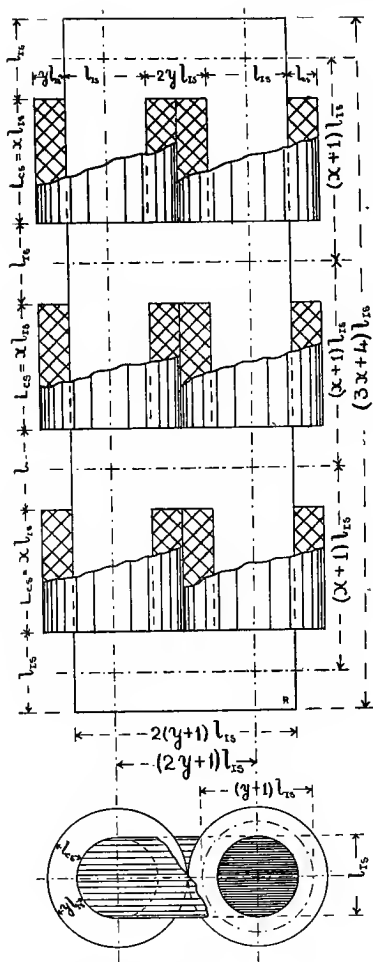


FIG. 10.59.—Three-Phase Tandem Core Transformer with Circular Coils.

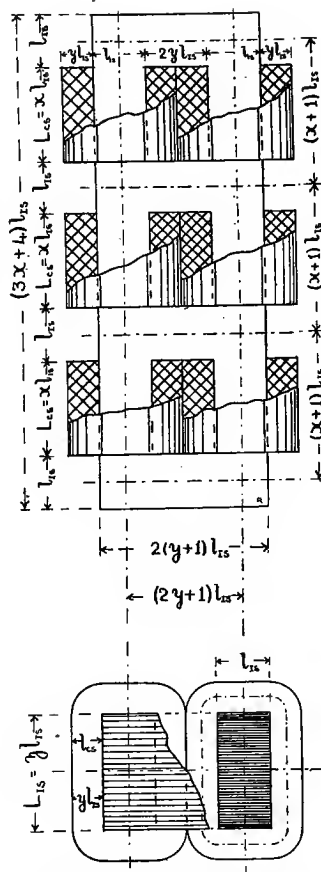


FIG. 10.61.—Three-Phase Tandem Core Transformer with Rectangular Coils.

wire piled up so as to fit into the sector in the centre of the ring, we have:—

$$L_I = d(l_{CS} + \frac{1}{2}l_{IS}) \times 2 \tan \pi/d = (d \tan \pi/d)(2y+1)l_{IS} \quad . \quad 10.25,$$

$$S_{CS} = d \times \frac{1}{2} \times 2 \tan (\pi/d) l_{CS}^2 = (d \tan \pi/d) y^2 l_{IS}^2 \quad . \quad 10.26,$$

$$S_{IS} = \frac{\pi}{4} l_{IS}^2, \text{ for circular coils} \quad . \quad 10.27,$$

$$S_{IS} = l_{IS} L_{IS} = 2 l_{IS}^2, \text{ for rectangular coils} \quad . \quad 10.28.$$

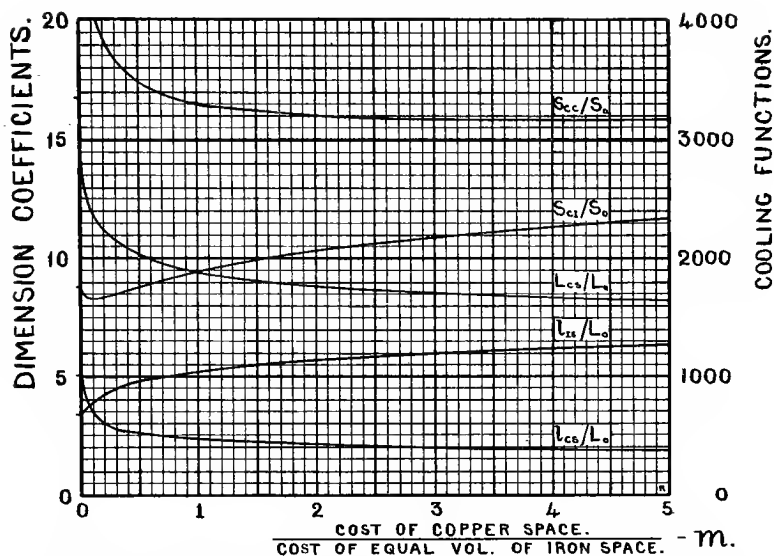


FIG. 10'60.—Dimension Coefficients for Three-Phase Tandem Core Transformer with Circular Coils.

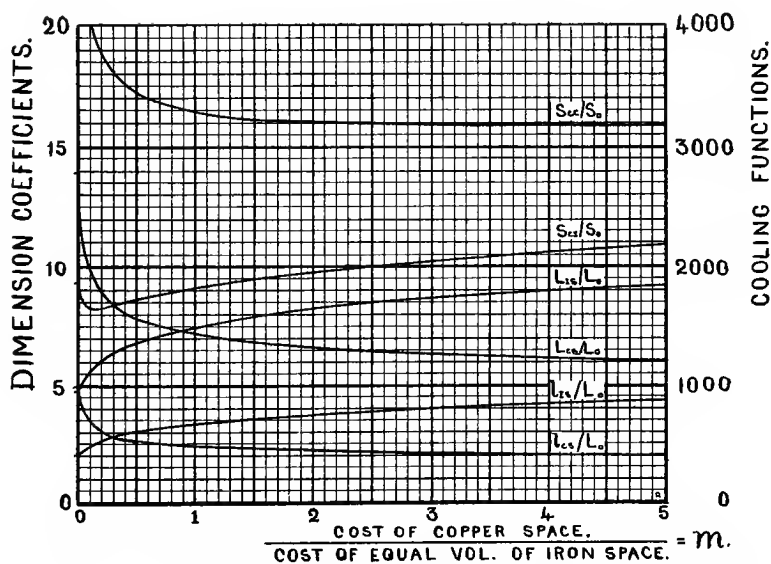


FIG. 10'62.—Dimension Coefficients for Three-Phase Tandem Core Transformer with Rectangular Coils.

The mean turn will be situated at one-third of the winding depth out from the inside, and so the mean length of one turn will be:—

$$L_C = \pi(\frac{2}{3}l_{CS} + l_{IS}) = \frac{\pi}{3}(2y + 3)l_{IS}, \text{ for circular coils} \quad 10\cdot29,$$

$$L_C = el_{CS} + 2l_{IS} + 2L_{IS} = (ey + 2 + 2z)l_{IS}, \text{ for rectangular coils} \quad 10\cdot30,$$

where e is a number to allow for the corners, which has been taken as 2 in getting the numerical values. If the corners were arcs of circles with their centres at the edge of the iron space, e would be $2\pi/3$; but the wire will actually bed more closely at the corners than elsewhere, and so e will be smaller than this. If the corners of the iron space be chamfered away so as to shorten the wires at the corners, e will be reduced still more, and might even be reduced to zero. The centre of the sloping sides of the coils is half-way up the winding depth, and so we must take $\frac{3}{2}e$ for the corners when reckoning the surface of these sides.

It will be noticed that none of the factors in the cost function which contain the variables are affected by the number of sides in the polygon. Hence the same proportions and dimension coefficients will give a minimum cost function whatever the number of sides may be. The cost functions, and also the iron cooling functions, are proportional to the values of $d \tan(\pi/d)$, which are given in the following table. This factor becomes π for a circular ring, but in that case, when reckoning the surface of the copper space, it is to be multiplied into the last pair of brackets, and $d \sec(\pi/d)$ is to be taken simply as the actual number of sectors into which the windings are divided.

TABLE 10·04.—COMPARISON OF DIFFERENT TYPES OF RING FOR EQUAL EFFICIENCIES.

Number of Sides, d .	3.	4.	5.	6.	8.	9.	12.	18.	Infinite.
	$3\sqrt{3} =$			$2\sqrt{3} =$					$\pi =$
$d \tan(\pi/d)$	5·196	4·000	3·633	3·464	3·314	3·276	3·215	3·174	3·142
Relative cost	1·654	1·273	1·156	1·103	1·055	1·043	1·024	1·010	1·000
Cosec (π/d)	1·155	1·414	1·701	2·000	2·613	2·924	3·864	5·759	...

Ring Transformers with Circular Coils.—Putting in these values for the lengths and cross-sections, the expression for the cost function becomes:—

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \frac{2\pi(d \tan \pi/d)(2y + 1)^{\frac{1}{2}}(2y + 3)^{\frac{1}{2}} \left[1 + \frac{4my^2(2y + 3)}{3(2y + 1)} \right]}{3\sqrt{3}y^3} \quad 10\cdot31.$$

For a minimum with different values of y ,

$$0 = \frac{\partial}{\partial y} \left\{ \left[\frac{(2y + 1)^5(2y + 3)^3}{y^6} \right]^{\frac{1}{2}} + \frac{4m}{3} \left[\frac{(2y + 1)^3(2y + 3)^5}{y^2} \right]^{\frac{1}{2}} \right\} \quad 10\cdot32.$$

$$\begin{aligned}
&= \left[\frac{(2y+1)^5(2y+3)^3}{y^6} \right]^{-\frac{1}{3}} \times \\
&\quad \left[\frac{y^6(2y+1)^5 \times 6(2y+3)^2 + y^6 \times 10y(2y+1)^4(2y+3)^3 - 6y^5(2y+1)^5(2y+3)^3}{y^{12}} \right] \\
&+ \frac{4m}{3} \left[\frac{(2y+1)^3(2y+3)^5}{y^2} \right]^{-\frac{1}{3}} \times \\
&\quad \left[\frac{y^2(2y+1)^3 \times 10(2y+3)^4 + y^2 \times 6y(2y+1)^2(2y+3)^5 - 2y(2y+1)^3(2y+3)^5}{y^4} \right] \\
&= 3(2y+1)[6y(2y+1) + 10y(2y+3) - 6(2y+1)(2y+3)] \\
&\quad + 4my^2(2y+3)[10y(2y+1) + 6y(2y+3) - 2(2y+1)(2y+3)] \\
&= 3(2y+1)[8y^2 - 12y - 18] + 4my^2(2y+3)[24y^2 + 12y - 6] \\
&= (2y+1)\{4y^2 - 6y - 9\} + 4my^2(2y+3)\{4y^2 + 2y - 1\}. \\
\therefore m &= \frac{(2y+1)\{-4y^2 + 6y + 9\}}{4y^2(2y+3)\{4y^2 + 2y - 1\}} \quad \dots \quad 10\cdot33,
\end{aligned}$$

$$\frac{\text{Total cost of copper}}{\text{Total cost of iron}} = m \frac{V_{CS}}{V_{IS}} = \frac{4my^2(2y+3)}{3(2y+1)} = \frac{\{-4y^2 + 6y + 9\}}{3\{4y^2 + 2y - 1\}} \quad 10\cdot34,$$

$$\begin{aligned}
\frac{\mathcal{L}_{\text{MIN}}}{\mathcal{L}_0} &= \frac{2\pi(d \tan \pi/d)(2y+1)^{\frac{1}{3}}(2y+3)^{\frac{2}{3}}}{3 \sqrt{3}y^3} \left[1 + \frac{\{-4y^2 + 6y + 9\}}{3\{4y^2 + 2y - 1\}} \right] \\
&= \frac{4\pi(d \tan \pi/d)(2y+1)^{\frac{1}{3}}(2y+3)^{\frac{2}{3}}\{4y^2 + 6y + 3\}}{9 \sqrt{3}y^3\{4y^2 + 2y - 1\}} \quad \dots \quad 10\cdot35.
\end{aligned}$$

For *any* proportions,

$$L_0 = \sqrt{\frac{S_{IS} S_{CS}}{L_I L_C}} = \left[\frac{3y^2}{4(2y+1)(2y+3)} \right]^{\frac{1}{3}} l_{IS} \quad \dots \quad 10\cdot36.$$

$$\therefore \frac{l_{IS}}{L_0} = \frac{2(2y+1)^{\frac{1}{3}}(2y+3)^{\frac{2}{3}}}{\sqrt{3}y} \quad \dots \quad 10\cdot37,$$

$$\frac{l_{CS}}{L_0} = y \frac{l_{IS}}{L_0} = \frac{2(2y+1)^{\frac{1}{3}}(2y+3)^{\frac{2}{3}}}{\sqrt{3}} \quad \dots \quad 10\cdot38,$$

$$\begin{aligned}
S_{CI} &= \pi \times (d \tan \pi/d)(2y+1)l_{IS}^2. \\
\therefore \frac{S_{CI}}{S_0} &= \frac{4\pi(d \tan \pi/d)(2y+1)^2(2y+3)}{3y^2} \quad \dots \quad 10\cdot39,
\end{aligned}$$

$$\begin{aligned}
S_{CC} &= 2\pi(d \tan \pi/d)y\{(y+1)/\sin \pi/d + 1\}l_{IS}^2. \\
\therefore \frac{S_{CC}}{S_0} &= \frac{8\pi(d \tan \pi/d)(2y+1)(2y+3)\{(y+1)/\sin \pi/d + 1\}}{3y} \quad 10\cdot40.
\end{aligned}$$

For the circular ring,

$$\frac{S_{CC}}{S_0} = \frac{8\pi(2y+1)(2y+3)[dy + (d + \pi)]}{3y} \quad \dots \quad 10\cdot41.$$

Ring Transformers with Rectangular Coils.—Here

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \frac{(d \tan \pi/d)(2y+1)^{\frac{2}{3}}(ey+2+2z)^{\frac{2}{3}}}{y^{\frac{2}{3}}z^{\frac{1}{3}}} \left[1 + my^2 \frac{(ey+2+2z)}{z(2y+1)} \right] \quad 10\cdot42.$$

For a minimum with different values of y ,

$$0 = \frac{\partial}{\partial y} \left\{ \left[\frac{(2y+1)^5(ey+2+2z)^3}{y^6} \right]^{\frac{1}{3}} + \frac{m}{z} \left[\frac{(2y+1)^3(ey+2+2z)^5}{y^2} \right]^{\frac{1}{3}} \right\} \quad 10\cdot43.$$

$$= z(2y+1)[3ey(2y+1) + 10y(ey+2+2z) - 6(2y+1)(ey+2+2z)] \\ + my^2(ey+2+2z)[5ey(2y+1) + 6y(ey+2+2z) - 2(2y+1)(ey+2+2z)]$$

$$\therefore z(2y+1)[3ey(2y+1) - 2(y+3)(ey+2+2z)] \\ = -my^2(ey+2+2z)[5ey(2y+1) + 2(y-1)(ey+2+2z)] \quad 10\cdot44.$$

For a minimum with different values of z ,

$$0 = \frac{\partial}{\partial z} \left\{ \left[\frac{(ey+2+2z)^3}{z} \right]^{\frac{1}{3}} + \frac{my^2}{(2y+1)} \left[\frac{(ey+2+2z)^5}{z^3} \right]^{\frac{1}{3}} \right\} \quad 10\cdot45$$

$$= z(2y+1)[6z - (ey+2+2z)] + my^2(ey+2+2z)[10z - 3(ey+2+2z)].$$

$$\therefore z(2y+1)\{4z - (ey+2)\} = -my^2(ey+2+2z)\{4z - 3(ey+2)\} \quad 10\cdot46.$$

Dividing each side by the corresponding one of equation 10·44 and multiplying across we get:—

$$\{4z - (ey+2)\}[5ey(2y+1) + 2(y-1)(ey+2+2z)] \\ = \{4z - 3(ey+2)\}[3ey(2y+1) - 2(y+3)(ey+2+2z)] \quad 10\cdot47.$$

$$\therefore 0 = ey(2y+1)[5\{4z - (ey+2)\} - 3\{4z - 3(ey+2)\}] \\ + 2(ey+2+2z)[(y-1)\{4z - (ey+2)\} + (y+3)\{4z - 3(ey+2)\}] \\ = ey(2y+1)[8z + 4ey + 8] + 2(ey+2+2z)[(8y+8)z - (4y+8)(ey+2)] \\ = ey(2y+1) + 4(y+1)z - 2(y+2)(ey+2).$$

$$\therefore 4(y+1)z = (3e+4)y + 8.$$

$$\therefore z = \frac{\{(3e+4)y + 8\}}{4(y+1)} \quad 10\cdot48,$$

$$(ey+2+2z) = \frac{2(y+1)(ey+2) + \{(3e+4)y + 8\}}{2(y+1)} \\ = \frac{\{2ey^2 + (5e+8)y + 12\}}{2(y+1)} \quad 10\cdot49,$$

$$\{4z - (ey+2)\} = \frac{\{(3e+4)y + 8\} - (y+1)(ey+2)}{(y+1)} \\ = \frac{\{-ey^2 + 2(e+1)y + 6\}}{(y+1)} \quad 10\cdot50,$$

$$\{4z - 3(ey+2)\} = \frac{\{(3e+4)y + 8\} - 3(y+1)(ey+2)}{(y+1)} = \frac{\{-3ey^2 - 2y + 2\}}{(y+1)} \quad 10\cdot51,$$

Putting these values in equation 10·46, we get:—

$$\frac{\{(3e+4)y + 8\}}{4(y+1)}(2y+1) \frac{\{-ey^2 + 2(e+1)y + 6\}}{(y+1)} \\ = -my^2 \frac{\{2ey^2 + (5e+8)y + 12\}}{2(y+1)} \frac{\{-3ey^2 - 2y + 2\}}{(y+1)}$$

$$\therefore m = \frac{(2y+1)\{(3e+4)y + 8\}\{-ey^2 + 2(e+1)y + 6\}}{2y^2\{2ey^2 + (5e+8)y + 12\}\{3ey^2 + 2y - 2\}} \quad 10\cdot52,$$

$$\begin{aligned}\frac{\text{Total cost of copper}}{\text{Total cost of iron}} &= m \frac{V_{CS}}{V_{IS}} = \frac{my^2(ey + 2 + 2z)}{z(2y + 1)} \quad (\text{from equation 10.42}) \\ &= - \frac{\{4z - (ey + 2)\}}{\{4z - 3(ey + 2)\}} \quad (\text{from equation 10.46}) \\ &= \frac{\{-ey^2 + 2(e + 1)y + 6\}}{\{3ey^2 + 2y - 2\}} \quad 10.53,\end{aligned}$$

$$\begin{aligned}\frac{\mathcal{L}_{\text{MIN}}}{\mathcal{L}_0} &= \frac{(d \tan \pi/d)(2y + 1) \left[\frac{\{2ey^2 + (5e + 8)y + 12\}}{2(y + 1)} \right]^{\frac{1}{2}}}{y^3 \left[\frac{\{(3e + 4)y + 8\}}{4(y + 1)} \right]^{\frac{1}{2}}} \times \\ &\quad \left[1 + \frac{\{-ey^2 + 2(e + 1)y + 6\}}{\{3ey^2 + 2y - 2\}} \right] \\ &= \frac{\sqrt{2}(d \tan \pi/d)(ey + 2)(2y + 1)^{\frac{1}{2}} \{2ey^2 + (5e + 8)y + 12\}^{\frac{1}{2}}}{y^3 \{(3e + 4)y + 8\}^{\frac{1}{2}} \{3ey^2 + 2y - 2\}} \quad 10.54.\end{aligned}$$

For any proportions,

$$\begin{aligned}L_0 &= \sqrt{\frac{S_{IS} S_{CS}}{L_I L_C}} = \left[\frac{y^2 z}{(2y + 1)(ey + 2 + 2z)} \right]^{\frac{1}{2}} l_{IS}. \\ \therefore l_{IS} &= \left[\frac{(2y + 1)(ey + 2 + 2z)}{y^2 z} \right]^{\frac{1}{2}} L_0 \quad 10.55.\end{aligned}$$

And for the cheapest proportions,

$$\frac{l_{IS}}{L_0} = \frac{\sqrt{2}(2y + 1)^{\frac{1}{2}} \{2ey^2 + (5e + 8)y + 12\}^{\frac{1}{2}}}{y \{(3e + 4)y + 8\}^{\frac{1}{2}}} \quad 10.56,$$

$$\frac{l_{CS}}{L_0} = y \frac{l_{IS}}{L_0} = \frac{\sqrt{2}(2y + 1)^{\frac{1}{2}} \{2ey^2 + (5e + 8)y + 12\}^{\frac{1}{2}}}{\{(3e + 4)y + 8\}^{\frac{1}{2}}} \quad 10.57,$$

$$\frac{L_{IS}}{L_0} = z \frac{l_{IS}}{L_0} = \frac{(2y + 1)^{\frac{1}{2}} \{(3e + 4)y + 8\}^{\frac{1}{2}} \{2ey^2 + (5e + 8)y + 12\}^{\frac{1}{2}}}{2 \sqrt{2} y (y + 1)} \quad 10.58,$$

$$S_{CI} = 2(d \tan \pi/d)(2y + 1)(1 + z) l_{IS}^2.$$

$$\therefore \frac{S_{CI}}{S_0} = \frac{(d \tan \pi/d)(2y + 1)^2 \{(3e + 8)y + 12\} \{2ey^2 + (5e + 8)y + 12\}}{y^2 (y + 1) \{(3e + 4)y + 8\}} \quad 10.59,$$

$$S_{CC} = 2(d \tan \pi/d) y \left\{ \frac{\{3ey^2 + 2 + 2z\}}{\sin \pi/d} + 2(1 + z) \right\} l_{IS}^2.$$

$$\begin{aligned}\therefore \frac{S_{CC}}{S_0} &= \frac{2(d \tan \pi/d)(2y + 1) \{2ey^2 + (5e + 8)y + 12\}}{y(y + 1) \{(3e + 4)y + 8\}} \times \\ &\quad \left[\frac{\{3ey^2 + 2(3e + 4)y + 12\}}{\sin \pi/d} + \{(3e + 8)y + 12\} \right] \quad 10.60.\end{aligned}$$

For the circular ring ;

$$\begin{aligned}\frac{S_{CC}}{S_0} &= \frac{2(2y + 1) \{2ey^2 + (5e + 8)y + 12\}}{y(y + 1) \{(3e + 4)y + 8\}} \times \\ &\quad [3dey^2 + \{2(3e + 4)d + (3e + 8)\pi\}y + 12(d + \pi)] \quad 10.61.\end{aligned}$$

TABLE 10-05.—PARTICULARS OF CHEAPEST RING TRANSFORMERS WITH RECTANGULAR COILS.*

Specific Cost Ratio.	m.	Proportional Dimensions.		Dimension Coefficients.	Iron Cooling Functions S_{ci}/S_o .						Copper Cooling Functions S_{ci}/S_o .						Cost Function $\mathcal{L}/\mathcal{L}_0$.						Total Cost of Copper. Total Cost of Iron.			
		y .	z .		$L_{is}/L_{o'}$.	$L_{cs}/L_{o'}$.	$L_{is}/L_{o'}$.	Square.	Pentagon.	Hexagon.	Octagon.	Dodecagon.	Circle.	Square.	Pentagon.	Hexagon.	Octagon.	Dodecagon.	Circle.							
0		3.7913	2.3957	1.8928	7.176	4.535	835	759	723	692	671	656	3529	3720	4058	4384	4584	4723	6501	558	506	433	462	448	438	0
0.1052	2.0	2.3333	2.3905	4.781	5.578	762	692	660	631	612	593	5247	2343	2534	2534	3012	4067	3953	883	883	802	764	731	710	693	0.3846
0.2084	1.5	2.3000	2.7241	4.086	6.265	784	712	679	649	630	615	1986	2061	2221	2627	3542	3427		1.283	1.283	1.166	1.111	1.063	1.031	1.007	0.7241
0.5000	1.3107	2.2836	2.9125	3.817	6.651	807	733	699	668	649	634	1905	1973	2123	2502	3370	3260		1.597	1.597	1.460	1.383	1.323	1.284	1.254	0.9543
0.5147	1.3	2.2826	2.9246	3.802	6.675	809	734	700	670	650	635	1901	1969	2118	2499	3361	3251		1.620	1.620	1.471	1.403	1.342	1.302	1.272	0.9702
0.5428	1.2808	2.2808	2.9471	3.775	6.722	812	737	703	673	653	638	1893	1961	2109	2488	3345	3236		1.663	1.663	1.546	1.440	1.378	1.337	1.306	1.0000
0.6848	1.2	2.2727	3.049	3.658	6.929	827	751	716	685	665	650	1864	1929	2074	2443	3281	3174		1.875	1.875	1.703	1.624	1.554	1.508	1.473	1.1416
0.9357	1.1	2.2619	3.194	3.513	7.254	852	774	738	706	685	669	1833	1894	2034	2398	3209	3105		2.230	2.230	2.026	1.932	1.848	1.793	1.752	1.3646
1.0000	1.0800	2.2596	3.226	3.484	7.200	858	779	743	710	689	673	1827	1888	2027	2385	3196	3093		2.318	2.318	2.105	2.008	1.920	1.863	1.821	1.4177
1.3235	1.0	2.2500	3.367	3.367	7.575	884	803	766	732	711	694	1808	1866	2002	2353	3149	3047		2.747	2.747	2.495	2.379	2.276	2.208	2.158	1.6667
1.6003	0.95	2.2436	3.466	3.298	7.776	904	821	783	749	727	710	1798	1855	1989	2336	3124	3023		3.101	3.101	2.816	2.685	2.569	2.493	2.435	1.8617
1.9614	0.90	2.2369	3.576	3.218	7.999	927	842	803	768	745	728	1790	1846	1978	2322	3102	3002		3.549	3.549	3.223	3.074	2.940	2.853	2.788	2.009
2.0000	0.8954	2.2362	3.587	3.211	8.021	929	844	805	770	747	730	1790	1845	1978	2321	3100	3000		3.597	3.597	3.267	3.115	2.980	2.893	2.825	2.123
2.4435	0.85	2.2297	3.698	3.143	8.246	954	866	826	790	767	749	1786	1840	1971	2311	3085	2985		4.132	4.132	3.752	3.578	3.423	3.321	3.245	2.393
3.0000	0.8069	2.2233	3.815	3.078	8.482	981	891	850	813	789	770	1784	1837	1968	2305	3074	2975		4.788	4.788	4.348	4.146	3.966	3.849	3.760	2.710
3.106	0.80	2.2232	3.835	3.068	8.522	986	896	854	817	792	774	1784	1837	1967	2304	3073	2974		4.910	4.910	4.460	4.253	4.068	3.947	3.857	2.768
4.043	0.75	2.2143	3.989	2.992	8.833	1023	929	886	848	822	803	1782	1843	1927	2302	3069	2969		5.990	5.990	5.440	5.187	4.962	4.815	4.704	3.261
5.449	0.70	2.2059	4.165	2.915	9.187	1068	970	925	884	858	839	1782	1843	1927	2302	3069	2969		7.557	7.557	6.863	6.545	6.261	6.075	5.936	3.940
7.671	0.65	2.1970	4.367	2.838	9.593	1122	1019	971	929	901	881	1804	1854	1982	2316	3079	2979		9.985	9.985	9.068	8.647	8.272	8.026	7.842	4.935
11.523	0.60	2.1875	4.600	2.760	10.063	1187	1078	1028	983	954	933	1822	1871	1999	2334	3100	3000		14.110	14.110	12.814	12.219	11.689	11.342	11.082	6.629
40.18	0.50	2.1667	5.204	2.602	11.275	1372	1246	1183	1137	1108	1077	1886	1934	2064	2405	3187	3084		43.960	43.960	39.924	38.071	36.417	35.337	34.526	17.00
∞	0.4343	2.514	6.747	2.496	12.364	1556	1413	1347	1289	1251	1222	1957	2005	2138	2488	3292	3186		∞	∞	∞	∞	∞	∞	∞	∞

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than this, owing to the closer bedding of the wires at the corners. The values of the coefficients for the different types of transformers are given in Table 10·06. General formulæ are found for the cheapest proportions. By putting in these coefficients, the particular formulæ for any type may be obtained. Only those for the more important cases are given, along with tables of numerical values, but the curves give the results for all the types.

TABLE 10·06.—COEFFICIENTS FOR LIMB TYPE TRANSFORMERS.

Type.	Figure No.		Coefficients.			
	C.C.	R.C.	<i>a.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>
Simple	10·27	10·29	2	2	4	1
Shell	10·31	10·33	2	2	2	1
Core	10·35	10·37	2	4	4	2
2-Phase Simple	10·39	10·41	(2 + $\sqrt{2}$)	4	2(2 + $\sqrt{2}$)	2
Do. Shell	10·43	10·45	4	(2 + $\sqrt{2}$)	(2 + $\sqrt{2}$)	2
Do. Tandem Core	10·47	10·49	4	2(2 + $\sqrt{2}$)	2(2 + $\sqrt{2}$)	4
3-Phase 3-Limb	10·51	10·53	3	8	6	3
Do. Shell	10·55	10·57	6	4	4	3
Do. Tandem Core	10·59	10·61	6	8	8	6

Circular Coils.						
Type.	Fig. No.	(<i>b</i> + 3 <i>c</i>).	(2 <i>b</i> + 5 <i>c</i>).	(<i>b</i> + 4 <i>c</i>).	(<i>a</i> + 2 <i>b</i>).	(<i>a</i> + <i>b</i> + 3 <i>c</i>).
Simple	10·27	14	24	18	6	16
Shell	10·31	8	14	10	6	10
Core	10·35	16	28	20	10	18
2-Phase Simple	10·39	2(8 + 3 $\sqrt{2}$)	2(14 + 5 $\sqrt{2}$)	4(5 + 2 $\sqrt{2}$)	(10 + $\sqrt{2}$)	(18 + 7 $\sqrt{2}$)
Do. Shell	10·43	4(2 + $\sqrt{2}$)	7(2 + $\sqrt{2}$)	5(2 + $\sqrt{2}$)	2(4 + $\sqrt{2}$)	4(3 + $\sqrt{2}$)
Do. Tandem Core	10·47	8(2 + $\sqrt{2}$)	14(2 + $\sqrt{2}$)	10(2 + $\sqrt{2}$)	4(3 + $\sqrt{2}$)	4(5 + 2 $\sqrt{2}$)
3-Phase 3-Limb	10·51	26	46	32	19	29
Do. Shell	10·55	16	28	20	14	22
Do. Tandem Core	10·59	32	56	40	22	38

Rectangular Coils.					
Type.	Fig. No.	<i>be.</i>	(2 <i>b</i> + <i>ce</i>).	(<i>a</i> + 2 <i>b</i>) <i>e.</i>	(2 <i>a</i> + 2 <i>b</i> + <i>ce</i>).
Simple	10·29	6	16	18	20
Shell	10·33	6	10	18	14
Core	10·37	12	20	30	24
2-Phase Simple	10·41	12	2(10 + 3 $\sqrt{2}$)	3(10 + $\sqrt{2}$)	8(3 + $\sqrt{2}$)
Do. Shell	10·45	3(2 + $\sqrt{2}$)	5(2 + $\sqrt{2}$)	6(4 + $\sqrt{2}$)	(18 + 5 $\sqrt{2}$)
Do. Tandem Core	10·49	6(2 + $\sqrt{2}$)	10(2 + $\sqrt{2}$)	12(3 + $\sqrt{2}$)	2(14 + 5 $\sqrt{2}$)
3-Phase 3-Limb	10·53	24	34	57	40
Do. Shell	10·57	12	20	42	32
Do. Tandem Core	10·61	24	40	66	52

Limb Transformers with Circular Coils.—Putting in these values for the lengths and the cross-sections, the expression for the cost function becomes,

$$\frac{\mathcal{L}_{\text{MIN}}}{\mathcal{L}_0} = \frac{2\pi(y+1)^{\frac{1}{2}}(ax+by+c)^{\frac{1}{2}}}{d^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{1}{2}}} \left[1 + \frac{4dmxy(y+1)}{(ax+by+c)} \right] \quad 10\cdot78.$$

For a minimum with different values of x ,

$$0 = \frac{\partial}{\partial x} \left\{ \left[\frac{(ax+by+c)^5}{x^3} \right]^{\frac{1}{2}} + 4dmxy(y+1) \left[\frac{(ax+by+c)^3}{x} \right]^{\frac{1}{2}} \right\} \quad 10\cdot79.$$

$$= (ax+by+c)[5ax-3(ax+by+c)] + 4dmxy(y+1)[3ax-(ax+by+c)].$$

$$\therefore (ax+by+c)\{2ax-3(by+c)\} = -4dmxy(y+1)\{2ax-(by+c)\} \quad 10\cdot80.$$

For a minimum with different values of y ,

$$0 = \frac{\partial}{\partial y} \left\{ \left[\frac{(y+1)^3(ax+by+c)^5}{y^3} \right]^{\frac{1}{2}} + 4dmx \left[\frac{(y+1)^5(ax+by+c)^3}{y} \right]^{\frac{1}{2}} \right\} \quad 10\cdot81.$$

$$= (ax+by+c)[3y(ax+by+c) + 5by(y+1) - 3(y+1)(ax+by+c)] \\ + 4dmxy(y+1)[5y(ax+by+c) + 3by(y+1) - (y+1)(ax+by+c)].$$

$$\therefore (ax+by+c)\{-3(ax+by+c) + 5by(y+1)\} = -4dmxy(y+1)[(4y-1) \times \\ (ax+by+c) + 3by(y+1)] \quad 10\cdot82.$$

Dividing each side by the corresponding one of equation 10·80 and multiplying across, we get:—

$$\{2ax-(by+c)\}\{-3(ax+by+c) + 5by(y+1)\} = \{2ax-3(by+c)\} \times \\ \{(4y-1)(ax+by+c) + 3by(y+1)\} \quad 10\cdot83.$$

$$\therefore -(ax+by+c)[3\{2ax-(by+c)\} + (4y-1)\{2ax-3(by+c)\}] \\ = by(y+1)[-5\{2ax-(by+c)\} + 3\{2ax-3(by+c)\}] \\ = by(y+1)[-4ax-4(by+c)].$$

$$\therefore (8y+4)ax - 12y(by+c) = 4by(y+1)$$

$$(2y+1)ax = y\{3(by+c) + b(y+1)\} = y\{4by + (b+3c)\}.$$

$$\therefore x = \frac{y\{4by + (b+3c)\}}{a(2y+1)} \quad 10\cdot84,$$

$$(ax+by+c) = \frac{y\{4by + (b+3c)\} + (2y+1)(by+c)}{(2y+1)} \\ = \frac{\{6by^2 + (2b+5c)y + c\}}{(2y+1)} \quad 10\cdot85,$$

$$\{2ax-3(by+c)\} = \frac{2y\{4by + (b+3c)\} - 3(2y+1)(by+c)}{(2y+1)} \\ = \frac{\{2by^2 - by - 3c\}}{(2y+1)} \quad 10\cdot86,$$

$$\{2ax-(by+c)\} = \frac{2y\{4by + (b+3c)\} - (2y+1)(by+c)}{(2y+1)} \\ = \frac{\{6by^2 + (b+4c)y - c\}}{(2y+1)} \quad 10\cdot87.$$

Putting these values in equation 10.80, we get :—

$$\begin{aligned} & \{6by^2 + (2b + 5c)y + c\} \{2by^2 - by - 3c\} \\ &= -\frac{4dm}{a} y^2(y+1) \{4by + (b+3c)\} \{6by^2 + (b+4c)y - c\}. \\ \therefore m &= \frac{a \{-2by^2 + by + 3c\} \{6by^2 + (2b+5c)y + c\}}{4dy^2(y+1) \{4by + (b+3c)\} \{6by^2 + (b+4c)y - c\}} \quad 10.88, \end{aligned}$$

$$\begin{aligned} \frac{\text{Total cost of copper}}{\text{Total cost of iron}} &= m \frac{V_{CS}}{V_{IS}} = \frac{4dmxy(y+1)}{(ax+by+c)} \quad (\text{from equation 10.78}) \\ &= -\frac{\{2ax - 3(by+c)\}}{\{2ax - (by+c)\}} \quad (\text{from equation 10.80}) \\ &= \frac{\{-2by^2 + by + 3c\}}{\{6by^2 + (b+4c)y - c\}} \quad 10.89, \end{aligned}$$

$$\begin{aligned} \frac{\mathcal{L}_{\text{MIN}}}{\mathcal{L}_0} &= \frac{2\pi a^{\frac{1}{2}}(y+1)^{\frac{1}{2}} \{6by^2 + (2b+5c)y + c\}^{\frac{1}{2}}}{d y^{\frac{1}{2}}(2y+1) \{4by + (b+3c)\}^{\frac{1}{2}}} \left[1 - \frac{\{2ax - 3(by+c)\}}{\{2ax - (by+c)\}} \right] \\ &= \frac{4\pi a^{\frac{1}{2}}(y+1)^{\frac{1}{2}}(by+c) \{6by^2 + (2b+5c)y + c\}^{\frac{1}{2}}}{d^{\frac{1}{2}} y^{\frac{1}{2}} \{4by + (b+3c)\}^{\frac{1}{2}} \{6by^2 + (b+4c)y - c\}} \quad 10.90. \end{aligned}$$

For any proportions,

$$\begin{aligned} L_0 &= \sqrt{\frac{S_{IS} S_{CS}}{L_I L_C}} = \left[\frac{dxy}{4(y+1)(ax+by+c)} \right] l_{IS}. \\ \therefore l_{IS} &= \left[\frac{4(y+1)(ax+by+c)}{dxy} \right]^{\frac{1}{2}} L_0 \quad 10.91; \end{aligned}$$

and for the cheapest proportions,

$$\frac{l_{IS}}{L_0} = \frac{2a^{\frac{1}{2}}(y+1)^{\frac{1}{2}} \{6by^2 + (2b+5c)y + c\}^{\frac{1}{2}}}{d^{\frac{1}{2}} y \{4by + (b+3c)\}^{\frac{1}{2}}} \quad 10.92,$$

$$\frac{L_{CS}}{L_0} = x \frac{l_{IS}}{L_0} = \frac{2(y+1)^{\frac{1}{2}} \{4by + (b+3c)\}^{\frac{1}{2}} \{6by^2 + (2b+5c)y + c\}^{\frac{1}{2}}}{a^{\frac{1}{2}} d^{\frac{1}{2}} (2y+1)} \quad 10.93,$$

$$\frac{l_{CS}}{L_0} = y \cdot \frac{l_{IS}}{L_0} = \frac{2a^{\frac{1}{2}}(y+1)^{\frac{1}{2}} \{6by^2 + (2b+5c)y + c\}^{\frac{1}{2}}}{d^{\frac{1}{2}} \{4by + (b+3c)\}^{\frac{1}{2}}} \quad 10.94.$$

The iron cooling function will have to be worked out for each case separately, but

$$\begin{aligned} S_{CC} &= 2\pi d(y+1)(x+y)l_{IS}^2 \\ &= \frac{2\pi dy(y+1) \{ \{4by + (b+3c)\} + a(2y+1) \}}{a(2y+1)} l_{IS}^2 \\ \therefore \frac{S_{CC}}{S_0} &= \frac{8\pi(y+1) \{ 2(a+2b)y + (a+b+3c) \} \{ 6by^2 + (2b+5c)y + c \}}{y(2y+1) \{ 4by + (b+3c) \}} \quad 10.95. \end{aligned}$$

Limb Transformers with Rectangular Coils.—In this case the cost function becomes :—

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \frac{(ax+by+c)^{\frac{1}{2}}(ey+2+2z)^{\frac{1}{2}}}{d^{\frac{1}{2}} x^{\frac{1}{2}} y^{\frac{1}{2}} z^{\frac{1}{2}}} \left[1 + \frac{d m x y (e y + 2 + 2 z)}{z(ax+by+c)} \right] \quad 10.96.$$

For a minimum with different values of x ,

$$0 = \frac{\partial}{\partial x} \left\{ \left[\frac{(ax + by + c)^5}{x^3} \right]^{\frac{1}{2}} + \frac{dmy(ey + 2 + 2z)}{z} \left[\frac{(ax + by + c)^3}{x} \right]^{\frac{1}{2}} \right\} \quad 10\cdot97.$$

$$= z(ax + by + c)[5ax - 3(ax + by + c)] + dmxxy(ey + 2 + 2z)[3ax - (ax + by + c)].$$

$$\therefore z(ax + by + c)\{2ax - 3(by + c)\} = -dmxy(ey + 2 + 2z)\{2ax - (by + c)\} \quad 10\cdot98.$$

For a minimum with different values of y ,

$$0 = \frac{\partial}{\partial y} \left\{ \left[\frac{(ax + by + c)^3(ey + 2 + 2z)^3}{y^3} \right]^{\frac{1}{2}} + \frac{dmx}{z} \left[\frac{(ax + by + c)^3(ey + 2 + 2z)^5}{y} \right]^{\frac{1}{2}} \right\} \quad 10\cdot99.$$

$$= z(ax + by + c)[5by(ey + 2 + 2z) + 3ey(ax + by + c) - 3(ax + by + c)(ey + 2 + 2z)] + dmxxy(ey + 2 + 2z)[3by(ey + 2 + 2z) + 5ey(ax + by + c) - (ax + by + c)(ey + 2 + 2z)].$$

$$\therefore z(ax + by + c)[-6(1 + z)(ax + by + c) + 5by(ey + 2 + 2z)] = -dmxy(ey + 2 + 2z)[\{4ey - 2(1 + z)\}(ax + by + c) + 3by(ey + 2 + 2z)] \quad 10\cdot100.$$

Dividing each side by the corresponding one of equation 10·98 and multiplying across, we get :—

$$\{2ax - (by + c)\}[-6(1 + z)(ax + by + c) + 5by(ey + 2 + 2z)] = \{2ax - 3(by + c)\}[\{4ey - 2(1 + z)\}(ax + by + c) + 3by(ey + 2 + 2z)] \quad 10\cdot101.$$

$$\therefore -(ax + by + c)[6(1 + z)\{2ax - (by + c)\} + \{4ey - 2(1 + z)\}\{2ax - 3(by + c)\}] = by(ey + 2 + 2z)[-5\{2ax - (by + c)\} + 3\{2ax - 3(by + c)\}].$$

$$\therefore -(ax + by + c)[8(ey + 1 + z)ax - 12ey(by + c)] = by(ey + 2 + 2z)[-4ax - 4(by + c)].$$

$$\therefore 2(ey + 1 + z)ax - 3ey(by + c) = by(ey + 2 + 2z) \quad 10\cdot102.$$

For a minimum with different values of z ,

$$0 = \frac{\partial}{\partial z} \left\{ \left[\frac{(ey + 2 + 2z)^3}{z} \right]^{\frac{1}{2}} + \frac{dmxy}{(ax + by + c)} \left[\frac{(ey + 2 + 2z)^5}{z^3} \right]^{\frac{1}{2}} \right\}$$

$$= z(ax + by + c)[6z - (ey + 2 + 2z)] + dmxxy(ey + 2 + 2z)[10z - 3(ey + 2 + 2z)] \quad 10\cdot103.$$

$$z(ax + by + c)\{4z - (ey + 2)\} = -dmxy(ey + 2 + 2z)\{4z - 3(ey + 2)\} \quad 10\cdot104.$$

Dividing each side by the corresponding one of equation 10·98 and multiplying across, we get :—

$$\{2ax - (by + c)\}\{4z - (ey + 2)\} = \{2ax - 3(by + c)\}\{4z - 3(ey + 2)\} \quad 10\cdot105.$$

$$\begin{aligned} \therefore 4z[\{2ax - (by + c)\} - \{2ax - 3(by + c)\}] \\ = (ey + 2)[\{2ax - (by + c)\} - 3\{2ax - 3(by + c)\}]. \\ \therefore z = \frac{(ey + 2)\{-ax + 2(by + c)\}}{2(by + c)} \quad 10\cdot106, \end{aligned}$$

$$\begin{aligned} (ey + 1 + z) &= \frac{2(ey + 1)(by + c) + (ey + 2)\{-ax + 2(by + c)\}}{2(by + c)} \\ &= \frac{-(ey + 2)ax + 2(2ey + 3)(by + c)}{2(by + c)} \quad 10\cdot107, \end{aligned}$$

$$(ey + 2 + 2z) = \frac{(ey + 2)\{-ax + 3(by + c)\}}{(by + c)} \quad 10\cdot108.$$

Putting these values in equation 10·102, we get :—

$$\begin{aligned} \{- (ey + 2)ax + 2(2ey + 3)(by + c)\}ax - 3ey(by + c)^2 \\ = by(ey + 2)\{-ax + 3(by + c)\}. \end{aligned}$$

$$\begin{aligned} \therefore - (ey + 2)a^2x^2 + \{2(2ey + 3)(by + c) \\ + by(ey + 2)\}ax - 3y(by + c)\{e(by + c) + b(ey + 2)\} = 0. \end{aligned}$$

$$\begin{aligned} \therefore (ey + 2)a^2x^2 - \{3(ey + 2)(by + c) \\ + y(2bey + 2b + ce)\}ax + 3y(by + c)(2bey + 2b + ce) = 0. \end{aligned}$$

$$\therefore \{ax - 3(by + c)\}[(ey + 2)ax - y(2bey + 2b + ce)] = 0.$$

$$\therefore ax = 3(by + c), \text{ which does not give a minimum,}$$

$$\text{or } x = \frac{y\{2bey + (2b + ce)\}}{a(ey + 2)} \quad 10\cdot109.$$

$$\begin{aligned} \therefore \{ax + by + c\} &= \frac{y\{2bey + (2b + ce)\} + (ey + 2)(by + c)}{(ey + 2)} \\ &= \frac{\{3bey^2 + 2(2b + ce)y + 2c\}}{(ey + 2)} \quad 10\cdot110, \end{aligned}$$

$$\begin{aligned} \{2ax - (by + c)\} &= \frac{2y\{2bey + (2b + ce)\} - (ey + 2)(by + c)}{(ey + 2)} \\ &= \frac{\{3bey^2 + (2b + ce)y - 2c\}}{(ey + 2)} \quad 10\cdot111, \end{aligned}$$

$$\begin{aligned} \{2ax - 3(by + c)\} &= \frac{2y\{2bey + (2b + ce)\} - 3(ey + 2)(by + c)}{(ey + 2)} \\ &= \frac{\{bey^2 - (2b + ce)y - 6c\}}{(ey + 2)} \quad 10\cdot112, \end{aligned}$$

$$z = \frac{-y\{2bey + (2b + ce)\} + 2(ey + 2)(by + c)}{2(by + c)} = \frac{\{(2b + ce)y + 4c\}}{2(by + c)} \quad 10\cdot113,$$

$$\begin{aligned}\{(ey + 2) + 2z\} &= \frac{(ey + 2)(by + c) + \{(2b + ce)y + 4c\}}{(by + c)} \\ &= \frac{\{bey^2 + 2(2b + ce)y + 6c\}}{(by + c)} \quad \dots \quad 10.114.\end{aligned}$$

Putting these values in equation 10.98, we get :—

$$\begin{aligned}& \frac{\{(2b + ce)y + 4c\}}{2(by + c)} \frac{\{3bey^2 + 2(2b + ce)y + 2c\}}{(ey + 2)} \frac{\{bey^2 - (2b + ce)y - 6c\}}{(ey + 2)} \\ &= -dm_y^2 \frac{\{2bey + (2b + ce)\}}{a(ey + 2)} \frac{\{bey^2 + 2(2b + ce)y + 6c\}}{(by + c)} \times \\ & \quad \frac{\{3by^2 + (2b + ce)y - 2c\}}{(ey + 2)} \\ \therefore m &= \frac{a\{(2b + ce)y + 4c\}\{-bey^2 + (2b + ce)y + 6c\}\{3bey^2 + 2(2b + ce)y + 2c\}}{2dy^2\{2bey + (2b + ce)\}\{bey^2 + 2(2b + ce)y + 6c\}\{3bey^2 + (2b + ce)y - 2c\}} \\ & \quad [10.115.\end{aligned}$$

$$\begin{aligned}\frac{\text{Total cost of copper.}}{\text{Total cost of iron.}} &= m \frac{V_{CS}}{V_{IS}} \\ &= \frac{dmxy(ey + 2 + 2z)}{z(ax + by + c)} \quad (\text{from equation 10.96}) \\ &= -\frac{\{2ax - 3(by + c)\}}{\{2ax - (by + c)\}} \quad (\text{from equation 10.98}) \\ &= \frac{\{-bey^2 + (2b + ce)y + 6c\}}{\{3bey^2 + (2b + ce)y - 2c\}} \quad \dots \quad 10.116.\end{aligned}$$

$$\begin{aligned}\frac{\mathcal{L}_{MIN}}{\mathcal{L}_O} &= \frac{\left[\frac{\{bey^2 + 2(2b + ce)y + 6c\}}{(by + c)} \right]^{\frac{2}{3}} \left[\frac{\{3bey^2 + 2(2b + ce)y + 2c\}}{(ey + 2)} \right]}{d^{\frac{2}{3}} \left[\frac{y^2\{2bey + (2b + ce)\}}{a(ey + 2)} \right]^{\frac{2}{3}} \left[\frac{\{(2b + ce)y + 4c\}}{2(by + c)} \right]^{\frac{1}{3}}} \times \\ & \quad \left[1 - \frac{\{2ax - 3(by + c)\}}{\{2ax - (by + c)\}} \right] \\ &= \frac{2\sqrt{2}a^{\frac{1}{3}}\{bey^2 + 2(2b + ce)y + 6c\}^{\frac{2}{3}}\{3bey^2 + 2(2b + ce)y + 2c\}^{\frac{2}{3}}}{d^{\frac{2}{3}}y^3\{2bey + (2b + ce)\}^{\frac{2}{3}}\{(2b + ce)y + 4c\}^{\frac{1}{3}}\{3bey^2 + (2b + ce)y - 2c\}} \quad 10.117.\end{aligned}$$

For any proportions,

$$\begin{aligned}\mathcal{L}_O &= \sqrt{\frac{\mathbf{S}_{IS}\mathbf{S}_{CS}}{\mathbf{L}_I\mathbf{L}_C}} = \left[\frac{dxyz}{(ey + 2 + 2z)(ax + by + c)} \right]^{\frac{1}{2}} l_{IS} \\ \therefore l_{IS} &= \left[\frac{(ey + 2 + 2z)(ax + by + c)}{dxyz} \right]^{\frac{1}{2}} \mathcal{L}_O \quad \dots \quad 10.118;\end{aligned}$$

and for the cheapest proportions,

$$\frac{l_{IS}}{\mathcal{L}_O} = \frac{\sqrt{2}a^{\frac{1}{3}}\{bey^2 + 2(2b + ce)y + 6c\}^{\frac{2}{3}}\{3bey^2 + 2(2b + ce)y + 2c\}^{\frac{2}{3}}}{d^{\frac{2}{3}}y\{2bey + (2b + ce)\}^{\frac{2}{3}}\{(2b + ce)y + 4c\}^{\frac{1}{3}}} \quad 10.119,$$

$$\frac{S_{CI}}{S_o} = [4(x+y+1) + z(6x+8y+6)] \frac{l_{IS}^2}{L_o^2}$$

$$= \frac{8(3y^2+10y+6)(9y^2+10y+2)[2(y+1)(9y^2+10y+2) + (5y+4)(15y^2+16y+3)]}{y^2(y+1)(3y+2)(5y+4)(6y+5)} \quad [10\cdot133,$$

$$\frac{S_{CC}}{S_o} = \frac{8(9y+7)(3y^2+10y+6)^2(9y^2+10y+2)}{y(y+1)(3y+2)(5y+4)(6y+5)} \quad . \quad . \quad 10\cdot134.$$

Two-Phase Shell Transformer with Rectangular Coils (fig. 10·45).—All the functions are the same as for single-phase except :—

$$x = \frac{(2 + \sqrt{2})y(6y+5)}{4(3y+2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 10\cdot135,$$

$$\frac{\mathcal{L}_{MIN}}{\mathcal{L}_o} = \frac{8(2 + \sqrt{2})(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{y^2(5y+4)^{\frac{1}{2}}(6y+5)^{\frac{1}{2}}(9y^2+5y-2)} \quad . \quad . \quad . \quad . \quad 10\cdot136,$$

$$\frac{L_{CS}}{L_o} = x \frac{l_{IS}}{L_o} = \frac{(2 + \sqrt{2})(6y+5)^{\frac{1}{2}}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{2(3y+2)(5y+4)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot137,$$

$$\frac{S_{CI}}{S_o} = \left[8x + 2(2 + \sqrt{2})(y+1) + z\{12x + 12y + (6 + \sqrt{2})\} \right] \frac{l_{IS}^2}{L_o^2}$$

$$= \frac{4(3y^2+10y+6)(9y^2+10y+2)[2(2 + \sqrt{2})(y+1)(9y^2+10y+2) + (5y+4)\{9(4 + \sqrt{2})y(y+1) + (6 + \sqrt{2})\}]}{y^2(y+1)(3y+2)(5y+4)(6y+5)}$$

$$= \frac{4(2 + \sqrt{2})(3y^2+10y+6)(9y^2+10y+2)[2(y+1)(9y^2+10y+2) + (3 - \sqrt{2})(5y+4)\{9y(y+1) + (11 - \sqrt{2})/7\}]}{y^2(y+1)(3y+2)(5y+4)(6y+5)} \quad [10\cdot138,$$

$$\frac{S_{CC}}{S_o} = \frac{4\{6(4 + \sqrt{2})y + (18 + 5\sqrt{2})\}(3y^2+10y+6)^2(9y^2+10y+2)}{y(y+1)(3y+2)(5y+4)(6y+5)}$$

$$= \frac{4(4 + \sqrt{2})\{6y + (31 + \sqrt{2})/7\}(3y^2+10y+6)^2(9y^2+10y+2)}{y(y+1)(3y+2)(5y+4)(6y+5)} \quad . \quad 10\cdot139.$$

Three-Phase Shell Transformer with Rectangular Coils (fig. 10·57).—Here also all the functions are the same as for single-phase except :—

$$x = \frac{2y(6y+5)}{3(3y+2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 10\cdot140,$$

$$\frac{\mathcal{L}_{MIN}}{\mathcal{L}_o} = \frac{32(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{y^2(5y+4)^{\frac{1}{2}}(6y+5)^{\frac{1}{2}}(9y^2+5y-2)} \quad . \quad . \quad . \quad . \quad 10\cdot141,$$

$$\frac{L_{CS}}{L_o} = x \frac{l_{IS}}{L_o} = \frac{4(6y+5)^{\frac{1}{2}}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{3(3y+2)(5y+4)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot142,$$

$$S_{CI} = [12x + 8y + 8 + z\{18x + 16y + 8\}] l_{IS}^2$$

$$\therefore \frac{S_{CI}}{S_o} = \frac{8(3y^2+10y+6)(9y^2+10y+2)[4(y+1)(9y^2+10y+2) + (5y+4)(30y^2+29y+4)]}{y^2(y+1)(3y+2)(5y+4)(6y+5)} \quad [10\cdot143,$$

$$\frac{S_{CC}}{S_o} = \frac{8(21y+16)(3y^2+10y+6)^2(9y^2+10y+2)}{y(y+1)(3y+2)(5y+4)(6y+5)} \quad . \quad . \quad . \quad . \quad 10\cdot144.$$

TABLE 10-07.—PARTICULARS OF CHEAPEST SHELL TRANSFORMERS WITH RECTANGULAR COILS.

Specific Cost Ratio.	Proportional Dimensions.				Dimension Coefficients.						Iron Cooling Functions S_{ci}/S_o .			Copper Cooling Functions S_{cc}/S_o .			Cost Functions ϵ/ϵ_o .			Total Cost of Copper.	Total Cost of Iron.	
	α .		y .	z .	ϵ_s/L_o .	L_{cs}/L_o .			ϵ_s/L_o .	ϵ_{cs}/L_o .	L_s/L_o .	1-Phase.	2-Phase.	3-Phase.	1-Phase.	2-Phase.	3-Phase.					
						1-Phase.	2-Phase.	3-Phase.														
	m.	1-Phase.	2-Phase.	3-Phase.	All.	All.	All.	All.	All.	All.	All.	All.	All.	All.	All.	All.	All.	All.	All.			All.
0	5.2122	4.4489	3.4748	2.4748	2.356	2.848	1.4817	12.547	9.878	7.035	6.038	2203	2659	3164	4078	940	1,606	1,881	0	0	0	0
0.0250	4.2500	3.6276	2.8533	2.0	2.333	3.048	1.9383	11.039	8.922	6.086	7.101	1225	2503	3164	3402	1,040	1,776	2,080	0.0909	0.0909	0.0909	0.0909
0.1250	3.2577	2.7577	2.1539	1.5	2.300	3.379	10.915	9.317	7.277	5.067	7.771	1243	2078	2581	2778	1,283	2,190	2,566	0.2621	0.2621	0.2621	0.2621
0.3113	2.5143	2.2314	1.7429	1.2	2.273	3.702	9.677	8.260	6.451	4.442	8.413	1238	1968	2352	2454	1,612	2,752	3,225	0.4528	0.4528	0.4528	0.4528
0.6023	2.0000	1.8778	1.4667	1.0	2.250	4.015	8.933	7.540	5.989	4.015	9.034	1257	1984	2370	2480	2,037	3,481	4,078	0.6667	0.6667	0.6667	0.6667
0.8723	1.9915	1.6998	1.3277	0.9	2.237	4.220	8.404	6.988	5.458	3.798	9.440	1279	2011	2399	2496	2,387	4,075	4,774	0.8243	0.8243	0.8243	0.8243
1.0000	1.8808	1.6105	1.2579	0.85	2.220	4.339	8.188	6.988	5.458	3.798	9.440	1279	2011	2399	2496	2,619	4,471	5,238	0.9236	0.9236	0.9236	0.9236
1.1460	1.8519	1.5907	1.2345	0.8333	2.227	4.382	8.115	6.927	5.410	3.652	9.676	1294	2031	2420	2496	2,708	4,623	5,416	0.9604	0.9604	0.9604	0.9604
1.2927	1.8148	1.5505	1.2110	0.8166	2.226	4.427	8.041	6.864	5.381	3.614	9.843	1306	2048	2438	2496	2,805	4,788	5,609	1.0000	1.0000	1.0000	1.0000
1.4448	1.7813	1.5209	1.1879	0.80	2.222	4.472	7.969	6.802	5.313	3.578	10.034	1318	2067	2446	2496	2,907	4,962	5,813	1.0412	1.0412	1.0412	1.0412
1.6048	1.7504	1.4907	1.1670	0.785	2.214	4.522	7.749	6.614	5.166	3.467	10.234	1336	2089	2483	2496	3,017	5,158	6,043	1.0856	1.0856	1.0856	1.0856
1.7748	1.7183	1.4407	1.0472	0.70	2.206	4.566	7.526	6.424	5.018	3.354	10.439	1358	2117	2518	2548	3,137	5,368	6,301	1.1367	1.1367	1.1367	1.1367
1.9548	1.6833	1.3945	0.9605	0.66	2.202	4.596	7.347	6.247	4.898	3.247	10.649	1383	2147	2548	2548	3,271	5,595	6,587	1.1937	1.1937	1.1937	1.1937
2.1448	1.6482	1.3282	0.9021	0.62	2.195	4.627	7.165	6.061	4.777	3.137	10.866	1409	2179	2571	2571	3,417	5,832	6,864	1.2567	1.2567	1.2567	1.2567
2.3448	1.6132	1.2607	0.8301	0.58	2.191	4.657	6.983	5.874	4.655	3.027	11.083	1436	2214	2624	2624	3,562	6,069	7,126	1.3257	1.3257	1.3257	1.3257
2.5548	1.5782	1.1925	0.7649	0.54	2.188	4.687	6.801	5.687	4.531	2.917	11.300	1464	2251	2637	2637	3,717	6,306	7,438	1.3967	1.3967	1.3967	1.3967
2.7748	1.5432	1.1245	0.6997	0.50	2.185	4.717	6.619	5.499	4.406	2.807	11.517	1492	2289	2674	2674	3,872	6,545	7,728	1.4697	1.4697	1.4697	1.4697
2.9948	1.5082	1.0565	0.6345	0.46	2.182	4.747	6.437	5.312	4.281	2.697	11.734	1520	2329	2717	2717	4,027	6,784	8,051	1.5427	1.5427	1.5427	1.5427
3.2148	1.4732	0.9807	0.5693	0.42	2.179	4.777	6.255	5.125	4.156	2.587	11.951	1548	2369	2760	2760	4,182	7,023	8,386	1.6157	1.6157	1.6157	1.6157
3.4348	1.4382	0.9107	0.5041	0.38	2.176	4.807	6.073	4.938	4.031	2.477	12.168	1576	2409	2806	2806	4,337	7,262	8,746	1.6887	1.6887	1.6887	1.6887
3.6548	1.4032	0.8407	0.4389	0.34	2.173	4.837	5.891	4.751	3.906	2.367	12.385	1604	2449	2847	2847	4,492	7,501	9,084	1.7617	1.7617	1.7617	1.7617
3.8748	1.3682	0.7707	0.3737	0.30	2.170	4.867	5.708	4.564	3.781	2.257	12.602	1632	2489	2888	2888	4,647	7,740	9,421	1.8347	1.8347	1.8347	1.8347
4.0948	1.3332	0.7007	0.3085	0.26	2.167	4.897	5.525	4.377	3.656	2.147	12.819	1660	2529	2929	2929	4,802	7,979	9,764	1.9077	1.9077	1.9077	1.9077
4.3148	1.2982	0.6307	0.2433	0.22	2.164	4.927	5.342	4.190	3.531	2.037	13.036	1688	2569	2970	2970	4,957	8,218	10,003	1.9807	1.9807	1.9807	1.9807
4.5348	1.2632	0.5607	0.1781	0.18	2.161	4.957	5.159	4.003	3.406	1.927	13.253	1716	2609	3011	3011	5,112	8,457	10,244	2.0537	2.0537	2.0537	2.0537
4.7548	1.2282	0.4907	0.1129	0.14	2.158	4.987	4.976	3.816	3.281	1.817	13.470	1744	2649	3052	3052	5,267	8,696	10,485	2.1267	2.1267	2.1267	2.1267
4.9748	1.1932	0.4207	0.0477	0.10	2.155	5.017	4.793	3.629	3.156	1.707	13.687	1772	2689	3093	3093	5,422	8,935	10,726	2.1997	2.1997	2.1997	2.1997
5.1948	1.1582	0.3507	0.0000	0.06	2.152	5.047	4.610	3.442	3.031	1.597	13.904	1800	2729	3134	3134	5,577	9,174	10,965	2.2727	2.2727	2.2727	2.2727
5.4148	1.1232	0.2807	0.0000	0.02	2.149	5.077	4.427	3.255	2.906	1.487	14.121	1828	2769	3175	3175	5,732	9,413	11,176	2.3457	2.3457	2.3457	2.3457
5.6348	1.0882	0.2107	0.0000	0.00	2.146	5.107	4.244	3.068	2.781	1.377	14.338	1856	2809	3216	3216	5,887	9,652	11,387	2.4187	2.4187	2.4187	2.4187
5.8548	1.0532	0.1407	0.0000	0.00	2.143	5.137	4.061	2.881	2.656	1.267	14.555	1884	2849	3257	3257	6,042	9,891	11,598	2.4917	2.4917	2.4917	2.4917
6.0748	1.0182	0.0707	0.0000	0.00	2.140	5.167	3.878	2.694	2.531	1.157	14.772	1912	2889	3298	3298	6,197	10,130	11,809	2.5647	2.5647	2.5647	2.5647
6.2948	0.9832	0.0000	0.0000	0.00	2.137	5.197	3.695	2.507	2.406	1.047	14.989	1940	2929	3339	3339	6,352	10,369	12,019	2.6377	2.6377	2.6377	2.6377
6.5148	0.9482	0.0000	0.0000	0.00	2.134	5.227	3.512	2.320	2.281	0.937	15.206	1968	2969	3380	3380	6,507	10,608	12,220	2.7097	2.7097	2.7097	2.7097
6.7348	0.9132	0.0000	0.0000	0.00	2.131	5.257	3.329	2.133	2.156	0.827	15.423	1996	3009	3421	3421	6,662	10,847	12,421	2.7827	2.7827	2.7827	2.7827
6.9548	0.8782	0.0000	0.0000	0.00	2.128	5.287	3.146	1.946	2.031	0.717	15.640	2024	3049	3462	3462	6,817	11,086	12,622	2.8557	2.8557	2.8557	2.8557
7.1748	0.8432	0.0000	0.0000	0.00	2.125	5.317	2.963	1.759	1.906	0.607	15.857	2052	3089	3503	3503	6,972	11,325	12,823	2.9287	2.9287	2.9287	2.9287
7.3948	0.8082	0.0000	0.0000	0.00	2.122	5.347	2.780	1.572	1.781	0.497	16.074	2080	3129	3544	3544	7,127	11,564	13,024	3.0017	3.0017	3.0017	3.0017
7.6148	0.7732	0.0000	0.0000	0.00	2.119	5.377	2.597	1.385	1.656	0.387	16.291	2108	3169	3585	3585	7,282	11,803	13,225	3.0747	3.0747	3.0747	3.0747
7.8348	0.7382	0.0000	0.0000	0.00	2.116	5.407	2.414	1.198	1.531	0.277	16.508	2136	3209	3626	3626	7,437	12,042	13,426	3.1477	3.1477	3.1477	3.1477
8.0548	0.7032	0.0000	0.0000	0.00	2.113	5.437	2.231	1.011	1.406	0.167	16.725	2164	3249	3667	3667	7,592	12,281	13,627	3.2207	3.2207	3.2207	3.2207
8.2748	0.6682	0.0000	0.0000	0.00	2.110	5.467	2.048	0.824	1.281	0.057	16.942	2192	3289	3708	3708	7,747	12,520	13,828	3.2937	3.2937	3.2937	3.2937
8.4948	0.6332	0.0000	0.0000	0.00	2.107	5.497	1.865	0.637	1.156	0.000	17.159	2220	3329	3749	3749	7,902	12,759	14,029	3.3667	3.3667	3.3667	3.3667
8.7148	0.5982	0.0000	0.0000	0.00	2.104	5.527	1.682	0.450	1.031	0.000	17.376	2248	3369	3790	3790	8,057	12,998	14,230	3.4397	3.4397	3.4397	3.4397
8.9348	0.5632	0.0000	0.0000	0.00	2.101	5.557	1.500	0.263	0.906	0.000	17.593	2276	3409	3831	3831	8,212	13,237	14,431	3.5127	3.5127	3.5127	3.5127
9.1548	0.5282	0.0000	0.0000	0.00	2.098	5.587	1.317	0.076	0.781	0.000	17.810	2304	3449	3872	3872	8,367	13,476	14,632	3.5857	3.5857	3.5857	3.5857
9.3748	0.4932	0.0000	0.0000	0.00	2.095	5.617	1.134	0.000	0.656	0.000	18.027	2332	3489	3913	3913	8,522	13,715	14,833	3.6587	3.6587	3.6587	3.6587
9.5948	0.4582	0.0000	0.0000	0.00	2.092	5.647	0.951	0.000	0.531	0.000	18.244	2360	352									

TABLE 10-08.—PARTICULARS OF CHEAPEST CORE TRANSFORMERS WITH CIRCULAR COILS.

Specific Cost Ratio m.	Proportional Dimensions.				Dimension Coefficients.				Iron Cooling Functions S_{ci}/S_o			Copper Cooling Functions S_{cu}/S_o			Cost Functions \pounds/\pounds_o			Total Cost of Copper Total Cost of Iron.	
	λ .			y .	l_{is}/L_o .	L_{cs}/L_o .			l_{cs}/L_o .	All.	1-Phase.	2-Phase.	3-Phase.	1-Phase.	2-Phase.	3-Phase.			
	1-Phase.	2-Phase.	3-Phase.	All.															
0	7.500	6.402	6.000	1.5000	3.333	25.000	21.339	16.667	5.0000	873	1454	1745	3142	5516	6807	727	1,241	1,454	0
0.1	4.566	3.898	3.044	0.8296	3.986	18.203	15.537	12.135	3.3071	821	1364	1643	1971	3454	4246	1,094	1,868	2,189	0.3371
0.2	3.945	3.367	2.630	0.6964	4.258	16.800	14.339	11.200	2.9614	836	1387	1672	1793	3139	3854	1,341	2,290	2,682	0.5072
0.5	3.207	2.737	2.138	0.5417	4.725	15.154	12.985	10.103	2.6599	883	1462	1765	1622	2837	3478	1,930	3,296	3,861	0.8516
0.6667	3.000	2.561	2.000	0.5000	4.899	14.697	12.545	9.798	2.4495	905	1498	1810	1583	2769	3393	2,216	3,783	4,433	1.0000
1.0	2.730	2.330	1.820	0.4466	5.166	14.103	12.038	9.402	2.3071	943	1559	1886	1541	2694	3299	2,746	4,687	5,491	1.255
1.5	2.485	2.121	1.657	0.3993	5.459	13.568	11.681	9.045	2.1799	989	1635	1979	1512	2642	3233	3,480	5,941	6,980	1.577
2.0	2.327	1.987	1.552	0.3694	5.682	13.223	11.287	8.815	2.0987	1027	1697	2055	1498	2617	3201	4,172	7,123	8,345	1.869
2.5	2.214	1.889	1.476	0.3481	5.862	12.974	11.074	8.649	2.0406	1060	1749	2120	1491	2605	3185	4,840	8,262	9,679	2.116
3.0	2.126	1.815	1.417	0.3320	6.013	12.734	10.912	8.623	1.9963	1088	1795	2177	1488	2598	3176	5,489	9,370	10,978	2.355
3.5	2.056	1.755	1.371	0.3192	6.144	12.631	10.782	8.421	1.9609	1113	1836	2227	1486	2596	3172	6,126	10,456	12,250	2.581
4.0	1.998	1.705	1.332	0.3086	6.260	12.505	10.674	8.337	1.9318	1136	1873	2272	1486	2595	3171	6,752	11,626	13,504	2.798
5.0	1.907	1.627	1.271	0.2922	6.455	12.307	10.505	8.205	1.8863	1176	1937	2351	1488	2598	3173	7,981	13,624	15,961	3.206
10	1.567	1.423	1.111	0.2500	7.071	11.786	10.059	7.367	1.7678	1309	2153	2618	1505	2627	3207	13,884	23,702	27,768	5.000
∞	1.167	0.996	0.778	0.1667	9.165	10.693	9.127	7.129	1.5276	1847	2901	3695	1642	2863	3489	∞	∞	∞	∞

$$\begin{aligned}\frac{\mathcal{E}_{\text{MIN}}}{\mathcal{E}_0} &= \frac{4\pi(y+1)^{\frac{1}{2}}(6y+1)^{\frac{1}{2}}}{y^3(6y-1)} \quad . \quad . \quad . \quad 10\cdot160, \\ \frac{L_{\text{CS}}}{L_0} &= x \frac{L_{\text{IS}}}{L_0} = \frac{16(y+1)^{\frac{1}{2}}(6y+1)^{\frac{1}{2}}}{3(2y+1)} \quad . \quad . \quad . \quad 10\cdot161, \\ S_{\text{CI}} &= \pi[6x + 8(y+1)]L_{\text{IS}}^2 \\ \therefore \frac{S_{\text{CI}}}{S_0} &= \frac{8\pi(y+1)^2(6y+1)^2}{y^2(2y+1)} \quad . \quad . \quad . \quad 10\cdot162, \\ \frac{S_{\text{CC}}}{S_0} &= \frac{4\pi(y+1)^2(6y+1)(22y+19)}{y(2y+1)} \quad . \quad . \quad 10\cdot163.\end{aligned}$$

Core Transformer with Rectangular Coils (fig. 10·37).—

$$\begin{aligned}x &= \frac{2y(6y+5)}{(3y+2)} \quad . \quad . \quad . \quad 10\cdot164, \\ z &= \frac{(5y+4)}{2(y+1)} \quad . \quad . \quad . \quad 10\cdot165, \\ m &= \frac{(5y+4)(-3y^2+5y+6)(9y^2+10y+2)}{2y^2(6y+5)(3y^2+10y+6)(9y^2+5y-2)} \quad . \quad . \quad 10\cdot166, \\ \frac{\text{Total cost of copper}}{\text{Total cost of iron}} &= \frac{(-3y^2+5y+6)}{(9y^2+5y-2)} \quad . \quad . \quad . \quad 10\cdot167, \\ \frac{\mathcal{E}_{\text{MIN}}}{\mathcal{E}_0} &= \frac{8\sqrt{2}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{y^3(5y+4)^{\frac{1}{2}}(6y+5)^{\frac{1}{2}}(9y^2+5y-2)} \quad . \quad . \quad 10\cdot168, \\ \frac{L_{\text{IS}}}{L_0} &= \frac{\sqrt{2}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{y(5y+4)^{\frac{1}{2}}(6y+5)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot169, \\ \frac{L_{\text{CS}}}{L_0} &= x \frac{L_{\text{IS}}}{L_0} = \frac{2\sqrt{2}(6y+5)^{\frac{1}{2}}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{(3y+2)(5y+4)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot170, \\ \frac{L_{\text{CS}}}{L_0} &= y \frac{L_{\text{IS}}}{L_0} = \frac{\sqrt{2}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{(5y+4)^{\frac{1}{2}}(6y+5)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot171, \\ \frac{L_{\text{IS}}}{L_0} &= z \frac{L_{\text{IS}}}{L_0} = \frac{(5y+4)^{\frac{1}{2}}(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{\sqrt{2}y(y+1)(6y+5)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot172, \\ \frac{S_{\text{CI}}}{S_0} &= 4(1+z)(x+2y+2) \frac{L_{\text{IS}}^2}{L_0^2} = \frac{8(7y+6)(3y^2+10y+6)(9y^2+10y+2)^2}{y^2(y+1)(3y+2)(5y+4)(6y+5)} \quad [10\cdot173, \\ \frac{S_{\text{CC}}}{S_0} &= \frac{24(3y^2+10y+6)^2(9y^2+10y+2)}{y(y+1)(3y+2)(6y+5)} \quad . \quad . \quad . \quad 10\cdot174.\end{aligned}$$

Two-Phase Tandem Core Transformer with Rectangular Coils (fig. 10·49).—The functions are the same as for single-phase except :—

$$\begin{aligned}x &= \frac{(2+\sqrt{2})y(6y+5)}{2(3y+2)} \quad . \quad . \quad . \quad 10\cdot175, \\ \frac{\mathcal{E}_{\text{MIN}}}{\mathcal{E}_0} &= \frac{4\sqrt{2}(2+\sqrt{2})(3y^2+10y+6)^{\frac{1}{2}}(9y^2+10y+2)^{\frac{1}{2}}}{y^3(5y+4)^{\frac{1}{2}}(6y+5)^{\frac{1}{2}}(9y^2+5y-2)} \quad . \quad . \quad 10\cdot176,\end{aligned}$$

TABLE 10-09.—PARTICULARS OF CHEAPEST CORE TRANSFORMERS WITH RECTANGULAR COILS.

Specific Cost Ratio m .	Proportional Dimensions.				Dimension Coefficients.						Iron Cooling Functions S_{CT}/S_o .			Copper Cooling Functions S_{CT}/S_o .			Cost Functions $\mathcal{E}/\mathcal{L}_o$.			Total Cost of Copper, Total Cost of Iron.			
	α .		g .	z .	t_{CS}/L_o .		L_{CS}/L_o .	t_{CS}/L_o .	L_{CS}/L_o .	1-Phase.	2-Phase.	3-Phase.	1-Phase.	2-Phase.	3-Phase.	1-Phase.	2-Phase.	3-Phase.					
	1-Phase.	2-Phase.			1-Phase.	2-Phase.																	
		All.				All.													All.		All.	All.	All.
0	10.424	8.398	6.950	2.4748	2.356	2.010	20.955	17.886	13.970	4.786	4.9747	13.970	17.886	13.970	4.786	4.9747	13.970	685	1.135	1.830	0		
0.0143	8.500	7.255	6.067	2.0	2.333	2.152	18.291	15.612	12.194	4.3035	4.5621	12.194	15.612	12.194	4.3035	4.5621	12.194	735	1.256	1.471	0.0009		
0.0642	6.402	5.615	4.308	1.5	2.300	2.389	15.437	13.176	10.291	3.5834	5.496	863	1404	1074	2017	3555	4415	907	1,549	1,814	0.2621		
0.1567	5.229	4.463	3.486	1.2	2.273	2.617	13.085	11.681	9.127	3.1408	5.949	893	1399	1065	1787	3149	3904	1,140	1.946	2.250	0.4528		
0.3014	4.400	3.766	2.938	1.0	2.250	2.639	12.493	10.763	8.329	2.8391	6.388	890	1421	1088	1654	2913	3618	1,442	2.461	2.854	0.6667		
0.4364	3.983	3.400	2.666	0.9	2.237	2.984	11.866	10.145	7.924	2.6856	6.675	897	1446	1175	1596	2810	3455	1,688	2.881	3.376	0.8243		
0.5325	3.774	3.321	2.516	0.85	2.230	3.068	11.579	9.883	7.719	2.6081	6.842	903	1463	1174	1569	2763	3418	1,852	3.161	3.704	0.9235		
0.5704	3.704	3.161	2.469	0.833	2.227	3.090	11.477	9.796	7.651	2.5921	6.901	913	1469	1174	1560	2748	3407	1,915	3.269	3.829	0.9604		
0.6124	3.633	3.101	2.422	0.8165	2.225	3.130	11.372	9.707	7.581	2.5858	6.964	918	1477	1179	1552	2733	3389	1,983	3.385	3.966	1.0000		
0.6574	3.564	3.042	2.376	0.80	2.222	3.162	11.270	9.619	7.513	2.5808	7.028	923	1484	1178	1544	2718	3371	2,055	3.509	4.111	1.0412		
0.8229	3.353	2.862	2.235	0.75	2.214	3.208	10.958	9.353	7.305	2.4512	7.287	941	1510	1183	1521	2679	3321	2,313	3.949	4.626	1.1835		
1.0475	3.142	2.681	2.094	0.70	2.206	3.358	10.644	9.085	7.086	2.3716	7.473	963	1543	1244	1501	2643	3276	2,647	4.519	5.294	1.3587		
1.1600	3.057	2.609	2.038	0.68	2.202	3.440	10.517	8.977	7.011	2.3695	7.577	973	1558	1245	1494	2680	3250	2,808	4.796	5.619	1.4407		
1.2392	2.972	2.536	1.981	0.66	2.199	3.496	10.390	8.868	6.926	2.3674	7.687	984	1574	1261	1488	2619	3245	2,993	5.109	5.985	1.5312		
1.4384	2.886	2.464	1.924	0.64	2.195	3.555	10.261	8.759	6.841	2.3650	7.803	996	1592	1281	1481	2608	3232	3,200	5.463	6.400	1.6313		
1.6117	2.801	2.391	1.867	0.62	2.191	3.617	10.133	8.649	6.755	2.3626	7.927	1009	1612	1303	1476	2598	3219	3,437	5.867	6.874	1.7428		
1.8147	2.716	2.318	1.811	0.60	2.188	3.684	10.004	8.539	6.689	2.3604	8.058	1023	1634	1323	1471	2589	3209	3,710	6.333	7.419	1.8679		
2.0538	2.630	2.246	1.753	0.58	2.184	3.754	9.874	8.428	6.583	2.3583	8.248	1039	1657	1355	1467	2582	3199	4,025	6.871	8.060	2.0080		
2.3384	2.544	2.172	1.686	0.56	2.180	3.829	9.745	8.317	6.496	2.3561	8.398	1056	1688	1385	1464	2576	3182	4,394	7.501	8.788	2.1606		
2.6302	2.458	2.098	1.639	0.54	2.175	3.910	9.612	8.204	6.408	2.3539	8.505	1075	1712	1412	1461	2672	3163	4,830	8.245	9.460	2.3339		
3.0095	2.372	2.025	1.581	0.52	2.171	3.996	9.480	8.092	6.320	2.3518	8.676	1096	1744	1444	1460	2669	3152	5,186	9.194	10.701	2.5675		
3.6065	2.286	1.951	1.524	0.50	2.167	4.089	9.346	7.978	6.231	2.3495	8.860	1120	1779	1460	1460	2668	3131	5,980	10.908	11.959	2.8182		
4.242	2.199	1.877	1.466	0.48	2.162	4.189	9.213	7.864	6.142	2.3473	9.068	1145	1818	1489	1460	2670	3121	6.768	11.597	13.066	3.1116		
5.049	2.112	1.803	1.408	0.46	2.158	4.298	9.078	7.749	6.052	1.9771	9.276	1174	1861	1519	1463	2673	3187	7.718	13.176	15.457	3.477		
6.096	2.025	1.728	1.350	0.44	2.153	4.416	8.943	7.633	5.962	1.9429	9.567	1206	1910	1525	1466	2680	3194	8.951	15.231	17.908	3.893		
9.394	1.850	1.579	1.233	0.40	2.143	4.685	8.668	7.398	5.779	1.8740	10.044	1233	2026	1573	1479	2692	3221	12.753	21.770	25.506	5.222		
1.4630	1.391	1.086	0.95	0.35	2.130	5.104	8.318	7.100	5.545	1.7865	10.871	1412	2222	2002	1508	2652	3283	23.690	40.442	47.380	8.660		
1.269	1.083	0.846			2.106	6.093	7.734	6.902	5.156	1.6413	12.832	1756	2742	3189	1604	2821	3489				∞		

$$\frac{L_{CS}}{L_0} = x \frac{l_{IS}}{L_0} = \frac{(2 + \sqrt{2})(6y + 5)^{\frac{1}{2}}(3y^2 + 10y + 6)^{\frac{1}{2}}(9y^2 + 10y + 2)^{\frac{1}{2}}}{\sqrt{2}(3y + 2)(5y + 4)^{\frac{1}{2}}} \quad 10\cdot177,$$

$$S_{CI} = [\{8x + 4(2 + \sqrt{2})(y + 1)\} + z\{8x + 12y + 2(4 + \sqrt{2})\}]l_{IS}^2.$$

$$\frac{S_{CI}}{S_0} = \frac{2(3y^2 + 10y + 6)(9y^2 + 10y + 2)[4(2 + \sqrt{2})(y + 1)(9y^2 + 10y + 2) + (5y + 4)\{6(7 + 2\sqrt{2})y^2 + (44 + 13\sqrt{2})y + 2(4 + \sqrt{2})\}]}{y^2(y + 1)(3y + 2)(5y + 4)(6y + 5)} \\ = \frac{(2 + \sqrt{2})(3y^2 + 10y + 6)(9y^2 + 10y + 2)[8(y + 1)(9y^2 + 10y + 2) + (10 - 3\sqrt{2})(5y + 4)\{6y^2 + (256 + 3\sqrt{2})y/41 + 2(24 - \sqrt{2})/41\}]}{y^2(y + 1)(3y + 2)(5y + 4)(6y + 5)} [10\cdot178,$$

$$\frac{S_{CC}}{S_0} = \frac{8\{6(3 + \sqrt{2})y + (14 + 5\sqrt{2})\}(3y^2 + 10y + 6)^2(9y^2 + 10y + 2)}{y(y + 1)(3y + 2)(5y + 4)(6y + 5)} \\ = \frac{8(3 + \sqrt{2})\{6y + (32 + \sqrt{2})/7\}(3y^2 + 10y + 6)^2(9y^2 + 10y + 2)}{y(y + 1)(3y + 2)(5y + 4)(6y + 5)} \quad 10\cdot179.$$

Three-Phase Tandem Core Transformer with Rectangular Coils (fig. 10·61).—

All the functions are the same as for single-phase except:—

$$x = \frac{4y(6y + 5)}{3(3y + 2)} \quad 10\cdot180,$$

$$\frac{\mathcal{L}_{MIN}}{\mathcal{L}_0} = \frac{16\sqrt{2}(3y^2 + 10y + 6)^{\frac{1}{2}}(9y^2 + 10y + 2)^{\frac{1}{2}}}{y^3(5y + 4)^{\frac{1}{2}}(6y + 5)^{\frac{1}{2}}(9y^2 + 5y - 2)} \quad 10\cdot181,$$

$$\frac{L_{CS}}{L_0} = x \frac{l_{IS}}{L_0} = \frac{4\sqrt{2}(6y + 5)^{\frac{1}{2}}(3y^2 + 10y + 6)^{\frac{1}{2}}(9y^2 + 10y + 2)^{\frac{1}{2}}}{3(3y + 2)(5y + 4)^{\frac{1}{2}}} \quad 10\cdot182,$$

$$S_{CI} = [\{12x + 16(y + 1)\} + z\{12x + 16y + 12\}]l_{IS}^2$$

$$\frac{S_{CI}}{S_0} = \frac{4(3y^2 + 10y + 6)(9y^2 + 10y + 2)[8(y + 1)(9y^2 + 10y + 2) + (5y + 4)(36y^2 + 37y + 6)]}{y^2(y + 1)(3y + 2)(5y + 4)(6y + 5)} [10\cdot183,$$

$$\frac{S_{CC}}{S_0} = \frac{8(33y + 26)(3y^2 + 10y + 6)^2(9y^2 + 10y + 2)}{y(y + 1)(3y + 2)(5y + 4)(6y + 5)} \quad 10\cdot184.$$

Three-Limb Three-Phase Transformer with Circular Coils

(fig. 10·51).—

$$x = \frac{2y(16y + 13)}{3(2y + 1)} \quad 10\cdot185,$$

$$m = \frac{(-8y^2 + 4y + 9)(24y^2 + 23y + 3)}{4y^2(y + 1)(16y + 13)(24y^2 + 16y - 3)} \quad 10\cdot186,$$

$$\frac{\text{Total cost of copper}}{\text{Total cost of iron}} = \frac{(-8y^2 + 4y + 9)}{(24y^2 + 16y - 3)} \quad 10\cdot187,$$

$$\frac{\mathcal{L}_{MIN}}{\mathcal{L}_0} = \frac{8\pi(y + 1)^{\frac{1}{2}}(4y + 3)(24y^2 + 23y + 3)^{\frac{1}{2}}}{y^3(16y + 13)^{\frac{1}{2}}(24y^2 + 16y - 3)} \quad 10\cdot188,$$

$$\frac{l_{IS}}{L_0} = \frac{2(y + 1)^{\frac{1}{2}}(24y^2 + 23y + 3)^{\frac{1}{2}}}{y(16y + 13)^{\frac{1}{2}}} \quad 10\cdot189,$$

$$\frac{L_{CS}}{L_0} = x \frac{l_{IS}}{L_0} = \frac{4(y + 1)^{\frac{1}{2}}(16y + 13)^{\frac{1}{2}}(24y^2 + 23y + 3)^{\frac{1}{2}}}{3(2y + 1)} \quad 10\cdot190,$$

$$\begin{aligned} \frac{l_{CS}}{L_0} &= y \frac{l_{IS}}{L_0} = \frac{2(y+1)^{\frac{1}{2}}(24y^2+23y+3)^{\frac{1}{2}}}{(16y+13)^{\frac{1}{2}}} \quad . \quad . \quad . \quad 10\cdot191, \\ S_{CI} &\doteq \pi(3x+8y+6)l_{IS}^2 \\ \therefore \frac{S_{CI}}{S_0} &= \frac{8\pi(y+1)(24y^2+23y+3)^2}{y^2(2y+1)(16y+13)} \quad . \quad . \quad . \quad 10\cdot192, \\ \frac{S_{CC}}{S_0} &= \frac{8\pi(y+1)^2(38y+29)(24y^2+23y+3)}{y(2y+1)(16y+13)} \quad . \quad . \quad 10\cdot193. \end{aligned}$$

TABLE 10·10.—PARTICULARS OF CHEAPEST THREE-LIMB THREE-PHASE TRANSFORMERS WITH CIRCULAR COILS.

Specific Cost Ratio. m.	Proportional Dimensions.		Dimension Coefficients.			Cooling Functions.		Cost Function.	Total Cost of Copper.
	x.	y.	l_s/L_0 .	L_{CS}/L_0 .	l_{CS}/L_0 .	S_{CI}/S_0 .	S_{CC}/S_0 .	£/£ ₀ .	Total Cost of Iron.
0	8·359	1·3397	3·412	28·523	4·5714	1529	4980	1,304	0
0·00717	7·577	1·2	3·517	26·643	4·2199	1489	4501	1,368	0·0449
0·02912	6·444	1·0	3·714	23·934	3·7139	1444	3871	1,522	0·1351
0·03701	5·232	0·8	4·004	21·189	3·2030	1424	3313	1,826	0·2814
0·15109	4·706	0·7	4·207	19·796	2·9448	1430	3065	2,097	0·3948
0·27314	4·109	0·6	4·473	18·882	2·6840	1454	2842	2,535	0·5591
0·37670	3·806	0·55	4·641	17·666	2·5526	1477	2742	2,865	0·6723
0·5325	3·500	0·50	4·840	16·941	2·4202	1509	2650	3,320	0·8182
0·6168	3·376	0·48	4·931	16·649	2·3669	1525	2616	3,552	0·8890
0·7187	3·252	0·46	5·029	16·355	2·3134	1544	2584	3,823	0·9692
0·7598	3·208	0·4529	5·066	16·251	2·2945	1552	2573	3,931	1·0000
0·8430	3·127	0·44	5·136	16·059	2·2597	1566	2554	4,143	1·0604
0·9964	3·001	0·42	5·252	15·761	2·2059	1591	2526	4,524	1·1654
1·1879	2·874	0·40	5·380	15·461	2·1518	1620	2500	4,984	1·2873
1·4304	2·746	0·38	5·520	15·159	2·0974	1654	2478	5,547	1·4307
1·7428	2·618	0·36	5·674	14·854	2·0428	1693	2458	6,247	1·6018
2·1529	2·488	0·34	5·847	14·546	1·9878	1738	2442	7,137	1·8095
2·7040	2·357	0·32	6·040	14·236	1·9326	1791	2430	8,293	2·067
3·466	2·225	0·30	6·257	13·921	1·8770	1854	2422	9,842	2·394
4·556	2·092	0·28	6·504	13·604	1·8211	1929	2420	11,992	2·824
6·191	1·957	0·26	6·787	13·282	1·7647	2019	2425	15,120	3·414
10·726	1·752	0·23	7·301	12·790	1·6792	2193	2449	23,500	4·871
22·491	1·543	0·20	7·963	12·286	1·5926	2436	2500	44,477	8·172
∞	1·203	0·15258	9·521	11·458	1·4527	3085	2671	∞	∞

Three-Limb Three-Phase Transformer with Rectangular Coils
(fig. 10·53).—

$$x = \frac{2y(24y+17)}{3(3y+2)} \quad . \quad . \quad . \quad 10\cdot194,$$

$$z = \frac{(17y+12)}{2(4y+3)} \quad . \quad . \quad . \quad 10\cdot195,$$

$$m = \frac{(17y+12)(-12y^2+17y+18)(18y^2+17y+3)}{2y^2(24y+17)(6y^2+17y+9)(36y^2+17y-6)} \quad . \quad . \quad 10\cdot196,$$

$$\frac{\text{Total cost of copper}}{\text{Total cost of iron}} = \frac{(-12y^2+17y+18)}{(36y^2+17y-6)} \quad . \quad . \quad 10\cdot197, .$$

$$\begin{aligned} \frac{\mathcal{L}_{\text{MIN}}}{\mathcal{L}_0} &= \frac{64 \sqrt{2}(6y^2 + 17y + 9)^{\frac{1}{2}}(18y^2 + 17y + 3)^{\frac{1}{2}}}{y^3(17y + 12)^{\frac{1}{2}}(24y + 17)^{\frac{1}{2}}(36y^2 + 17y - 6)} \quad . \quad . \quad 10\cdot198, \\ \frac{l_{\text{IS}}}{L_0} &= \frac{2 \sqrt{2}(6y^2 + 17y + 9)^{\frac{1}{2}}(18y^2 + 17y + 3)^{\frac{1}{2}}}{y(17y + 12)^{\frac{1}{2}}(24y + 17)^{\frac{1}{2}}} \quad . \quad . \quad 10\cdot199, \\ \frac{L_{\text{CS}}}{L_0} &= x \frac{l_{\text{IS}}}{L_0} = \frac{4 \sqrt{2}(24y + 17)^{\frac{1}{2}}(6y^2 + 17y + 9)^{\frac{1}{2}}(18y^2 + 17y + 3)^{\frac{1}{2}}}{3(3y + 2)(17y + 12)^{\frac{1}{2}}} \quad 10\cdot200, \\ \frac{l_{\text{CS}}}{L_0} &= y \frac{l_{\text{IS}}}{L_0} = \frac{2 \sqrt{2}(6y^2 + 17y + 9)^{\frac{1}{2}}(18y^2 + 17y + 3)^{\frac{1}{2}}}{(17y + 12)^{\frac{1}{2}}(24y + 17)^{\frac{1}{2}}} \quad . \quad 10\cdot201, \\ \frac{L_{\text{IS}}}{L_0} &= z \frac{l_{\text{IS}}}{L_0} = \frac{\sqrt{2}(17y + 12)^{\frac{1}{2}}(6y^2 + 17y + 9)^{\frac{1}{2}}(18y^2 + 17y + 3)^{\frac{1}{2}}}{y(4y + 3)(24y + 17)^{\frac{1}{2}}} \quad 10\cdot202, \\ S_{\text{CI}} &= [(6x + 16y + 12) + z(6x + 16y + 10)]l_{\text{IS}}^2. \\ \therefore \frac{S_{\text{CI}}}{S_0} &= \frac{8(6y^2 + 17y + 9)(18y^2 + 17y + 3)[4(25y + 18)(18y^2 + 17y + 3) - (3y + 2)(17y + 12)]}{y^2(3y + 2)(4y + 3)(17y + 12)(24y + 17)} \quad [10\cdot203, \\ S_{\text{CC}} &= \frac{32(57y + 40)(6y^2 + 17y + 9)^2(18y^2 + 17y + 3)}{y(3y + 2)(4y + 3)(17y + 12)(24y + 17)} \quad . \quad 10\cdot204. \end{aligned}$$

TABLE 10.11.—PARTICULARS OF CHEAPEST THREE-LIMB THREE-PHASE TRANSFORMERS WITH RECTANGULAR COILS.

Specific Cost Ratio.	Proportional Dimensions.			Dimension Coefficients.				Cooling Functions.		Cost Function.	Total Cost of Copper.
	<i>x</i> .	<i>y</i> .	<i>z</i> .	l_{IS}/L_0 .	L_{CS}/L_0 .	l_{CS}/L_0 .	L_{IS}/L_0 .	S_{CI}/S_0 .	S_{CC}/S_0 .	$\mathcal{L}/\mathcal{L}_0$.	Total Cost of Iron.
0	11.492	2.1232	2.092	2.170	24.941	4.6079	4.541	1654	4831	1.229	0
0.0371	8.164	1.5	2.083	2.434	19.848	3.6512	5.071	1527	3661	1.485	0.1642
0.2055	5.467	1.0	2.071	2.860	15.637	2.8604	5.925	1494	2902	2.195	0.4894
0.3018	4.928	0.9	2.068	2.998	14.772	2.6980	6.200	1506	2777	2.515	0.6131
0.4587	4.388	0.8	2.065	3.167	13.896	2.5334	6.538	1530	2662	2.985	0.7807
0.7028	3.892	0.7083	2.061	3.361	13.081	2.3805	6.925	1567	2571	3.645	0.9965
0.7071	3.886	0.7071	2.061	3.364	13.070	2.3784	6.931	1568	2570	3.656	1.0000
0.9496	3.576	0.65	2.058	3.511	12.555	2.2819	7.225	1602	2521	4.264	1.1836
1.1213	3.414	0.62	2.057	3.598	12.282	2.2307	7.400	1625	2498	4.675	1.3019
1.2590	3.305	0.60	2.056	3.661	12.100	2.1964	7.525	1642	2484	4.996	1.3916
1.4201	3.197	0.58	2.055	3.728	11.916	2.1620	7.658	1660	2471	5.364	1.4917
1.6100	3.088	0.56	2.053	3.799	11.731	2.1275	7.801	1681	2460	5.789	1.6042
1.8354	2.979	0.54	2.052	3.875	11.546	2.0927	7.954	1704	2446	6.283	1.7314
2.1055	2.871	0.52	2.051	3.957	11.360	2.0577	8.117	1730	2438	6.863	1.8764
2.4329	2.762	0.50	2.050	4.045	11.172	2.0226	8.293	1758	2434	7.551	2.044
2.8339	2.653	0.48	2.049	4.140	10.984	1.9873	8.482	1790	2429	8.379	2.238
3.3319	2.544	0.46	2.048	4.243	10.794	1.9518	8.688	1826	2426	9.388	2.467
3.9600	2.435	0.44	2.046	4.355	10.604	1.9160	8.911	1866	2425	10.633	2.741
4.787	2.326	0.42	2.045	4.476	10.412	1.8801	9.154	1912	2426	12.208	3.074
5.828	2.217	0.40	2.044	4.610	10.218	1.8439	9.420	1963	2431	14.238	3.488
7.260	2.107	0.38	2.042	4.756	10.023	1.8074	9.713	2022	2439	16.933	4.017
10.577	1.943	0.35	2.040	5.006	9.728	1.7522	10.211	2127	2458	23.048	5.156
24.527	1.669	0.30	2.036	5.529	9.227	1.6580	11.255	2364	2517	48.018	9.410
∞	1.314	0.2355	2.030	6.516	8.563	1.5346	13.222	2871	2671	∞	∞

is calculated by dividing the input by the primary voltage. The space-factors for the iron and copper must be estimated, figures 10.03 and 10.04 being of assistance. The iron space-factor is largely determined by the type of core adopted and the number of ventilating ducts required, but it will be found advisable with small transformers to take it rather low and reckon part of the space lost between the core and coils as belonging to the iron space. Otherwise, the actual length of the mean turn comes out rather

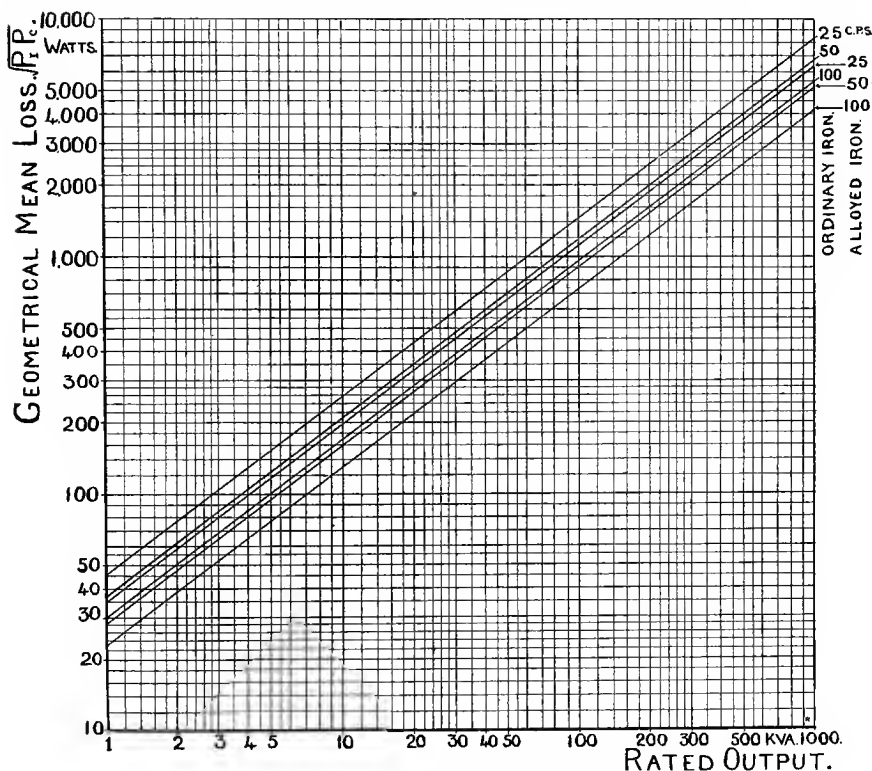


FIG. 10.63. —Maximum Permissible Geometrical Mean Loss for Different Ratings.

greater than what it is assumed to be in the preliminary design. This is particularly important in the case of transformers of the ring types. The copper space-factor for small transformers should also be taken rather lower than is given by the curves, which were not worked out with special reference to this method of design.

In the absence of actual works figures, the specific cost ratio when alloyed iron is employed may be taken as twice the ratio of the copper and iron space-factors, and 3.5 times that ratio with ordinary iron.

The loss-length must now be taken from the curves (figs. 10·05, 10·06). This length is quite constant with flux densities above about 0·65 millivolt-seconds per square inch (10,000 C.G.S.), but increases considerably when the flux density gets low. If the iron losses be about normal, the constant value, or a little over it, may be taken; but if they are low for the output, the probable flux density should be guessed at, and the corresponding loss-length used. If the actual flux density comes out very different from this, it may be necessary to make a fresh estimate and go through the preliminary design again, but this will not often be required unless it is desired to obtain the specified losses with considerable accuracy. It is as well to keep a little margin on hand to allow for extra joint losses. The increase of the loss-length with diminishing flux densities makes it very expensive to reduce the losses, and especially the iron losses, below a certain point.

The fundamental length, volume, and surface can now be calculated, and then, after the corresponding coefficients have been taken from the curves or tables, all the leading dimensions and particulars are easily fixed. It should not be forgotten that these dimensions refer to the conventional iron and copper spaces. The conventional dissipation intensity is put early in the schedule, in order that it may be seen at the very beginning whether the temperature is likely to rise too much. It is also useful in showing how much subdivision of the coils and iron is desirable from the thermal standpoint.

The "mass of standard of equal cost" is the mass of standard iron costing the same as the transformer does; it is useful in comparing the costs of transformers using different materials. Throughout the designs it is assumed that alloyed iron costs $1\frac{3}{4}$ times an equal *mass* of the standard, and copper three times.

The winding equations allot half the net copper section to each winding, thus setting the extra exciting current in the primary against the extra turns required in the secondary to compensate for the voltage drop. It will generally be found that this over-compensates for the latter, but that can be adjusted when getting out the final design. The modifications in the formula for polyphase transformers, or if the space is to be divided in any other proportion, are obvious; for example, with concentric coils, it might be desirable to give a greater share to the outer coil to make up for its greater length of turn.

The flux density is worked out in the preliminary design as a check on the loss-length chosen, and on the probable magnetising current. If these are unsatisfactory, a fresh design should be worked out before going on to the final design. If the specified losses and assumed space-factors be normal

for the rating, the designer ought to be able to keep the temperature rise within the usual limits, and the magnetising current will also be about normal. Should the temperature rise be too great, increase all the lengths in the cube root of the ratio in which the temperature rise has to be reduced (equation 10.13). The magnetising current can also be reduced by increasing the fundamental length, while the efficiency is raised at the same time. The following considerations should be borne in mind when it is necessary to do this. Neglecting the change in the loss-length, the flux density varies as

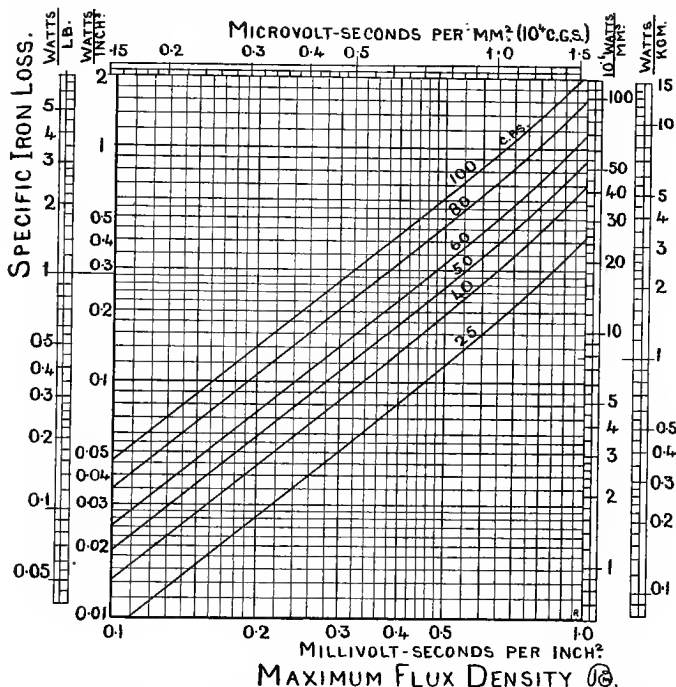


FIG. 10-64.—Logarithmic Curves for Specific Iron Loss with Sankey's "Lohys" Iron
15 mils thick. (From makers' figures.)

the $\frac{5}{4}$ power of the iron loss and as the $\frac{3}{4}$ power of the copper loss (equation 10-207). For a given value of their product, it is proportional to the fourth root of the ratio of the iron to the copper loss; while if the iron and copper losses are made equal to one another, or kept in any fixed ratio, the flux density is proportional to the square of the total loss (equation 10-208). It also varies inversely as the $\frac{5}{2}$ power of the loss-length. The number of turns remains the same when the losses are varied, so long as their ratio is not altered.

The magnetising current can also be lowered by altering the proportions,

making the core shorter and thicker (within limits) while the winding depth is made greater. When special proportions are required for this or any other reason, the general equations 10·91 and 10·118 should be employed to get the width of the iron space, and the other dimensions should be calculated from their assumed ratios to this one. The rest of the procedure is the same as with the cheapest proportions.

For simplicity, when working out the exciting current from the preliminary design, the length of the flux has been taken the same as

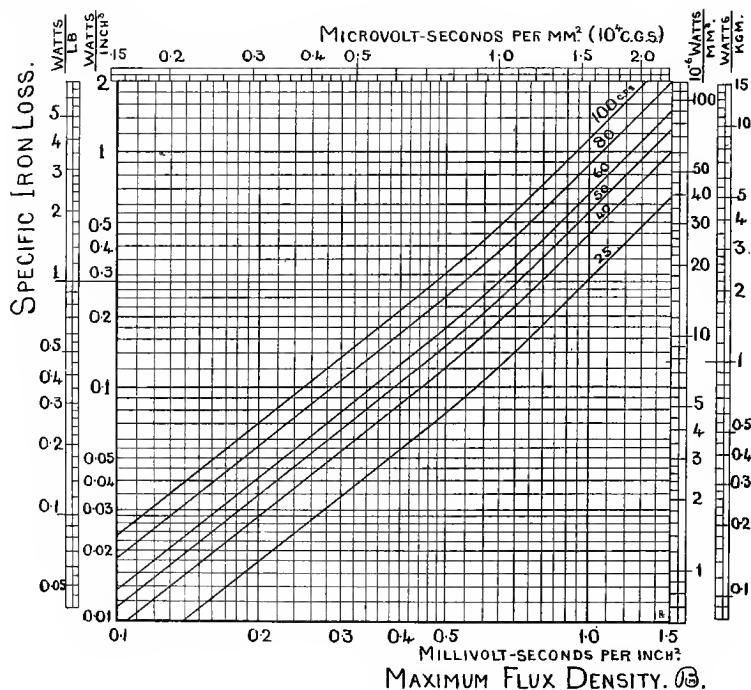


FIG. 10·65.—Logarithmic Curves for Specific Iron Loss with Sankey's "Stalloy" Iron 20 mils thick. (From makers' figures.)

that of the iron in single-phase transformers, and one-third of it for three-phase. This will give results rather on the high side. The leakage inductance is also worked out approximately by setting the clearances between the coils and the boundaries of the copper space against the fact that the whole of that between the coils is effective for leakage, and taking one-third of the depth of the copper space as the equivalent leakage thickness. This will, as a rule, lead to an underestimate of the leakage reactance, but a more accurate result can be afterwards made when the final design is settled, although even that can only be an approximation. It is not, as a

rule, necessary to work out the reactance for the preliminary design; it is only required when a guide is required as to the amount of subdivision desirable. For equal leakage with sandwiched coils, as with concentric ones, each winding should have about as many sections as the length of the copper space contains twice the depth of the copper space (*i.e.* $x/2y$), the outside section being split into two halves, one at each end.

Whenever possible, it is desirable to adopt standard sizes of wire, and to avoid all sorts of odd dimensions to be worked to in the shops. After having got a satisfactory preliminary design, a final one should be made in which standard wires are adopted, and the dimensions in general altered into round numbers. The designer must use his art in determining how far the preliminary design may be departed from, and which way to do so. As a result of these modifications, the performance of the finished transformer will, as a rule, differ somewhat from that assumed in the beginning, and the losses should always be estimated after everything is settled. To facilitate doing so, figs. 10-64 and 10-65 are given. These show the specific iron loss for "Lohys" and "Stalloy" as guaranteed by the makers. They are plotted logarithmically in order to give the same accuracy at any place, and incidentally they also show how far the Steinmetz law is followed for these materials.

Application to Choking Coils.—Let the two windings of a transformer of 1 : 1 ratio be joined in series the same way round, while an air-gap is put in the magnetic circuit of such a length as to keep the flux with full load current down to its original value. We then obtain a choking coil which takes the same current and has the same losses as the original transformer, while it absorbs double the voltage of one winding. The rating of the apparatus as a choking coil is thus twice that as a transformer. Hence the normal losses and design for a choking coil will be the same as those of a transformer of half its rating. The power-factor of the choking coil is the ratio of the losses to the volt-ampere input, and will be a minimum when the losses are a minimum, which is for that load which makes the copper and iron losses alike.

The design is worked out in the same way as for a transformer, except that the whole space goes to one winding and the gap has to be inserted. If half of this be placed at the centre of each coil, its correct length can be very accurately estimated beforehand, and the stray flux will be comparatively small. If it be outside the coils, the gap will have to be longer than calculated, while the stray flux will be very considerable and may cause trouble with eddies.

Application to Auto-Transformers.—An auto-transformer is not in

reality a special kind of transformer, but a special way of using one. The two windings are connected in series with the supply across the ends and the load between one end and the joint, or *vice versa*. If the parts into which the total voltage be divided be V_1 and V_2 , the latter being the load voltage, while the currents in the two parts are I_1 and I_2 , the load for which it is rated (assuming that the voltage is being reduced) will be

$$\begin{aligned}(I_1 + I_2)V_2 &= I_1V_2 + I_2V_2 \\ &= I_1V_2 + I_1V_1 \\ &= I_1V_1 \times (V_1 + V_2)/V_1 \quad . \quad . \quad . \quad 10\cdot209.\end{aligned}$$

Hence the rating as an auto-transformer is $(V_1 + V_2)/V_1$ times its ordinary rating, and this increase becomes very great when the supply and load voltages do not differ much. To design an auto-transformer, it is therefore best to treat it as an ordinary transformer with the rating altered to suit. The normal losses and size are the same as for an ordinary rating $V_1/(V_1 + V_2)$ times that required, but the nominal efficiency will be higher, for it will be reckoned on the larger effective output. The voltage drop and the magnetising current will also be reduced in the same ratio; the former because a drop of V_2 is necessarily accompanied by a rise of V_1 when the total is kept constant, and the magnetising current is reduced by the fact that it goes through both windings instead of one only. Table 10·12 gives the values for the ratio $(V_1 + V_2)/V_1$. The rating is unaltered if the positions of the load and supply are interchanged, but in that case the magnetising current is the same as with an ordinary transformer.

TABLE 10·12.—COMPARISON OF AUTO- AND ORDINARY
TRANSFORMER RATINGS.

$\frac{\text{Supply voltage}}{\text{Load voltage}}$	$\frac{V_1 + V_2}{V_2}$	4 : 1	3 : 1	2 : 1	3 : 2	4 : 3	5 : 4	6 : 5	10 : 9	1 : 1
$\frac{\text{Auto rating}}{\text{Ordinary rating}}$	$\frac{V_1 + V_2}{V_1}$	1·333	1·50	2	3	4	5	6	10	∞

Application to Multi-Voltage Transformers.—When more than two windings, or sections of one winding, are required, the aggregate volt-ampereage should be taken as the sum of the greatest volt-ampereages for each section, whether these occur simultaneously or not. The greatest input to the primary will of course be determined by the greatest simultaneous loads for the other windings. The normal losses may be taken the same as those in a transformer rated at half the grand aggregate. The

procedure is then exactly as before, but the copper section should be divided between the windings in the proportion which their output or input bears to the aggregate, the corresponding fraction being taken instead of $\frac{1}{2}$ in the equations for the number of turns and section of wire. If the maximum loads do not occur simultaneously, the average temperature rise will be somewhat less than is permissible, but it is unsafe to take advantage of this fact by reducing the dimensions much, for fear of overheating the loaded parts.

Preliminary Design for a Choking Coil of the Rectangular Coil Core Type to Absorb 1 K.V.A. at 100 Volts, 10 Amperes, and 100 Cycles per second, using Stalloy Iron.

Loss allowed in iron	(given)	$P_I = 12.5 \text{ watts} = 1.25 \text{ per cent.}$
„ „ copper	„	$P_C = 12.5 \text{ „} = 1.25 \text{ „}$
„ geom. mean	„	$\sqrt{P_I P_C} = 12.5 \text{ „} = 1.25 \text{ „}$
Input	(given)	$IV = 1000 \text{ volt-amperes.}$
Power-factor	$\cos \phi = (P_I + P_C)/IV$	$= 0.025$
Potential difference	(given)	$V = 100 \text{ volts.}$
Current	„	$I = 10 \text{ amperes.}$
Frequency	(given)	$f = 100 \text{ cycles per second.}$
Loss-length for 20 mils Stalloy (from curve)		$L_L = 2.40 \text{ mils.}$
Resistivity of copper (hot A.C) (assumed)		$\rho_C = 0.90 \times 10^{-6} \text{ ohm-inches.}$
Space-factor for iron	(estimated)	$\sigma_I = 0.85.$
„ „ copper	„	$\sigma_C = 0.45.$
„ geom. mean	„	$\sqrt{\sigma_I \sigma_C} = 0.619.$
Specific cost ratio		$m = 2\sigma_C/\sigma_I = 1.06.$
Fundamental length—		
$L_o = \frac{IV L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{1000 \text{ volt-amps.} \times 2.40 \text{ mils}}{0.619 \times 12.5 \text{ watts}} = 0.310 \text{ inches.}$		
Fundamental surface		$S_o = L_o^2 = 0.096 \text{ inches}^2.$
„ volume		$V_o = L_o^3 = 0.0298 \text{ inches}^3.$
Conventional cooling surface of iron—		
$S_{CI} = (S_{CI}/S_o)S_o$	$= 964 \times 0.096$	$\text{inches}^2 = 92.5 \text{ inches}^2.$
Conventional cooling surface of copper—		
$S_{CC} = (S_{CC}/S_o)S_o$	$= 1500 \times 0.096$	„ $= 144$ „
Conventional dissipation intensity for iron—		
P_I/S_{CI}	$= 12.5 \text{ watts} \div 92.5$	„ $= 0.135 \text{ watts per inch}^2.$
Conventional dissipation intensity for copper—		
P_C/S_{CC}	$= 12.5 \text{ watts} \div 144$	„ $= 0.087$ „

Width of iron space—

$$l_{IS} = (l_{IS}/L_O)L_O = 3.40 \times 0.310 \quad \text{inches} = 1.05 \text{ inches.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_O)L_O = 10.60 \times 0.310 \quad \text{,,} = 3.28 \quad \text{,,}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_O)L_O = 2.36 \times 0.310 \quad \text{,,} = 0.73 \quad \text{,,}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_O)L_O = 7.50 \times 0.310 \quad \text{,,} = 2.32 \quad \text{,,}$$

Length of iron—

$$L_I = (4l_{IS} + 2L_{CS} + 4l_{CS}) = (4.20 + 6.56 + 2.92) \text{ inches} = 13.68 \text{ inches.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 3l_{CS} + 2L_{IS}) = (2.10 + 2.19 + 4.64) \quad \text{,,} = 8.93 \quad \text{,,}$$

Section of iron space—

$$S_{IS} = l_{IS}L_{IS} = 1.05 \text{ ins.} \times 2.32 \quad \text{inches} = 2.44 \text{ inches}^2.$$

Section of copper space—

$$S_{CS} = 2L_{CS}l_{CS} = 6.56 \quad \text{,,} \times 0.73 \quad \text{,,} = 4.80 \quad \text{,,}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.85 \times 2.44 \quad \text{inches}^2 = 2.07 \quad \text{,,}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.45 \times 4.80 \quad \text{,,} = 2.16 \quad \text{,,}$$

Volume of iron—

$$V_I = L_I S_I = 13.68 \text{ ins.} \times 2.07 \quad \text{inches}^2 = 28.3 \text{ inches}^3.$$

Volume of copper—

$$V_C = L_C S_C = 8.93 \quad \text{,,} \times 2.16 \quad \text{,,} = 19.3 \quad \text{,,}$$

Mass of iron—

$$M_I = D_I V_I = \frac{0.282 \text{ lb.}}{\text{inch}^3} \times 28.3 \quad \text{inches}^3 = 8.0 \text{ lbs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{0.322 \text{ lb.}}{\text{inch}^3} \times 19.3 \quad \text{,,} = 6.2 \quad \text{,,}$$

Mass of standard iron of equal total cost—

$$M_S = 1.75 M_I + 3 M_C = (14.0 + 18.6) \quad \text{lbs.} = 32.6 \quad \text{,,} = 14.8 \text{ kgs.}$$

Turns in winding—

$$N = \frac{1}{I} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{10 \text{ amps.}} \left\{ \frac{12.5 \text{ watts} \times 2.16 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 8.93 \text{ ins.}} \right\}^{\frac{1}{2}} = 184 \text{ turns} \\ = 2 \times 92 \text{ turns.}$$

Section of wire—

$$S_W = S_C / N = 2.16 \text{ ins.}^2 \div 184 = 11.7 \times 10^{-3} \text{ inches}^2.$$

Current density (R.M.S.)—

$$f = I / S_W = 10 \text{ amps.} \div 11.7 \times 10^{-3} \text{ ins.}^2 = 855 \text{ amps. per inch}^2.$$

Flux density (maximum)—

$$\beta = \frac{V}{4.44 f N S_I} = \frac{100 \text{ volts}}{4.44 \times 100 \text{ per sec.} \times 184 \times 2.07 \text{ ins.}^2} = 0.59 \text{ M.V.S. per in.}^2$$

$$= 9.150 \text{ C.G.S.}$$

Excitation (R.M.S.)—

$$X = NI = 184 \text{ turns} \times 10 \text{ amps.} = 1840 \text{ amp.-turns.}$$

Length of gap required—

$$l_G = \frac{\sqrt{2 \mu_A X}}{\sigma_I \beta} = \frac{45 \text{ mils} \times 10^{-6} \text{ V.S. per in.}^2}{\text{amp.-turns}} \times \frac{1840 \text{ amp.-turns}}{0.85 \times 0.59 \text{ M.V.S. per in.}^2} = 164 \text{ mils.}$$

$$= 2 \times 82 \text{ mils.}$$

Design for a Choking Coil of the Rectangular Coil Core Type to absorb 1 K.V.A. at 100 Volts, 10 Amperes, and 100 Cycles per Second, using Stalloy Iron. (Plate 3.)

Rated voltage	V = 100 volts.
„ current	I = 10 amperes.
Number of turns	N = 184 turns.
„ coils	2.
„ turns per coil	92.
„ layers per coil	5.
„ turns per layer	19, 19, 19, and 16.
Conductor, diameter, bare	10 S.W.G. 128 mils.
„ „ covered	142 „
„ section	$S_W = 12.87 \times 10^{-3} \text{ inches}^2$.
„ total section of copper	$S_C = NS_W = 2.37 \text{ inches}^2$.
„ current density	$I = I/S_W = 778 \text{ amps. per inch}^2$.
Depth of winding	$l_{CS} = 0.71 \text{ inches.}$
Axial length of coil, allowing one extra diameter for spiral	2.84 „
Perimeter of coil section, after taping	$L_{PC} = 7.26 \text{ „}$
Former, dimensions of section	$1 \frac{3}{16} \times 2 \frac{9}{16} \text{ inches.}$
„ perimeter of section	7.50 „
Allowance for corners	2.13 „
Length of mean turn	$L_C = 9.63 \text{ „}$
„ wire	$L_W = NL_C = 1770 \text{ „}$
Cooling surface of coils	$S_{CC} = 2L_{PC}L_C = 14.52'' \times 9.63'' = 140 \text{ inches}^2$.
Volume of copper	$V_C = L_C S_C = 22.8 \text{ inches}^3$.
Mass of copper	$M_C = (0.322 \text{ lb. per inch}^3) V_C = 7.34 \text{ lbs.}$

Effective resistance, hot—

$$R = 0.90 \times 10^{-6} \text{ ohm-ins.} \times L_W/S_W = 0.124 \text{ ohm.}$$

Copper loss

$$P_C = RI^2 = 12.4 \text{ watts} = 1.24 \%$$

Dissipation intensity for coils

$$P_C/S_{CC} = 0.088 \text{ watts per inch}^2.$$

Estimated temperature rise—

$$t_{RC} = (240^\circ \text{ C. per watt per in.}^2) \times P_C/S_{CC} = 21^\circ \text{ C.}$$

Width of core

$$l_{IS} = 1 \text{ inch.}$$

Distance between yokes

$$L_{CS} = 3 \text{ inches.}$$

„ „ cores

$$2l_{CS} = 2 \text{ „}$$

Length of iron—

$$L_I = (4l_{IS} + 2L_{CS} + 2 \times 2l_{CS}) = (4 + 6 + 4) \text{ ins.} = 14 \text{ inches.}$$

„ flux—

$$L_F = (3l_{IS} + 2L_{CS} + 2 \times 2l_{CS}) = (3 + 6 + 4) \text{ „} = 13 \text{ „}$$

Thickness of sheets

$$20 \text{ mils.}$$

Number of sheets in each core, Γ -shaped

$$100.$$

Iron depth, net

$$2.00 \text{ inches.}$$

„ „ gross

$$L_{IS} = 2.35 \text{ „}$$

„ section, net

$$S_I = 2.00 \text{ inches}^2.$$

Cooling surface of iron—

$$S_{CI} = 2L_I(l_{IS} + L_{IS}) = 28 \text{ ins.} \times (1 + 2.35) \text{ ins.} = 93.8 \text{ inches}^2.$$

Volume of iron

$$V_I = L_I S_I = 28.0 \text{ inches}^3.$$

Mass of iron

$$M_I = (0.282 \text{ lb. per in.}^3) V_I = 7.90 \text{ lbs.}$$

Mass of standard iron of equal total cost—

$$M_S = 1.75 M_I + 3 M_C = (13.8 + 22.0) \text{ lbs.} = 35.8 \text{ „} = 16.2 \text{ kgs.}$$

Relative cost mass—

$$\begin{aligned} &= M_S \times \{ \sqrt{P_I P_C / \frac{1}{2} IV} \}^3 \times (f/50 \text{ C.P.S.})^{\frac{3}{2}} \\ &= 35.8 \text{ lbs.} \times \{ \sqrt{12.6 \times 12.4 / 500} \}^3 \times 2\sqrt{2} = 1.58 \times 10^{-3} \text{ lbs.} \\ &= 0.716 \text{ grams.} \end{aligned}$$

Flux density (maximum)—

$$\begin{aligned} \mathcal{B} &= \frac{V}{4fNS_I} = \frac{100 \text{ volts}}{4 \times 60 \times 100 \text{ per sec.} \times 184 \times 2.00 \text{ ins.}^2} = 0.613 \text{ M.V.S. per inch}^2. \\ &= 9,500 \text{ lines per cm.}^2 \end{aligned}$$

Iron loss per unit volume (from curves for Stalloy)—

$$P_I/V_I = 0.45 \text{ watts per inch}^3.$$

„ total

$$P_I = (P_I/V_I) V_I = 12.6 \text{ watts} = 1.26 \%$$

Dissipation intensity for iron

$$P_I/S_{CI} = 0.135 \text{ watts per inch}^2.$$

Estimated temperature rise for iron—

$$t_{RI} = (240^\circ \text{ C. per watt/in.}^2) \times P_I/S_{CI} = 32^\circ \text{ C.}$$

Magnetising field (maximum) required for iron (from curve)—

$$H = 6.7 \text{ amp.-turns per inch.}$$

Excitation (R.M.S.) required for iron $X_I = \frac{1}{\sqrt{2}} H L_F = 62 \text{ amp.-turns.}$

„ „ due to current $X_M = NI = 1840$ „


„ „ to be used by gaps $X_G = X_M - X_I = 1778$ „

Length of air-gap required—

$$l_G = \frac{\sqrt{2} \mu_A X_G}{\sigma_I \beta_I} = \frac{45 \text{ mils} \times \text{M.V.S. per in.}^2}{.1000 \text{ amp.-turns}} \times \frac{1778 \text{ amp.-turns}}{0.85 \times 0.613 \text{ M.V.S. per in.}^2} = 154 \text{ mils}$$

$$= 2 \times 77 \text{ mils.}$$

Power-factor $\cos \phi = (P_I + P_C)/IV = (12.6 + 12.4) \text{ watt}/1000 \text{ V.A.} = 0.0250.$

The desired gap is fixed by inserting wooden distance-pieces of the correct thickness between the two parts of the core, and it can be adjusted by trial to give the right inductance exactly, if that is necessary, by inserting press-pahn packing-pieces. It is most convenient to stamp the sheets in the form of a  to save labour in assembling, and those constituting each half of the core should be securely bound together by cord or tape. They must be thoroughly impregnated if silent running be required. The two halves may be clamped together by tie-bars, or the case may be designed as shown, so as to act as a clamp as well. The coils can be secured by wooden wedges between them and the cores and yokes.

Preliminary Design for an Auto-Transformer of the Rectangular Coil Core Type to give 2 K.V.A. at 210/105 Volts, 9.8/19 Amperes, and 50 Cycles per Second, using Stalloy Iron.

Loss allowed in iron	(given)	$P_I = 20 \text{ watts} = 0.97 \text{ per cent.}$
„ „ copper	„	$P_C = 40 \text{ „} = 1.94 \text{ „}$
„ geom. mean		$\sqrt{P_I P_C} = 28.3 \text{ „} = 1.37 \text{ „}$
Output of secondary winding	(estimated)	$I_2 V_2 = 970 \text{ volt-amperes.}$
Input to primary winding	„	$I_1 V_1 = 1030 \text{ „}$
Aggregate volt-ampereage	„	$(I_1 V_1 + I_2 V_2) = 2000 \text{ „}$
Terminal P.D. of primary winding	(given)	$V_1 = 105 \text{ volts.}$
„ secondary „ „		$V_2 = 105 \text{ „}$
Current in primary winding	(estimated)	$I_1 = 9.80 \text{ amperes.}$
„ secondary „	(given)	$I_2 = 9.24 \text{ „}$

Frequency	(given)	f	= 50 cycles per second.
Loss-length for 20 mils Stalloy (from curve)		L_L	= 3.20 mils.
Resistivity of copper (hot A.C.) (assumed)		ρ_C	= 0.90×10^{-6} ohm-inches.

Space-factor for iron	(estimated)	σ_I	= 0.85.
„ „ copper	„	σ_C	= 0.40.
„ geom. mean	„	$\sqrt{\sigma_I \sigma_C}$	= 0.583.
Specific cost ratio	„	$m = 2\sigma_C/\sigma_I$	= 0.94.

Fundamental length—

$$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{2000 \text{ volt-amps.} \times 3.20 \text{ mils}}{0.583 \times 28.3 \text{ watts}} = 0.388 \text{ inches.}$$

Fundamental surface	$S_0 = L_0^2$	= 0.151 inches ² .
„ volume	$V_0 = L_0^3$	= 0.0584 inches ³ .

Conventional cooling surface of iron—

$$S_{CI} = (S_{CI}/S_0) S_0 = 950 \times 0.151 \text{ inches}^2 = 143 \text{ inches}^2.$$

Conventional cooling surface of copper—

$$S_{CC} = (S_{CC}/S_0) S_0 = 1510 \times 0.151 \text{ „} = 228 \text{ „}$$

Conventional dissipation intensity for iron—

$$P_I/S_{CI} = 20 \text{ watts} \div 143 \text{ „} = 0.140 \text{ watts per inch}^2.$$

Conventional dissipation intensity for copper—

$$P_C/S_{CC} = 40 \text{ watts} \div 228 \text{ „} = 0.175 \text{ „}$$

Width of iron space—

$$l_{IS} = (l_{IS}/L_0) L_0 = 3.33 \times 0.388 \text{ inches} = 1.29 \text{ inches.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_0) L_0 = 10.75 \times 0.388 \text{ „} = 4.17 \text{ „}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_0) L_0 = 2.40 \times 0.388 \text{ „} = 0.93 \text{ „}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_0) L_0 = 7.37 \times 0.388 \text{ „} = 2.86 \text{ „}$$

Length of iron—

$$L_I = (4l_{IS} + 2L_{CS} + 4l_{CS}) = (5.16 + 8.34 + 3.72) \text{ inches} = 17.22 \text{ inches.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 3l_{CS} + 2L_{IS}) = (2.58 + 2.79 + 5.72) \text{ „} = 11.09 \text{ „}$$

Section of iron space—

$$S_{IS} = l_{IS} L_{IS} = 1.29 \text{ ins.} \times 2.86 \text{ inches} = 3.69 \text{ inches}^2.$$

Section of copper space—

$$S_{CS} = 2L_{CS} l_{CS} = 8.34 \text{ „} \times 0.93 \text{ „} = 7.75 \text{ „}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.85 \times 3.69 \text{ inches}^2 = 3.14 \text{ „}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.40 \times 7.75 \text{ „} = 3.10 \text{ „}$$

Volume of iron—

$$V_I = L_I S_I = 17.22 \text{ ins.} \times 3.14 \text{ inches}^2 = 54.1 \text{ inches}^3.$$

Volume of copper—

$$V_C = L_C S_C = 11.09 \text{ „} \times 3.10 \text{ „} = 34.4 \text{ „}$$

Mass of iron—

$$M_I = D_I V_I = \frac{0.282 \text{ lb.}}{\text{inch}^3} \times 54.1 \text{ inches}^3 = 15.2 \text{ lbs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{0.322 \text{ lb.}}{\text{inch}^3} \times 34.4 \text{ „} = 11.1 \text{ „}$$

Mass of standard iron of equal total cost—

$$M_S = 1.75 M_I + 3 M_C = (26.6 + 33.3) \text{ lbs.} = 59.9 \text{ „} = 27.2 \text{ kgs.}$$

Turns in primary winding—

$$N_1 = \frac{1}{2I_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{19.6 \text{ amps.}} \left\{ \frac{40 \text{ watts} \times 3.10 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 11.09 \text{ ins.}} \right\}^{\frac{1}{2}} = 180 \text{ turns.}$$

$$= 2 \times 90 \text{ „}$$

Turns in secondary winding—

$$N_2 = \frac{1}{2I_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{18.5 \text{ amps.}} \left\{ \frac{40 \text{ watts} \times 3.10 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 11.09 \text{ ins.}} \right\}^{\frac{1}{2}} = 190 \text{ „}$$

$$= 2 \times 95 \text{ „}$$

Section of wire for primary—

$$S_{W1} = S_C / 2N_1 = 3.10 \text{ ins.}^2 \div 360 = 8.62 \times 10^{-3} \text{ inches}^2.$$

Section of wire for secondary—

$$S_{W2} = S_C / 2N_2 = 3.10 \text{ „} \div 380 = 8.16 \times 10^{-3} \text{ „}$$

Flux density (maximum)—

$$\mathfrak{B} \div \frac{V_1}{4\pi f N_1 S_I} = \frac{105 \text{ volts}}{4.44 \times 50 \text{ per sec.} \times 180 \times 3.14 \text{ ins.}^2} = 0.838 \text{ M.V.S. per in.}^2$$

$$= 13,000 \text{ C.G.S.}$$

Current density (R.M.S.)—

$$\hat{I} = I_1 / S_{W1} = I_2 / S_{W2} = 9.80 \text{ amps.} \div 8.62 \times 10^{-3} \text{ ins.}^2 = 1140 \text{ amps. per in.}^2$$

Design for an Auto-Transformer of the Rectangular Coil Core Type to give 2 K.V.A. at 210/105 Volts, 9·8/19 Amperes, and 50 Cycles per Second, using Stalloy Iron. (Plate 4.)

		Primary.	Secondary.	Total.	Units.
Rated voltage	V	105	105	210	volts.
„ current	I	9·80	9·24	19·0	amperes.
Number of turns	N	178	186	364	turns.
„ coils		2	2		
„ turns per coil		89	93		
„ layers per coil		3	3		
„ turns per layer		31, 31, 27	31		
Conductor, diameter, bare	{	11	11	...	S.W.G.
		116	116	...	mils.
„ „ covered		130	130	...	„
„ section	S_W	10·57	10·57	...	10^{-3} ins. ²
„ total section of copper	$S_C = NS_W$	1·88	1·97	3·85	inches ² .
„ current density	$f = I/S_W$	927	874	900	amps. per in. ²
Depth of winding	l_{CS}	0·390	0·390	0·780	inches.
Axial length of coil, allowing one extra diameter for spiral		4·16	4·16	...	„
Perimeter of coil section, after taping	L_{PC}	9·30	9·30	...	„
Former, dimensions of section		$4\frac{1}{2} \times 2\frac{5}{8}$	$3\frac{3}{8} \times 1\frac{1}{2}$...	inches.
„ perimeter of section		14·25	9·75	...	„
Allowance for corners		1·2	1·2	...	„
Length of mean turn	L_C	15·45	10·95	13·15	„
„ wire	$L_W = NL_C$	2750	2040	4790	„
Cooling surface of coils	$S_{CC} = 2L_{PC}L_C$	287	204	491	inches ² .
Volume of copper	$V_C = L_C S_C$	29·1	21·5	50·6	inches ³ .
Mass of copper	$M_C = (0·322 \text{ lbs. per inch}^3)V_C$	9·36	6·94	16·30	lbs.
Effective resistance, hot— $R = 0·90 \times 10^{-6} \text{ ohm-ins.} \times L_W/S_W$		0·235	0·174	0·438	ohms.
Copper loss	$P_C = RI^2$	22·5	14·8	37·3	watts.
Dissipation intensity for coils	P_C/S_{CC}	0·079	0·073	0·076	watts per in. ²
Estimated temperature rise— $t_{RC} = (240^\circ \text{ C. per watt/in.}^2) \times P_C/S_{CC}$		19	17	18	° C.

Width of core	$l_{IS} = 1\frac{3}{8}$ inches.
Distance between yokes	$L_{CS} = 4\frac{3}{8}$ „
„ „ cores	$2l_{CS} = 2\frac{3}{8}$ „
Length of iron—	
$L_I = (4l_{IS} + 2L_{CS} + 2 \times 2l_{CS}) = (5\frac{1}{2} + 8\frac{3}{4} + 4\frac{3}{4})$ ins. = 19.0 inches.	
„ of flux—	
$L_F = (3l_{IS} + 2L_{CS} + 2 \times 2l_{CS}) = (4\frac{1}{8} + 8\frac{3}{4} + 4\frac{3}{4})$ „ = 17.6 „	
Thickness of sheets	= 20 mils.
Number of sheets in each core (L-shaped)	= 135.
Iron depth, net	= 2.70 inches.
„ „ gross	$L_{IS} = 3.18$ „
„ section, net	$S_I = 3.71$ inches ² .
Cooling surface of iron—	
$S_{CI} = 2L_I(l_{IS} + L_{IS}) = 38$ ins. \times (1.38 + 3.18) ins. = 174 inches ² .	
Volume of iron	$V_I = L_I S_I = 70.5$ inches ³ .
Mass of iron	$M_I = (0.282 \text{ lb. per in.}^3) V_I = 19.9$ lbs.
„ standard iron of equal total cost—	
$M_S = 1.75 M_I + 3 M_C = (34.8 + 48.9)$ lbs. = 83.7 „ = 38.0 kgs.	
Relative cost mass—	
$= M_S \times \{ \sqrt{P_I P_C} / \frac{1}{2} (I_1 V_1 + I_2 V_2) \}^3 \times (f/50 \text{ C.P.S.})^2$	
$= 83.7 \text{ lbs.} \times \{ \sqrt{20.4 \times 37.3/1000} \}^3$	$= 1.76 \times 10^{-3} \text{ lbs.}$
	$= 0.798 \text{ grams.}$
Flux density (maximum)—	
$\mathcal{B} = \frac{V_I}{4f N_I S_I} = \frac{105 \text{ volts}}{4.44 \times 50 \text{ per sec.} \times 178 \times 3.71 \text{ ins.}^2} = 0.716 \text{ M.V.S. per inch}^2.$	
	$= 11,100 \text{ C.G.S.}$
Iron loss per unit volume (from curves for Stalloy)—	
$P_I/V_I = 0.29$ watts per inch ³ .	
„ total	$P_I = (P_I/V_I) V_I = 20.4$ watts = 0.99 %.
Dissipation intensity for iron	$P_I/S_{CI} = 0.117$ watts per inch ² .
Estimated temperature rise for iron—	
$t_{RI} = (240^\circ \text{ C. per watt/in.}^2) \times P_I/S_{CI} = 28^\circ \text{ C.}$	
Magnetising field (maximum) required for iron (from curve)—	
	$\mathcal{H} = 9.0$ amp.-turns per inch.
Excitation (R.M.S.) required for iron	$\chi_I = \frac{1}{\sqrt{2}} \mathcal{H} L_F = 112$ amp.-turns.
„ „ „ „ two lap joints—	
$\chi_J = 2 \times (30 \text{ amp.-turns per M.V.S./in.}^2) \mathcal{B} = 43$ „	
„ (R.M.S.) required for whole	$\chi_M = \chi_I + \chi_J = 155$ „

Exciting current, idle component	$I_{IO} = X_M / (N_1 + N_2) = 0.426$	amperes.
„ „ working component	$I_{WO} = P_I / (V_1 + V_2) = 0.097$	„
„ „ total—		
	$I_O = (I_{IO}^2 + I_{WO}^2)^{\frac{1}{2}} = (0.182 + 0.010)^{\frac{1}{2}}$	amps. = 0.436 „
		= 4.45 per cent.

Equivalent leakage thickness between coils	$(\frac{a}{16} - 0.390)$	ins. = 0.172 inches.
„ „ $\frac{1}{3}$ of winding depths		= 0.260 „
„ „ total		$l_{TL} = 0.432$ „

Total equivalent resistance, referred to secondary—

$$R_{T_2} = P_C / I_2^2 = 0.438 \text{ ohms.}$$

„ leakage reactance, referred to secondary—

$$\begin{aligned} \mathcal{L}_{TL_2} \omega &= 2\pi \mu_a f N_2^2 (L_C / 2 L_{CS}) l_{TL} \\ &= \frac{0.20 \times 10^{-6} \text{ henry}}{\text{inch}} \times \frac{50}{\text{sec.}} \times 186^2 \times \frac{13.15}{8.75} \times 0.432 \text{ ins.} = 0.225 \text{ „} \end{aligned}$$

Total equivalent impedance, referred to secondary—

$$Z_{T_2} = (R_{T_2}^2 + \mathcal{L}_{TL_2}^2 \omega^2)^{\frac{1}{2}} = (0.192 + 0.051)^{\frac{1}{2}} \text{ ohms} = 0.493 \text{ „}$$

Internal lag, $\cos \phi = R_{T_2} / Z_{T_2} = 0.889$.

$$\phi = 27^\circ.$$

Voltage drop at unity power-factor	$= \frac{1}{2} R_{T_2} I_2$	= 2.02 volts = 1.92 %
„ zero (lagging) „	$= \frac{1}{2} \mathcal{L}_{TL_2} \omega I_2$	= 1.04 „ = 0.99 „
„ 0.899 „ „	$= \frac{1}{2} Z_{T_2} I_2$	= 2.28 „ = 2.18 „

EFFICIENCY AT UNITY POWER-FACTOR.

Load . . .	500	1000	1500	2000	2500	watts.
Iron loss . .	21	21	21	21	21	„
Copper loss . .	3	10	21	38	60	„
Input . . .	524	1031	1542	2059	2581	„
Total loss . .	4.58	3.00	2.72	2.87	3.14	per cent.
Efficiency . .	95.42	97.00	97.28	97.13	96.86	„

The actual design comes out appreciably larger than the preliminary one, partly because we have adopted a larger size of wire than necessary, leading to smaller copper losses, but partly also because the space-factor assumed has not given sufficient clearance between the coils, and the increased distance thus required between the cores has necessitated a greater section of iron in order to keep the iron losses down to the specified amount.

Plate 4 shows the general arrangement, and calls for no special comment. Both in this and in the previous design the efficiency is considerably higher than the limit for the rating.

Preliminary Design for an Oil-Insulated Transformer of the Hexagonal Ring Type with Rectangular Coils to give 5 K.V.A. at 2000/200 Volts, 2·6/25 Amperes, and 80 Cycles per Second, using Stalloy Iron.

Loss allowed in iron	(given)	$P_I = 60$ watts = 1·16 %
„ „ copper	„	$P_C = 120$ „ = 2·32 „
„ geom. mean	„	$\sqrt{P_I P_C} = 84·9$ „ = 1·64 „
Output of secondary winding	(given)	$I_2 V_2 = 5000$ volt-amperes.
Input to primary winding	(estimated)	$I_1 V_1 = 5180$ „
Aggregate volt-ampereage	„	$(I_1 V_1 + I_2 V_2) = 10,180$ „
Terminal P.D. of primary winding	(given)	$V_1 = 2000$ volts.
„ secondary	„	$V_2 = 200$ „
Current in primary winding	(estimated)	$I_1 = 2·59$ amperes.
„ secondary	(given)	$I_2 = 25·0$ „
Frequency	(given)	$f = 80$ cycles per second.
Loss-length for 20 mils Stalloy (from curve)		$L_L = 2·75$ mils.
Resistivity of copper (hot, A.C.) (assumed)		$\rho_C = 0·90 \times 10^{-6}$ ohm-inches.
Space-factor for iron	(estimated)	$\sigma_I = 0·70$.
„ „ copper	„	$\sigma_C = 0·33$.
„ geom. mean.	„	$\sqrt{\sigma_I \sigma_C} = 0·481$.
Specific cost ratio	„	$m = 2\sigma_C/\sigma_I = 0·943$.
Fundamental length—		
$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{10,180 \text{ volt-amps.} \times 2·75 \text{ mils}}{0·481 \times 84·9 \text{ watts}} = 0·686 \text{ inches.}$		
Fundamental surface		$S_0 = L_0^2 = 0·470$ inches ² .
„ volume		$V_0 = L_0^3 = 0·322$ inches ³ .
Conventional cooling surface of iron—		
$S_{CI} = (\sigma_{CI}/S_0)S_0$	$= 738 \times 0·470$	inches ² = 347 inches ² .
Conventional cooling surface of copper—		
$S_{CC} = (\sigma_{CC}/S_0)S_0$	$= 2034 \times 0·470$	„ = 955 „
Conventional dissipation intensity for iron—		
P_I/S_{CI}	$= 60 \text{ watts} \div 347$	„ = 0·173 watts per inch ²
Conventional dissipation intensity for copper—		
P_C/S_{CC}	$= 120 \text{ watts} \div 955$	„ = 0·126 „ „

Width of iron space—

$$l_{IS} = (l_{IS}/L_O)L_O = 3.19 \times 0.686 \quad \text{inches} = 2.19 \text{ inches.}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_O)L_O = 3.52 \times 0.686 \quad \text{,,} = 2.41 \quad \text{,,}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_O)L_O = 7.22 \times 0.686 \quad \text{,,} = 4.95 \quad \text{,,}$$

Length of iron—

$$L_I = 2\sqrt{3}(l_{IS} + 2l_{CS}) = \sqrt{12}(2.19 + 4.82) \quad \text{inches} = 24.3 \text{ inches.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 2l_{CS} + 2L_{IS}) = (4.38 + 4.82 + 9.90) \quad \text{,,} = 19.1 \quad \text{,,}$$

Section of iron space—

$$S_{IS} = l_{IS}L_{IS} = 2.19 \text{ ins.} \times 4.95 \quad \text{inches} = 10.8 \text{ inches}^2.$$

Section of copper space—

$$S_{CS} = 2\sqrt{3}l_{CS}^2 = \sqrt{12} \times 2.41^2 \quad \text{inches}^2 = 20.1 \quad \text{,,}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.70 \times 10.8 \quad \text{,,} = 7.59 \quad \text{,,}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.33 \times 20.1 \quad \text{,,} = 6.64 \quad \text{,,}$$

Volume of iron—

$$V_I = L_I S_I = 24.3 \text{ ins.} \times 7.59 \quad \text{inches}^3 = 184 \text{ inches}^3.$$

Volume of copper—

$$V_C = L_C S_C = 19.1 \quad \text{,,} \times 6.64 \quad \text{,,} = 127 \quad \text{,,}$$

Mass of iron—

$$M_I = D_I V_I = \frac{0.282 \text{ lb.}}{\text{inch}^3} \times 184 \quad \text{inches}^3 = 52.0 \text{ lbs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{0.322 \text{ lb.}}{\text{inch}^3} \times 127 \quad \text{,,} = 40.8 \quad \text{,,}$$

Mass of standard of equal cost—

$$M_S = 1.75M_I + 3M_C = (91.0 + 122.4) \quad \text{lbs.} = 213 \quad \text{,,} = 96.5 \text{ kgs.}$$

Turns in primary winding—

$$N_1 = \frac{1}{2I_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{5.18 \text{ amps.}} \left\{ \frac{120 \text{ watts} \times 6.64 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 19.1 \text{ ins.}} \right\}^{\frac{1}{2}} = 1314 \text{ turns}$$

$$= 6 \times 219 \quad \text{,,}$$

Turns in secondary winding—

$$N_2 = \frac{1}{2I_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{50 \text{ amps.}} \left\{ \frac{120 \text{ watts} \times 6.64 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 19.1 \text{ ins.}} \right\}^{\frac{1}{2}} = 136 \quad \text{,,}$$

$$= 6 \times 22.7 \quad \text{,,}$$

Section of wire for primary—

$$S_{W1} = S_C/2N_1 = 6.64 \text{ ins.}^2 \div 2628 = 2.53 \times 10^{-3} \text{ inches}^2.$$

Section of wire for secondary—

$$S_{W2} = S_C/2N_2 = 6.64 \quad \text{,,} \div 272 = 24.4 \times 10^{-3} \quad \text{,,}$$

Flux density (maximum)—

$$\beta = \frac{V_1}{4\pi f N_1 S_I} = \frac{2000 \text{ volts}}{4 \cdot 44 \times 80 \text{ per sec.} \times 1314 \times 7 \cdot 59 \text{ ins.}^2} = 0 \cdot 566 \text{ M.V.S. per inch}^2$$

$$= 8780 \text{ C.G.S.}$$

Current density (R.M.S.)—

$$\bar{I} = I_1 / S_{W1} = I_2 / S_{W2} = 2 \cdot 59 \text{ amps.} \div 2 \cdot 53 \times 10^{-3} \text{ ins.}^2 = 1020 \text{ amps. per inch}^2.$$

Design for an Oil-Insulated Transformer of the Hexagonal Ring Type with Rectangular Coils to give 5 K.V.A. at 2000/200 Volts, 2·6/25 Amperes, and 80 Cycles per Second, using Stalloy Iron.
(Plate 6.)

		Primary.	Secondary.	Total.	Units.
Rated voltage	V	2000	200	...	volts.
„ current	I	2·59	25·0	...	amperes.
Number of turns	N	1224	126	...	turns
„ coils		6	6		
„ turns per coil		204	21		
„ layers per coil		17	2		
„ turns per layer		20·4	11·10		
Conductor, diameter, bare	{	17	7	...	S.W.G.
		56	176	...	mils.
„ „ covered		68	190	...	„
„ section S_W		2·463	24·33	...	10^{-3} ins. ²
„ total section of copper $S_C = N S_W$		3·02	3·07	6·09	inches ² .
„ current density $\bar{I} = I / S_W$		1050	1028	1040	amps. per inch ² .
Depth of winding	l_{CS}	1·156	0·380	1·55	inches.
Axial length of coil, allowing one extra diameter for spiral	{	0·34	2·09	...	„
		1·43	2·28	...	„
Perimeter of coil section, after taping	L_{PC}	4·4	5·2	...	„
Former, dimensions of section		$6\frac{1}{4} \times 3\frac{1}{2}$	$5\frac{1}{8} \times 2\frac{3}{8}$...	inches.
„ perimeter of section		19·5	15·0	...	„
Allowance for corners		2·3	1·2	...	„
Length of mean turn	L_C	21·8	16·2	19·0	„
„ wire $L_W = N L_C$		26,700	2040	...	„
Cooling surface of coils	$S_{CC} = 6 L_{PC} L_C$	6×96	6×84	6×180	inches ² .
Volume of copper	$V_C = L_C S_C$	65·8	49·7	115·5	inches ³ .
Mass of copper $M_C = (0 \cdot 322 \text{ lbs. per inch}^3) V_C$		21·2	16·0	37·2	lbs.

	Primary.	Secondary.	Total.	Units.
Effective resistance, hot— $R = 0.90 \times 10^{-6} \text{ ohm-ins.} \times L_W/S_W$	9.74	0.0755	0.180	ohms.
Copper loss $P_C = RI^2$	65.3	47.2	112.5	watts.
Dissipation intensity for coils P_C/S_{CC}	0.114	0.094	0.104	watts per inch ² .
Estimated temperature rise— $t_{RC} = (200^\circ \text{ C. per watt/in.}^2) \times P_C/S_{CC}$	23	19	21	° C.
Width of core	$l_{IS} = 2.00 \text{ inches.}$			
Radius of inscribed circle	$l_{CS} = 2.44 \text{ „}$			
Side of hexagon, inner	$\frac{2}{\sqrt{3}} l_{CS} = 2.82 \text{ „}$			
„ outer	$\frac{2}{\sqrt{3}} (l_{IS} + l_{CS}) = 5\frac{1}{8} \text{ „}$			
Length of iron and of flux $L_F = L_I = 2\sqrt{3}(l_{IS} + 2l_{CS})$	$= 23.8 \text{ „}$			
Thickness of sheets	20 mils.			
Number of sheets in each core	200.			
Iron depth, net	4.00 inches.			
„ „ gross— 2 packets $2\frac{1}{4}$ ins. thick + 1 duct $\frac{1}{4}$ in. wide	$L_{IS} = 4.75 \text{ „}$			
„ section, net	$S_I = 8.00 \text{ inches}^2.$			
Cooling surface of iron— $S_{CI} = 23.8 \text{ ins.} \times (4 \times 2.00 + 4 \times 2\frac{1}{4}) \text{ ins.} = 405 \text{ inches}^2.$				
Volume of iron $V_I = L_I S_I$	$= 190 \text{ inches}^3.$			
Mass of iron $M_I = (0.282 \text{ lbs. per inch}^3) V_I$	$= 53.7 \text{ lbs.}$			
„ standard iron of equal total cost— $M_S = 1.75 M_I + 3 M_C = (94 + 112) \text{ lbs.} = 206 \text{ „} = 93.4 \text{ kgs.}$				
Relative cost mass— $= M_S \times \{\sqrt{P_I P_C} / \frac{1}{2} (I_1 V_1 + I_2 V_2)\}^3 \times (f/50 \text{ C.P.S.})^{\frac{1}{2}}$ $= 206 \text{ lbs.} \times \{\sqrt{59 \times 112.5} / 5086\}^3 \times (1.6)^{\frac{1}{2}}$ $= 1.72 \times 10^{-3} \text{ lbs.}$ $= 0.78 \text{ grams.}$				
Flux density (maximum)— $\beta = \frac{V_1}{4ffN_I S_I} = \frac{2000 \text{ volts}}{4 \times 50 \times 80 \text{ per sec.} \times 1224 \times 8.00 \text{ ins.}^2} = 0.575 \text{ M.V.S. per inch}^2.$ $= 8900 \text{ C.G.S.}$				
Iron loss per unit volume (from curves for Stalloy)— $P_I/V_I = 0.31 \text{ watts per inch}^3.$				
„ total $P_I = (P_I/V_I) V_I = 59 \text{ watts.}$				
Dissipation intensity for iron P_I/S_{CI}	$= 0.145 \text{ watts per inch}^2.$			
Estimated temperature rise for iron— $t_{RI} = (200^\circ \text{ C. per watt/inch}^2) \times P_I/S_{CI} = 29^\circ \text{ C.}$				

Magnetising field (maximum) required for iron (from curve)—

$$\mathcal{H} = 6.0 \text{ amp.-turns per inch.}$$

Excitation (R.M.S.) required for iron $\mathcal{X}_I = \frac{1}{\sqrt{2}} \mathcal{H} L_F = 101 \text{ amp.-turns.}$

„ „ „ 3 lap joints—

$$\mathcal{X}_J = 3 \times (30 \text{ amp.-turns per M.V.S./in.}^2) \beta = 52 \quad ,$$

„ „ required, whole $\mathcal{X}_M = \mathcal{X}_I + \mathcal{X}_J = 153 \quad ,$

Exciting current, idle component $I_{IO} = \mathcal{X}_M / N_I = 0.125 \text{ amperes.}$

„ „ working component $I_{WO} = P_I / V_I = 0.030 \quad ,$

„ „ total—

$$I_O = (I_{IO}^2 + I_{WO}^2)^{1/2} = (0.0156 + 0.0009)^{1/2} \text{ amps.} = 0.128 \quad ,$$

$$= 4.95 \text{ per cent.}$$

Equivalent leakage thickness between coils $= 0.18 \text{ inches.}$

„ „ $\frac{1}{2}$ of winding depths $= 0.52 \quad ,$

„ „ total $l_{TL} = 0.70 \quad ,$

Total equivalent resistance, referred to secondary—

$$R_{T_2} = P_C / I_2^2 = 0.180 \text{ ohms.}$$

„ „ leakage reactance, referred to secondary—

$$\mathcal{L}_{TL_2} \omega = 2\pi \mu_a f N_2^2 (L_C / L_F) l_{TL}$$

$$= \frac{0.20 \times 10^{-6} \text{ henry}}{\text{inch}} \times \frac{80}{\text{sec.}} \times 126^2 \times \frac{19.0}{23.8} \times 0.70 \text{ ins.} = 0.142 \quad ,$$

Total equivalent impedance, referred to secondary—

$$Z_{T_2} = (R_{T_2}^2 + L_{T_2}^2 \omega^2)^{1/2} = (0.033 + 0.020)^{1/2} \text{ ohms} = 0.229 \quad ,$$

Internal lag $\cos \phi = R_{T_2} / Z_{T_2} = 0.786 \quad \phi = 38^\circ.$

Voltage drop at unity power-factor $= R_{T_2} I_2 = 4.50 \text{ volts} = 2.25 \text{ per cent.}$

„ „ zero (lagging) power-factor $= \mathcal{L}_{TL_2} \omega I_2 = 3.55 \quad , = 1.78 \quad ,$

„ „ 0.786 „ „ $= Z_{T_2} I_2 = 5.73 \quad , = 2.86 \quad ,$

EFFICIENCY AT UNITY POWER-FACTOR.

Load	1250	2500	3750	5000	6250	watts.
Iron loss	59	59	59	59	59	„
Copper loss	7	28	64	113	177	„
Input	1316	2587	3873	5172	6486	„
Total loss	5.01	3.37	3.18	3.32	3.64	per cent.
Efficiency	94.99	96.63	96.82	96.68	96.36	„

Preliminary Design for an Oil-Insulated Transformer of the Rectangular Coil Core Type to give 10 K.V.A. at 6350/230 Volts, 1·63/43·5 Amperes, and 50 Cycles per Second, using Stalloy Iron.

Loss allowed in iron	(given)	P_I	= 160 watts = 1·55 per cent.
„ „ copper	„	P_C	= 160 „ = 1·55 „
„ geom. mean	„	$\sqrt{P_I P_C}$	= 160 „ = 1·55 „
Output of secondary winding	(given)	$I_2 V_2$	= 10,000 volt-amperes.
Input to primary winding	(estimated)	$I_1 V_1$	= 10,320 „
Aggregate volt-amperage	„	$(I_1 V_1 + I_2 V_2)$	= 20,320 „
Terminal P.D. of primary winding	(given)	V_1	= 6350 volts.
„ secondary „ „	„ „	V_2	= 230 „
Current in primary winding	(estimated)	I_1	= 1·63 amperes.
„ secondary „	(given)	I_2	= 43·5 „
Frequency	(given)	f	= 50 cycles per second.
Loss-length for 20 mils Stalloy	(from curve)	L_L	= 0·085 mm.
Resistivity of copper (hot, A.C.)	(assumed)	ρ_C	= 23×10^{-9} ohm-mm.
Space-factor for iron	(estimated)	σ_I	= 0·80.
„ „ copper	„	σ_C	= 0·50.
„ geom. mean	„	$\sqrt{\sigma_I \sigma_C}$	= 0·632.
Specific cost ratio	„	$m = 2\sigma_C/\sigma_I$	= 1·25.
Fundamental length—			
$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{20,320 \text{ volt-amps.} \times 0·085 \text{ mm.}}{0·632 \times 160 \text{ watts}} = 17·1 \text{ mm.}$			
Fundamental surface		$S_0 = L_0^2$	= 292 mm. ²
„ volume		$V_0 = L_0^3$	= 5000 mm. ³
Conventional cooling surface of iron—			
$S_{CI} = (S_{CI}/S_0) S_0$	= 980 × 292	mm. ²	= 0·286 metres ² .
Conventional cooling surface of copper—			
$S_{CC} = (S_{CC}/S_0) S_0$	= 1490 × 292	„	= 0·436 „
Conventional dissipation intensity for iron—			
P_I/S_{CI}	= 160 watts ÷ 0·286 metres ²		= 560 watts per metre ² .
Conventional dissipation intensity for copper—			
P_C/S_{CC}	= 160 watts ÷ 0·436 metres ²		= 367 „ „

Width of iron space—

$$l_{IS} = (l_{IS}/L_0)L_0 = 3.49 \times 17.1 \quad \text{mm.} = 59.6 \text{ mm.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_0)L_0 = 10.40 \times 17.1 \quad \text{,,} = 178 \quad \text{,,}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_0)L_0 = 2.31 \times 17.1 \quad \text{,,} = 39.5 \quad \text{,,}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_0)L_0 = 7.65 \times 17.1 \quad \text{,,} = 131 \quad \text{,,}$$

Length of iron—

$$L_I = (4l_{IS} + 2L_{CS} + 4l_{CS}) = (238 + 356 + 158) \quad \text{mm.} = 752 \text{ mm.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 3l_{CS} + 2L_{IS}) = (119 + 119 + 262) \quad \text{,,} = 500 \quad \text{,,}$$

Section of iron space—

$$S_{IS} = l_{IS}L_{IS} = 59.6 \text{ mm.} \times 131 \quad \text{mm.} = 7,810 \text{ mm.}^2$$

Section of copper space—

$$S_{CS} = 2L_{CS}l_{CS} = 356 \quad \text{,,} \times 39.5 \quad \text{,,} = 14,070 \quad \text{,,}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.80 \quad \times 7810 \quad \text{mm.}^2 = 6,248 \quad \text{,,}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.50 \quad \times 14,070 \quad \text{,,} = 7,035 \quad \text{,,}$$

Volume of iron—

$$V_I = L_I S_I = 752 \text{ mm.} \times 6248 \quad \text{mm.}^2 = 4.70 \times 10^6 \text{ mm.}^3$$

Volume of copper—

$$V_C = L_C S_C = 500 \quad \text{,,} \times 7035 \quad \text{,,} = 3.52 \times 10^6 \quad \text{,,}$$

Mass of iron—

$$M_I = D_I V_I = \frac{7.80 \text{ kgs.}}{10^6 \text{ mm.}^3} \times 4.70 \times 10^6 \text{ mm.}^3 = 36.7 \text{ kgs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{8.90 \text{ kgs.}}{10^6 \text{ mm.}^3} \times 3.52 \times 10^6 \quad \text{,,} = 31.3 \quad \text{,,}$$

Mass of standard of equal cost—

$$M_S = 1.75 M_I + 3 M_C = (64.2 + 93.9) \text{ kgs.} = 158 \quad \text{,,} \\ = 349 \text{ lbs.}$$

Relative cost mass—

$$= M_S \times \{ \sqrt{P_I P_C} / \frac{1}{2} (I_1 V_1 + I_2 V_2) \}^3 \times (f/50 \text{ C.P.S.})^{\frac{1}{2}} \\ = 158 \text{ kgs.} \times \{ 160/10,160 \}^3 = 0.618 \text{ grams} \\ = 1.37 \times 10^{-3} \text{ lbs.}$$

Turns in primary winding—

$$N_1 = \frac{1}{2I_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{3.26 \text{ amps.}} \left\{ \frac{160 \text{ watts} \times 7035 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 500 \text{ mm.}} \right\}^{\frac{1}{2}} = 3030 \text{ turns.} \\ = 2 \times 1515 \quad \text{,,}$$

Turns in secondary winding—

$$N_2 = \frac{1}{2I_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{87.0 \text{ amps.}} \left\{ \frac{160 \text{ watts} \times 7035 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 500 \text{ mm.}} \right\}^{\frac{1}{2}} = 114 \quad \text{,,} \\ = 2 \times 57 \quad \text{,,}$$

Section of wire for primary—

$$S_{W_1} = S_C / 2N_1 = 7035 \text{ mm.}^2 \div 6060 = 1.16 \text{ mm.}^2$$

Section of wire for secondary—

$$S_{W_2} = S_C / 2N_2 = 7035 \text{ ,, } \div 228 = 30.8 \text{ ,,}$$

Flux density (maximum)—

$$\mathcal{B} = \frac{V_1}{4ffN_1S_I} = \frac{6350 \text{ volts}}{4 \cdot 44 \times 50 \text{ per sec.} \times 3030 \times 6248 \text{ mm.}^2} = 1.51 \times 10^{-6} \text{ V.S. per mm.}^2$$

$$= 15,100 \text{ C.G.S.}$$

Current density (R.M.S.)—

$$f = I_1 / S_{W_1} = I_2 / S_{W_2} = 1.63 \text{ amps.} \div 1.16 \text{ mm.}^2 = 1.40 \text{ amps. per mm.}^2$$

Magnetising field (maximum) required for iron (from curve)—

$$\mathcal{H} = 1.20 \text{ amp.-turns. per mm.}$$

Excitation (R.M.S.) required for iron $\chi_I = \frac{1}{\sqrt{2}} \mathcal{H} L_F = 639 \text{ amp.-turns.}$

„ „ „ „ 4 lap joints—

$$\chi_J = 4 \times (20 \text{ amp.-turns per } 10^{-6} \text{ V.S./mm.}^2) \mathcal{B} = 121 \text{ ,,}$$

$$\text{„ (R.M.S.) required for whole } \chi_M = \chi_I + \chi_J = 760 \text{ ,,}$$

Exciting current, idle component $I_{IO} = \chi_M / N_1 = 0.251 \text{ amperes.}$

„ working component $I_{WO} = P_I / V_1 = 0.0252 \text{ ,,}$

„ total—

$$I_O = (I_{IO}^2 + I_{WO}^2)^{1/2} = (0.0630 + 0.0006)^{1/2} \text{ amps.} = 0.252 \text{ ,,}$$

$$= 15.4 \text{ per cent.}$$

Total equivalent resistance, referred to secondary—

$$R_{T_2} = P_C / I_2^2 = 0.0846 \text{ ohms.}$$

„ leakage reactance, referred to secondary—

$$\mathcal{L}_{TL_2} \omega = \frac{2\pi \mu_a f N_2^2 (L_C / 2 L_{CS}) \times l_{CS} / 3}{8 \times 10^{-9} \text{ henry} \times \frac{50}{\text{mm.}} \times \frac{50}{\text{sec.}} \times 114^2 \times \frac{500}{356} \times \frac{39.5 \text{ mm.}}{3}} = 0.0962 \text{ ,,}$$

Total equivalent impedance, referred to secondary—

$$Z_{T_2} = (R_{T_2}^2 + \mathcal{L}_{TL_2}^2 \omega^2)^{1/2} = (7160 + 9280)^{1/2} \times 10^{-3} \text{ ohms} = 0.128 \text{ ,,}$$

Internal lag $\cos \phi = R_{T_2} / Z_{T_2} = 0.66 \quad \phi = 49^\circ.$

Voltage drop at unity power-factor $= R_{T_2} I_2 = 3.68 \text{ volts} = 1.60 \text{ per cent.}$

„ zero (lagging) power-factor $= \mathcal{L}_{TL_2} \omega I_2 = 4.18 \text{ ,, } = 1.82 \text{ ,,}$

„ 0.66 „ „ $= Z_{T_2} I_2 = 5.57 \text{ ,, } = 2.42 \text{ ,,}$

EFFICIENCY AT UNITY POWER-FACTOR.

Load .	2500	5000	7500	10,000	12,500	watts.
Iron loss .	160	160	160	160	160	„
Copper loss .	10	40	90	160	250	„
Input .	2670	5200	7750	10,320	12,910	„
Total loss .	6.37	3.85	3.23	3.10	3.18	per cent.
Efficiency .	93.63	96.15	96.77	96.90	96.82	„

This design has been worked out for the same rating as the one on Plate 7, which also uses alloyed iron, and with which it should be compared. It will be noticed that this design is appreciably cheaper, partly owing to a difference in the qualities of the iron, but largely due to the fact that the one in the plate uses square coils instead of rectangular.

The next design is also for the same rating, but uses Lohys iron instead of Stalloy. It costs about 75 per cent. more than the one with Stalloy, the equivalent standard masses being 349 and 615 lbs., but its magnetising current is much less, owing to the lower flux density at which it is worked. The saving by using Stalloy, however, is more than enough to permit of an increase of size sufficient to bring the exciting current down to the same figure, when there will be the advantage in its favour of considerably reduced losses.

Only the preliminary designs have been worked out for these two; the final designs will not differ greatly from that in Plate 7, except in the actual dimensions.

Preliminary Design for an Oil-Insulated Transformer of the Rectangular Coil Core Type to give 10 K.V.A. at 6350/230 Volts, 1·63/43·5 Amperes, and 50 Cycles per Second, using Lohys Iron.

Loss allowed in iron	(given)	$P_I = 160$ watts = 1·55 per cent.
„ „ copper	„	$P_C = 160$ „ = 1·55 „
„ geom. mean		$\sqrt{P_I P_C} = 160$ „ = 1·55 „
Output of secondary winding	(given)	$I_2 V_2 = 10,000$ volt-amperes.
Input to primary	„ (estimated)	$I_1 V_1 = 10,320$ „
Aggregate volt-ampereage	„	$(I_1 V_1 + I_2 V_2) = 20,320$ „
Terminal P.D. of primary winding	(given)	$V_1 = 6350$ volts.
„ „ secondary	„ „	$V_2 = 230$ „
Current in primary winding	(estimated)	$I_1 = 1·63$ amperes.
„ secondary	(given)	$I_2 = 43·5$ „
Frequency	(given)	$f = 50$ cycles per sec.
Loss-length for 15 mils Lohys	(from curve)	$L_L = 0·110$ mm.
Resistivity of copper (hot, A.C.)	(assumed)	$\rho_C = 23 \times 10^{-6}$ ohm-mm.

Space-factor for iron	(estimated)	$\sigma_I = 0.80$
„ „ copper	„	$\sigma_C = 0.50$
„ geom. mean	„	$\sqrt{\sigma_I \sigma_C} = 0.632$
Specific cost ratio	„	$m = 3.5 \sigma_C / \sigma_I = 2.19$

Fundamental length—

$$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{20,320 \text{ volt-amps.} \times 0.110 \text{ mm.}}{0.632 \times 160 \text{ watts}} = 22.1 \text{ mm.}$$

Fundamental surface

$$S_0 = L_0^2 = 488 \text{ mm.}^2$$

„ volume

$$V_0 = L_0^3 = 10,800 \text{ mm.}^3$$

Conventional cooling surface of iron—

$$S_{CI} = (S_{CI}/S_0) S_0 = 1050 \times 488 \text{ mm.}^2 = 0.512 \text{ metre}^2.$$

Conventional cooling surface of copper—

$$S_{CC} = (S_{CC}/S_0) S_0 = 1460 \times 488 \text{ „} = 0.712 \text{ „}$$

Conventional dissipation intensity for iron—

$$P_I/S_{CI} = 160 \text{ watts} \div 0.512 \text{ metre}^2 = 313 \text{ watts per metre}^2.$$

Conventional dissipation intensity for copper—

$$P_C/S_{CC} = 160 \text{ „} \div 0.712 \text{ „} = 225 \text{ „ „}$$

Width of iron space—

$$l_{IS} = (l_{IS}/L_0) L_0 = 3.80 \times 22.1 \text{ mm.} = 84.0 \text{ mm.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_0) L_0 = 9.80 \times 22.1 \text{ „} = 217 \text{ „}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_0) L_0 = 2.15 \times 22.1 \text{ „} = 47.5 \text{ „}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_0) L_0 = 8.33 \times 22.1 \text{ „} = 184 \text{ „}$$

Length of iron—

$$L_I = (4l_{IS} + 2L_{CS} + 4l_{CS}) = (336 + 434 + 190) \text{ mm.} = 960 \text{ mm.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 3L_{CS} + 2L_{IS}) = (168 + 143 + 368) \text{ „} = 679 \text{ „}$$

Section of iron space—

$$S_{IS} = l_{IS} L_{IS} = 84.0 \text{ mm.} \times 184 \text{ mm.} = 15,450 \text{ mm.}^2$$

Section of copper space—

$$S_{CS} = 2L_{CS} l_{CS} = 434 \text{ „} \times 47.5 \text{ „} = 20,600 \text{ „}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.80 \times 15,450 \text{ mm.}^2 = 12,360 \text{ „}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.50 \times 20,600 \text{ „} = 10,300 \text{ „}$$

Volume of iron—

$$V_I = L_I S_I = 960 \text{ mm.} \times 12,360 \text{ mm.}^2 = 11.87 \times 10^6 \text{ mm.}^3$$

Volume of copper—

$$V_C = L_C S_C = 679 \text{ „} \times 10,300 \text{ „} = 7.00 \times 10^6 \text{ „}$$

Mass of iron—

$$M_I = D_I V_I = \frac{7.80 \text{ kgs.}}{10^6 \text{ mm.}^3} \times 11.87 \times 10^6 \text{ mm.}^3 = 92.5 \text{ kgs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{8.90 \text{ kgs.}}{10^6 \text{ mm.}^3} \times 7.00 \times 10^6 \text{ mm.}^3 = 62.3 \text{ kgs.}$$

Mass of standard of equal cost—

$$M_S = M_I + 3M_C = (92.5 + 186.9) \text{ kgs.} = 280 \text{ kgs.} = 615 \text{ lbs.}$$

Relative cost mass—

$$= M_S \times \left\{ \sqrt{\frac{P_I P_C}{I_1 I_2}} \left(\frac{1}{2} (I_1 V_1 + I_2 V_2) \right) \right\}^3 \times (f/50 \text{ C.P.S.})^2 \\ = 280 \text{ kgs.} \times \{160/10,160\}^3 = 1.095 \text{ grams} \\ = 2.42 \times 10^{-3} \text{ lbs.}$$

Turns in primary winding—

$$N_1 = \frac{1}{2I_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{3.26 \text{ amps.}} \left\{ \frac{160 \text{ watts} \times 10,300 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 679 \text{ mm.}} \right\}^{\frac{1}{2}} = 3156 \text{ turns.} \\ = 2 \times 1578 \text{ ,,}$$

Turns in secondary winding—

$$N_2 = \frac{1}{2I_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{87.0 \text{ amps.}} \left\{ \frac{160 \text{ watts} \times 10,300 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 679 \text{ mm.}} \right\}^{\frac{1}{2}} = 118 \text{ ,,} \\ = 2 \times 59 \text{ ,,}$$

Section of wire for primary—

$$S_{W1} = S_C / 2N_1 = 10,300 \text{ mm.}^2 \div 6312 = 1.63 \text{ mm.}^2$$

Section of wire for secondary—

$$S_{W2} = S_C / 2N_2 = 10,300 \text{ mm.}^2 \div 236 = 43.6 \text{ mm.}^2$$

Flux density (maximum)—

$$\mathcal{B} = \frac{V_1}{4f N_1 S_I} = \frac{6350 \text{ volts}}{4 \times 50 \text{ per sec.} \times 3156 \times 12,360 \text{ mm.}^2} = 0.736 \times 10^{-6} \text{ V.S. per mm.}^2 \\ = 7360 \text{ G.G.S.}$$

Current density (R.M.S.)—

$$I = I_1 / S_{W1} = I_2 / S_{W2} = 1.63 \text{ amps.} \div 1.63 \text{ mm.}^2 = 1.00 \text{ amps. per mm.}^2$$

Magnetising field (maximum) required for iron (from curve)—

$$\mathcal{H} = 0.21 \text{ amp.-turns per mm.}$$

Excitation (R.M.S.) required for iron $\chi_I = \frac{1}{\sqrt{2}} \mathcal{H} L_F = 143 \text{ amp.-turns.}$

4 lap joints—
 ,, $\chi_J = 4 \times (20 \text{ amp.-turns per } 10^{-6} \text{ V.S./mm.}^2) \mathcal{B} = 59 \text{ ,,}$
 ,, (R.M.S.) required, whole $\chi_M = \chi_I + \chi_J = 202 \text{ ,,}$

Exciting current, idle component $I_{IO} = \chi_M / N_1 = 0.0641 \text{ amperes.}$

,, working component $I_{WO} = P_I / V_1 = 0.0252 \text{ ,,}$

total—
 $I_O = (I_{IO}^2 + I_{WO}^2)^{\frac{1}{2}} = (4100 + 634)^{\frac{1}{2}} \times 10^{-3} \text{ amps.} = 0.0689 \text{ ,,}$
 $= 4.23 \text{ per cent.}$

Total equivalent resistance, referred to secondary—

$$R_{T_2} = P_C / I_2^2 = 0.0846 \text{ ohm.}$$

leakage reactance, referred to secondary—

$$\mathcal{L}_{TL_2} \omega = 2\pi \mu_a f N_2^2 (L_C / 2 L_{CS}) \times l_{CS} / 3$$

$$= \frac{8 \times 10^{-9} \text{ henry}}{\text{mm.}} \times \frac{50}{\text{sec.}} \times 118^2 \times \frac{679}{434} \times \frac{47.5 \text{ mm.}}{3} = 0.138 \quad ,$$

Total equivalent impedance, referred to secondary—

$$Z_{T_2} = (R_{T_2}^2 + \mathcal{L}_{TL_2}^2 \omega^2)^{\frac{1}{2}} = (0.0072 + 0.0191)^{\frac{1}{2}} \text{ ohms} = 0.162 \quad ,$$

$$\text{Internal lag} \quad \cos \phi = R_{T_2} / Z_{T_2} = 0.52 \quad \phi = 59^\circ.$$

$$\text{Voltage drop at unity power-factor} \quad = R_{T_2} I_2 = 3.68 \text{ volts} = 1.60 \text{ per cent.}$$

$$, , \text{ zero (lagging) power-factor} \quad = \mathcal{L}_{TL_2} \omega I_2 = 6.00 \quad , = 2.61 \quad ,$$

$$, , 0.52 \quad , , \quad = Z_{T_2} I_2 = 7.05 \quad , = 3.07 \quad ,$$

EFFICIENCY AT UNITY POWER-FACTOR.

Load	2500	5000	7500	10,000	12,500	watts.
Iron loss	160	160	160	160	160	„
Copper loss	10	40	90	160	250	„
Input	2670	5200	7750	10,320	12,910	„
Total loss	6.37	3.85	3.23	3.10	3.18	per cent.
Efficiency	93.63	96.15	96.77	96.90	96.82	„

Preliminary Design for an Oil-Insulated Transformer of the Rectangular Coil Core Type to give 100 K.V.A. at 6000/2200 Volts, 17/45.5 Amperes, and 50 Cycles per Second, using Stalloy Iron.

Loss allowed in iron	(given)	P_I	= 900 watts = 0.885 per cent.
„ „ copper	„	P_C	= 900 „ = 0.885 „
„ geom. mean	„	$\sqrt{P_I P_C}$	= 900 „ = 0.885 „

Output of secondary winding	(given)	$I_2 V_2$	= 100 kilo-volt-amperes.
Input to primary winding	(estimated)	$I_1 V_1$	= 101.8 „
Aggregate volt-ampereage	„	$(I_1 V_1 + I_2 V_2)$	= 201.8 „

Terminal P.D. of primary winding	(given)	V_1	= 6000 volts.
„ secondary „ „	„	V_2	= 2200 „

Current in primary winding	(estimated)	I_1	= 17.0 amperes.
„ secondary „	(given)	I_2	= 45.5 „
Frequency	(given)	f	= 50 cycles per second.
Loss-length for 20 mils Stalloy (from curve)		L_L	= 3.20 mils.
Resistivity of copper (hot, A.C.) (assumed)		ρ_C	= 0.90×10^{-6} ohm-inches.

Space-factor for iron	(estimated)	σ_I	= 0.80
„ „ copper	„	σ_C	= 0.34
„ geom. mean	„	$\sqrt{\sigma_I \sigma_C}$	= 0.521
Specific cost ratio	„	$m = 2\sigma_C/\sigma_I$	= 0.85

Fundamental length—

$$L_O = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \rho_C} \sqrt{P_I P_C}} = \frac{201.8 \text{ K.V.A.} \times 3.20 \text{ mils}}{0.521 \times 900 \text{ watts}} = 1.38 \text{ inches.}$$

Fundamental surface

$$S_O = L_O^2 = 1.90 \text{ inches}^2.$$

„ volume

$$V_O = L_O^3 = 2.62 \text{ inches}^3.$$

Conventional cooling surface of iron—

$$S_{CI} = (S_{CI}/S_O) S_O = 943 \times 1.90 \text{ inches}^2 = 1790 \text{ inches}^2.$$

Conventional cooling surface of copper—

$$S_{CC} = (S_{CC}/S_O) S_O = 1520 \times 1.90 \text{ „} = 2890 \text{ „}$$

Conventional dissipation intensity for iron—

$$P_I/S_{CI} = 900 \text{ watts} \div 1790 \text{ „} = 0.502 \text{ watts per inch}^2.$$

Conventional dissipation intensity for copper—

$$P_C/S_{CC} = 900 \text{ watts} \div 2890 \text{ „} = 0.312 \text{ „}$$

Width of iron space—

$$l_{IS} = (l_{IS}/L_O) L_O = 3.28 \times 1.38 \text{ inches} = 4.52 \text{ inches.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_O) L_O = 10.92 \times 1.38 \text{ „} = 15.08 \text{ „}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_O) L_O = 2.45 \times 1.38 \text{ „} = 3.38 \text{ „}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_O) L_O = 7.28 \times 1.38 \text{ „} = 10.05 \text{ „}$$

Length of iron—

$$L_I = (4l_{IS} + 2L_{CS} + 4l_{CS}) = (18.08 + 30.16 + 13.52) \text{ inches} = 61.8 \text{ inches.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 3l_{CS} + 2L_{IS}) = (9.04 + 10.14 + 20.10) \text{ „} = 39.3 \text{ „}$$

Section of iron space—

$$S_{IS} = l_{IS} L_{IS} = 4.52 \text{ ins.} \times 10.05 \text{ inches} = 45.5 \text{ inches}^2.$$

Section of copper space—

$$S_{CS} = 2L_{CS} l_{CS} = 30.16 \text{ „} \times 3.38 \text{ „} = 101.8 \text{ „}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.80 \times 45.5 \text{ inches}^2 = 36.4 \text{ „}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.34 \times 101.8 \text{ „} = 34.6 \text{ „}$$

Volume of iron—

$$V_I = L_I S_I = 61.8 \text{ ins.} \times 36.4 \text{ inches}^2 = 2250 \text{ inches}^3.$$

Volume of copper—

$$V_C = L_C S_C = 39.3 \text{ „} \times 34.6 \text{ „} = 1360 \text{ „}$$

Mass of iron—

$$M_I = D_I V_I = \frac{0.282 \text{ lb.}}{\text{inch}^3} \times 2250 \text{ inches}^3 = 634 \text{ lbs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{0.322 \text{ lb.}}{\text{inch}^3} \times 1360 \text{ „} = 438 \text{ „}$$

Mass of standard of equal cost—

$$M_S = 1.75 M_I + 3 M_C = (1108 + 1314) \text{ lbs.} = 2422 \text{ „} = 1100 \text{ kgs.}$$

Turns in primary winding—

$$N_1 = \frac{1}{2l_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{34.0 \text{ amps.}} \left\{ \frac{900 \text{ watts} \times 34.6 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 39.3 \text{ ins.}} \right\}^{\frac{1}{2}} = 874 \text{ turns.}$$

$$= 2 \times 437 \text{ „}$$

Turns in secondary winding—

$$N_2 = \frac{1}{2l_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{91.0 \text{ amps.}} \left\{ \frac{900 \text{ watts} \times 34.6 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 39.3 \text{ ins.}} \right\}^{\frac{1}{2}} = 326 \text{ „}$$

$$= 2 \times 163 \text{ „}$$

Section of wire for primary—

$$S_{W1} = S_C / 2N_1 = 34.6 \text{ ins.}^2 \div 1748 = 19.8 \times 10^{-3} \text{ inches}^2.$$

Section of wire for secondary—

$$S_{W2} = S_C / 2N_2 = 34.6 \text{ ins.}^2 \div 652 = 53.1 \times 10^{-3} \text{ „}$$

Flux density (maximum)—

$$\mathcal{B} = \frac{V_1}{4.44 f N_1 S_I} = \frac{6000 \text{ volts}}{4.44 \times 50 \text{ per sec.} \times 874 \times 36.4 \text{ ins.}^2} = 0.850 \text{ M.V.S. per inch}^2.$$

$$= 13,180 \text{ C.G.S.}$$

Current density (R.M.S.)—

$$I = I_1 / S_{W1} = I_2 / S_{W2} = 17.0 \text{ amps.} \div 19.8 \times 10^{-3} \text{ ins.}^2 = 859 \text{ amps. per inch}^2.$$

Design for an Oil-Insulated Transformer of the Rectangular Coil Core Type, to give 100 K.V.A. at 6000/2200 Volts, 17/45.5 Amperes, and 50 Cycles per Second, using Stalloy Iron. (Plate 10.)

		Primary.	Secondary.	Total.	Units.
Rated voltage	V	6000	2200	...	volts.
„ current	1	17.0	45.5	...	amperes.
Number of turns	N	864	320	...	turns.
„ coils		8	2	...	
„ turns per coil		108	160	...	
„ layers per coil		9	5	...	
„ turns per layer		12	32	...	
Conductor, dimensions, bare		200 × 100	360 × 150	...	mils.
„ „ covered		230 × 130	390 × 180	...	„
„ section	S_W	20.0	54.0	...	10^{-3} ins. ²
„ total section of copper	$S_C = N S_W$	17.3	17.3	34.6	inches ² .
„ current density	$I = I/S_W$	850	843	847	amps. per in. ²
Depth of winding	l_{CS}	1.17	0.90	2.07	inches.
Axial length of coil, allowing one extra space for spiral		2.99	12.9	...	„
Perimeter of coil section after taping	L_{PC}	8.52	27.8	...	„
Former, dimensions of section		$8\frac{1}{4} \times 13\frac{3}{4}$	$5\frac{1}{4} \times 10\frac{3}{4}$...	inches.
„ perimeter of section		44.0	32.0	...	„
Allowance for corners		3.5	2.7	...	„
Length of mean turn	L_C	47.5	34.7	41.1	„
„ wire	$L_W = N L_C$	41,000	11,100	...	„
Cooling surface of coils	S_{CC}	3240	1930	5170	inches ² .
Volume of copper	$V_C = L_C S_C$	820	600	1420	inches ³ .
Mass of copper	$M_C = (0.322 \text{ lbs. per in.}^3) V_C$	264	193	457	lbs.
Effective resistance, hot— $R = 0.90 \times 10^{-6} \text{ ohm-ins.} \times L_W/S_W$		1.84	0.185	0.443	ohms.
Copper loss	$P_C = R I^2$	534	383	917	watts.
Dissipation intensity for coils	P_C/S_{CC}	0.165	0.198	0.177	watts per in. ²
Estimated temperature rise— $t_{RC} = (200^\circ \text{ C. per watt/in.}^2) \times P_C/S_{CC}$		33	40	36	° C.

Width of core	$l_{IS} = 4\frac{1}{2}$ inches.
Distance between yokes	$L_{CS} = 15$ „
„ „ cores	$2l_{CS} = 6\frac{3}{4}$ „
<hr/>	
Length of iron—	
$L_I = (4l_{IS} + 2L_{CS} + 2 \times 2l_{CS}) = (18 + 30 + 13.5)$ ins. = 61.5 inches.	
Length of flux—	
$L_F = (3l_{IS} + 2L_{CS} + 2 \times 2l_{CS}) = (13.5 + 30 + 13.5)$ „ = 57.0 „	
<hr/>	
Thickness of sheets	20 mils.
Number of sheets in each core	$4 \times 100 = 400$.
Iron depth, net	8.00 inches.
„ „ gross—	
4 packets $2\frac{1}{4}$ ins. thick + 3 ducts $\times \frac{1}{8}$ in. each	$L_{IS} = 10.00$ „
Iron section, net	$S_I = 36.0$ inches ² .
<hr/>	
Cooling surface of iron—	
$S_{CI} = 8(4\frac{1}{2} + 2\frac{1}{4})$ ins. $\times 61.5$ ins. = 3320 inches.	
Volume of iron	$V_I = L_I S_I = 2214$ inches ³ .
Mass of iron	$M_I = (0.282 \text{ lb. per in.}^3) V_I = 625$ lbs.
„ standard of equal cost—	
$M_S = 1.75 M_I + 3 M_C = (1092 + 1371)$ lbs. = 2463 „ = 1120 kgs.	
Relative cost mass—	
$= M_S \times \{ \sqrt{P_I P_C} / \frac{1}{2} (I_1 V_1 + I_2 V_2) \}^3 \times (f/50 \text{ C.P.S.})^2$	
$= 2463 \text{ lbs.} \times \{ \sqrt{875 \times 917 / 100,900} \}^3$	$= 1.73 \times 10^{-3}$ lbs.
	$= 0.785$ grams.
<hr/>	
Flux density (maximum)—	
$\mathcal{B} = \frac{V_1}{4 f N_1 S_I} = \frac{6000 \text{ volts}}{4 \cdot 44 \times 50 \text{ per sec.} \times 864 \times 36.0 \text{ ins.}^2} = 0.870 \text{ M.V.S. per inch}^2$	
	$= 13,500 \text{ C.G.S.}$
Iron loss per unit volume (from curves for Stalloy)—	
$P_I / V_I = 0.395$ watts per inch ³ .	
„ total	$P_I = (P_I / V_I) V_I = 875$ watts.
Dissipation intensity for iron	$P_I / S_{CI} = 0.263$ watts per inch ² .
Estimated temperature rise for iron—	
$t_{RI} = (200^\circ \text{ C. per watt/in.}^2) \times P_I / S_{CI} = 52^\circ \text{ C.}$	
<hr/>	
Magnetising field (maximum) required for iron (from curve)—	
	$\mathcal{H} = 14$ amp.-turns per inch.
Excitation (R.M.S.) required for iron	$\chi_I = \frac{1}{\sqrt{2}} \mathcal{H} L_F = 565$ amp.-turns.
„ „ „ 4 lap joints—	
$\chi_J = 4 \times (30 \text{ amp.-turns per M.V.S./in.}^2) \mathcal{B} = 105$ „	
„ required for whole	$\chi_M = \chi_I + \chi_J = 670$ „

Exciting current, idle component	$I_{I0} = X_M/N_1 = 0.775$ amperes.
„ „ working component	$I_{W0} = P_I/V_1 = 0.146$ „
„ „ total—	
	$I_0 = (I_{I0}^2 + I_{W0}^2)^{1/2} = (0.60 + 0.02)^{1/2}$ amps. = 0.787 „
	= 4.64 per cent.

Equivalent leakage thickness, between coils	0.60 inches.
„ „ $\frac{1}{3}$ of winding depths	0.69 „
„ „ total	$l_{TL} = 1.29$ „

Total equivalent resistance, referred to secondary—	
	$R_{T2} = P_C/I_2^2 = 0.444$ ohms.
„ leakage reactance, referred to secondary—	
$\mathcal{L}_{TL2}\omega = 2\pi\mu_a f N_2^2 (L_C/2L_{CS}) \times l_{TL}$	
$= \frac{0.20 \times 10^{-6} \text{ henry}}{\text{inch}} \frac{50}{\text{sec.}} \times 320^2 \times \frac{41.1}{30.0} \times 1.29 \text{ ins.} = 1.81$	„

Total equivalent impedance, referred to secondary—	
$Z_{T2} = (R_{T2}^2 + \mathcal{L}_{TL2}^2\omega^2)^{1/2} = (0.197 + 3.29)^{1/2}$ ohms = 1.87 „	
Internal lag $\cos \phi = R_{T2}/Z_{T2} = 0.24$	$\phi = 76^\circ$.

Voltage drop at unity power-factor	$= R_{T2}I_2 = 20.2$ volts = 0.92 per cent.
„ zero (lagging) power-factor	$= \mathcal{L}_{TL2}\omega I_2 = 82.4$ „ = 3.74 „
„ 0.24 „ „	$= Z_{T2}I_2 = 85.1$ „ = 3.87 „

EFFICIENCY AT UNITY POWER-FACTOR.

Load	25,000	50,000	75,000	100,000	125,000	watts.
Iron loss	875	875	875	875	875	„
Copper loss	57	229	514	917	1,430	„
Input	25,932	51,104	76,389	101,792	127,305	„
Total loss	3.60	2.16	1.82	1.76	1.81	per cent.
Efficiency	96.40	97.84	98.18	98.24	98.19	„

Preliminary Design for an Oil-Insulated Transformer of the Hexagonal Ring Type with Rectangular Coils, to give 100 K.V.A. at 2000/200 Volts, 50.7/500 Amperes, and 50 Cycles per Second, using Stalloy Iron.

Loss allowed in iron (given)	$P_I = 500$ watts = 0.494 per cent.
„ „ copper „	$P_C = 800$ „ 0.790 „
„ geom. mean „	$\sqrt{P_I P_C} = 633$ „ 0.625 „

Output of secondary winding (given)	$I_2 V_2 = 100$	kilo-volt-amperes.
Input to primary winding (estimated)	$I_1 V_1 = 101.3$	„
Aggregate volt-amperage „	$(I_1 V_1 + I_2 V_2) = 201.3$	„

Terminal P.D. of primary winding (given)	$V_1 = 2000$	volts.
„ secondary „ „	$V_2 = 200$	„

Current in primary winding (estimated)	$I_1 = 50.7$	amperes.
„ secondary „ (given)	$I_2 = 500$	„

Frequency (given)	$f = 50$	cycles per second.
Loss-length for 20 mils Stalloy (from curve)	$L_L = 3.70$	mils.
Resistivity of copper (hot, A.C.) (assumed)	$\rho_C = 0.90 \times 10^{-6}$	ohm-inches.

Space-factor for iron (estimated)	$\sigma_I = 0.80$.
„ „ copper „	$\sigma_C = 0.50$.
„ geom. mean „	$\sqrt{\sigma_I \sigma_C} = 0.633$.
Specific cost ratio „	$m = 2\sigma_C/\sigma_I = 1.25$.

Fundamental length—

$$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{201.3 \text{ K.V.A.} \times 3.70 \text{ mils}}{0.633 \times 633 \text{ watts}} = 1.86 \text{ inches.}$$

Fundamental surface	$S_0 = L_0^2 = 3.46$	inches ² .
„ volume	$V_0 = L_0^3 = 6.45$	inches ³ .

Conventional cooling surface of iron—

$$S_{CI} = (S_{CI}/S_0) S_0 = 761 \times 3.46 \quad \text{inches}^2 = 2630 \text{ inches}^2$$

Conventional cooling surface of copper—

$$S_{CC} = (S_{CC}/S_0) S_0 = 2008 \times 3.46 \quad \text{„} = 6960 \quad \text{„}$$

Conventional dissipation intensity for iron—

$$P_I/S_{CI} = 500 \text{ watts} \div 2630 \quad \text{„} = 0.190 \text{ watts per inch}^2.$$

Conventional dissipation intensity for copper—

$$P_C/S_{CC} = 800 \text{ watts} \div 6960 \quad \text{„} = 0.115 \quad \text{„}$$

Width of iron space—

$$l_{IS} = (l_{IS}/L_0) L_0 = 3.34 \times 1.86 \quad \text{inches} = 6.21 \text{ inches.}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_0) L_0 = 3.39 \times 1.86 \quad \text{„} = 6.30 \quad \text{„}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_0) L_0 = 7.51 \times 1.86 \quad \text{„} = 14.0 \quad \text{„}$$

Length of iron—

$$L_I = 2\sqrt{3}(l_{IS} + 2l_{CS}) = \sqrt{12}(6.21 + 12.60) \text{ inches} = 65.2 \text{ inches.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 2l_{CS} + 2L_{IS}) = (12.42 + 12.60 + 28.0) \text{ ,,} = 53.0 \text{ ,,}$$

Section of iron space—

$$S_{IS} = l_{IS}L_{IS} = 6.21 \text{ inches} \times 14.0 \text{ inches} = 87.0 \text{ inches}^2.$$

Section of copper space—

$$S_{CS} = 2\sqrt{3}l_{CS}^2 = \sqrt{12} \times 6.30^2 \text{ inches}^2 = 137.5 \text{ ,,}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.80 \times 87.0 \text{ ,,} = 69.6 \text{ ,,}$$

Section of copper—

$$S_C = \sigma_C S_{IS} = 0.50 \times 137.5 \text{ ,,} = 68.8 \text{ ,,}$$

Volume of iron—

$$V_I = L_I S_I = 65.2 \text{ inches} \times 69.6 \text{ inches}^2 = 4530 \text{ inches}^3.$$

Volume of copper—

$$V_C = L_C S_C = 53.0 \text{ ,,} \times 68.8 \text{ ,,} = 3640 \text{ ,,}$$

Mass of iron—

$$M_I = D_I V_I = \frac{0.282 \text{ lb.}}{\text{inch}^3} \times 4530 \text{ inches}^3 = 1278 \text{ lbs.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{0.322 \text{ lb.}}{\text{inch}^3} \times 3640 \text{ ,,} = 1172 \text{ ,,}$$

Mass of standard of equal cost—

$$M_S = 1.75 M_I + 3 M_C = (2240 + 3520) \text{ ,,} = 5760 \text{ ,,}$$

Turns in primary winding—

$$N_1 = \frac{1}{2l_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{101.4 \text{ amps.}} \left\{ \frac{800 \text{ watts} \times 68.8 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 53.0 \text{ ins.}} \right\}^{\frac{1}{2}} = 335 \text{ turns.}$$

$$= 6 \times 56 \text{ ,,}$$

Turns in secondary winding—

$$N_2 = \frac{1}{2l_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{1000 \text{ amps.}} \left\{ \frac{800 \text{ watts} \times 68.8 \text{ ins.}^2}{0.90 \times 10^{-6} \text{ ohm-ins.} \times 53.0 \text{ ins.}} \right\}^{\frac{1}{2}} = 34.0 \text{ ,,}$$

$$= 6 \times 5.67 \text{ ,,}$$

Section of wire for primary—

$$S_{W1} = S_C / 2N_1 = 68.8 \text{ inches}^2 \div 670 = 0.1028 \text{ inches}^2.$$

Section of wire for secondary—

$$S_{W2} = S_C / 2N_2 = 68.8 \text{ ,,} \div 68.0 = 1.011 \text{ ,,}$$

Flux density (maximum)—

$$\mathcal{B} = \frac{V}{4ffN_1 S_I} = \frac{2000 \text{ volts}}{4 \times 50 \text{ per sec.} \times 335 \times 69.6 \text{ ins.}^2} = 0.386 \text{ M.V.S. per inch}^2.$$

$$= 5980 \text{ C.G.S.}$$

Current density (R.M.S.)—

$$I = I_1 / S_{W1} = I_2 / S_{W2} = 494 \text{ amps. per inch}^2.$$

**Design for an Oil-Insulated Transformer of the Hexagonal Ring Type
with Rectangular Coils to give 100 K.V.A. at 2000/200 Volts,
50·7/500 Amperes, and 50 Cycles per Second, using Stalloy Iron.**

		Primary.	Secondary.	Total.	Units.
Rated voltage	V	2000	200	...	volts.
„ current	I	50·7	500	...	amperes.
Number of turns	N	324	33	...	turns.
„ coils		6	6	...	
„ turns per coil		54	5·5	...	
„ layers per coil		4	3	...	
„ turns per layer		15-12	3, 2, & 1 or 0	...	
Conductor, dimensions, bare		0·34 × 0·29	1·20 × 0·85	...	inches.
„ „ covered		0·40 × 0·35	1·25 × 0·90	...	„
„ section	S_W	0·0985	1·020	...	inches ² .
„ total section of copper— $S_C = NS_W$		31·9	33·7	65·6	„
„ current density— $I = I/S_W$		{ 515	490	502 }	amps. per in. ²
Depth of winding	l_{CS}	1·40	2·70	4·10	inches.
Axial length of coil		{ 4·80 6·00	1·25 3·75	„ „
Perimeter of coil section, after taping	L_{PC}	14·4	12·9	...	„
Former, dimensions of section		14·3 × 6·35	18·0 × 10·05	...	inches.
„ perimeter of section		41·3	56·1	...	„
Allowance for corners		4·2	5·4	...	„
Length of mean turn	L_C	45·5	61·5	53·7	„
„ wire	$L_W = NL_C$	14,750	2030	...	„
Cooling surface of coils— $S_{CC} = 6L_{PC}L_C$		3930	4760	8690	inches ² .
Volume of copper	$V_C = L_C S_C$	1450	2070	3520	inches ³ .
Mass of copper— $M_C = (0·322 \text{ lbs. per inch}^3)V_C$		467	666	1133	lbs.
Effective resistance, hot— $R = 0·90 \times 10^{-6} \text{ ohm-ins.} \times L_W/S_W$		0·135	$1·79 \times 10^{-3}$	$3·18 \times 10^{-3}$	ohms.
Copper loss	$P_C = RI^2$	347	447	794	watts.
Dissipation intensity for coils— P_C/S_{CC}		{ 0·088	0·094	0·091 }	watts per in. ²
Estimated temperature rise— $t_{RC} = (200^\circ \text{C. per watt/in.}^2) \times P_C/S_{CC}$		18	19	18	° C.

Width of core	$l_{IS} = 6$ inches.
Radius of inscribed circle	$l_{CS} = 6.34$ „
Side of hexagon, inner	$\frac{2}{\sqrt{3}}l_{CS} = 7.32$ „
„ outer	$\frac{2}{\sqrt{3}}(l_{IS} + l_{CS}) = 14.25$ „
Length of iron and of flux	$L_F = L_I = 2\sqrt{3}(l_{IS} + 2l_{CS}) = 64.7$ „
Thickness of sheets	20 mils.
Number of sheets in each core	560.
Iron depth, net	11.2 inches.
„ „ gross	$L_{IS} = 14.0$ „
„ section, net	$S_I = 67.2$ inches ² .

Cooling surface of iron—

$$S_{CI} = 2L_I(l_{IS} + L_{IS}) = 129.4 \text{ ins. } (6 + 14.0) \text{ ins.} = 2590 \text{ inches}^2.$$

$$\text{Volume of iron} \quad V_I = L_I S_I = 4350 \text{ inches}^3.$$

$$\text{Mass of iron} \quad M_I = (0.282 \text{ lb. per inch}^3) V_I = 1225 \text{ lbs.}$$

„ standard of equal cost—

$$M_S = 1.75 M_I + 3 M_C = (2145 + 3399) \text{ lbs.} = 5544 \text{ „ } = 2515 \text{ kgs.}$$

Relative cost mass—

$$\begin{aligned} &= M_S \times \{ \sqrt{P_I P_C} / \frac{1}{2} (I_1 V_1 + I_2 V_2) \}^3 (f/50 \text{ C.P.S.})^2 \\ &= 5544 \text{ lbs.} \times \{ \sqrt{505 \times 794} / 100,650 \}^3 = 1.38 \times 10^{-3} \text{ lbs.} \\ &= 0.625 \text{ grams.} \end{aligned}$$

Flux density (maximum)—

$$\begin{aligned} \beta &= \frac{V_1}{4ffN_I S_I} = \frac{2000 \text{ volts}}{4.44 \times 50 \text{ per sec.} \times 324 \times 67.2 \text{ ins.}^2} = 0.414 \text{ M.V.S. per inch}^2. \\ &= 6420 \text{ C.G.S.} \end{aligned}$$

Iron loss per unit volume (from curves for Stalloy)—

$$P_I / V_I = 0.116 \text{ watts per inch}^3.$$

„ total

$$P_I = (P_I / V_I) V_I = 505 \text{ watts.}$$

Dissipation intensity for iron

$$P_I / S_{CI} = 0.195 \text{ watts per inch}^2.$$

Estimated temperature rise for iron—

$$t_{RI} = (200^\circ \text{ C. per watt/in.}^2) \times P_I / S_{CI} = 39^\circ \text{ C.}$$

Magnetising field (maximum) required for iron (from curve)—

$$H = 3.8 \text{ amp.-turns per inch.}$$

$$\text{Excitation (R.M.S.) required for iron} \quad \chi_I = \frac{1}{\sqrt{2}} H L_F = 174 \text{ amp.-turns.}$$

„ „ 6 lap joints—

$$\chi_J = 6 \times (30 \text{ amp.-turns per M.V.S./ins.}^2) \beta = 75 \text{ „}$$

$$\text{„ (R.M.S.) required, whole} \quad \chi_M = \chi_I + \chi_M = 249 \text{ „}$$

Exciting current, idle component	$I_{I_0} = \chi_M/N_1 = 0.770$ amperes.
„ working component	$I_{W_0} = P_1/V_1 = 0.253$ „
„ total—	
$I_0 = (I_{I_0}^2 + I_{W_0}^2)^{1/2} = (0.590 + 0.064)^{1/2}$ amps.	$= 0.810$ „
	$= 1.60$ per cent.

Equivalent leakage thickness, between coils—
 $(1.85 - 1.40)$ ins. $= 0.45$ inches.

„ „ „ $\frac{1}{3}$ of winding depths	$= 1.37$ „
„ „ „ total	$l_{TL} = 1.82$ „

Total equivalent resistance, referred to secondary—

$$R_{T_2} = P_C/I_2^2 = 3.18 \times 10^{-3} \text{ ohms.}$$

„ leakage reactance, referred to secondary—

$$\mathcal{L}_{TL_2} \omega = 2\pi \mu_a f N_2^2 (L_C/L_F) l_{TL}$$

$$= \frac{0.20 \times 10^{-6} \text{ henry}}{\text{inch}} \times \frac{50}{\text{sec.}} \times 33^2 \times \frac{53.7}{64.7} \times 1.82 \text{ ins.} = 16.4 \times 10^{-3} \text{ „}$$

Total equivalent impedance, referred to secondary—

$$Z_{T_2} = (R_{T_2}^2 + \mathcal{L}_{TL_2}^2 \omega^2)^{1/2} = (10.1 + 269)^{1/2} \times 10^{-3} \text{ ohms} = 16.7 \times 10^{-3} \text{ „}$$

Internal lag $\cos \phi = R_{T_2}/Z_{T_2} = 0.190$ $\phi = 79^\circ$.

Voltage drop at unity power-factor	$= R_{T_2} I_2 = 1.59$ volts $= 0.80$ per cent.
„ „ zero (lagging) power-factor	$= \mathcal{L}_{TL_2} \omega I_2 = 8.20$ „ $= 4.10$ „
„ „ 0.190 „ „	$= Z_{T_2} I_2 = 8.35$ „ $= 4.18$ „

EFFICIENCY AT UNITY POWER-FACTOR.

Load	25,000	50,000	75,000	100,000	125,000	watts.
Iron loss	505	505	505	505	505	„
Copper loss	50	199	446	794	1,240	„
Input	25,555	50,704	75,951	101,300	126,745	„
Total loss	2.17	1.39	1.25	1.28	1.38	per cent.
Efficiency	97.83	98.61	98.75	98.72	98.62	„

Like the previous ring transformer, this has been designed for an extra low iron loss. With a little more ventilation in the core, its rating could be easily doubled, the voltage and current being increased together. When we compare this design with the 100 K.V.A. core design already given, we see that the ratio of the equivalent standard masses is 2.25 : 1, while the cube of that of the geometrical mean losses is 1 : 2.85. The increased cost is entirely due to the increased efficiency required in this case, and it shows how expensive it is to increase the efficiency by quite a small amount when it is already high; the change in the maximum efficiencies is only from 98.25 to 98.75. To do this without changing type would raise the cost in the ratio 2.85 : 1, so that the hexagon shows a saving of about 26 1/2 per cent., although

a small part of this is due to the larger copper space-factor adopted for it owing to the lower voltage rating. The relative cost mass gives a means of comparing them after eliminating the effect of the difference in efficiency.

Preliminary Design for an Oil-Insulated Three-Phase Transformer of the Three-Limb Type with Circular Coils, to give 75 K.V.A. at 6300/310 Line Volts, 7'05/140 Line Amperes, and 50 Cycles per Second, using Lohys Iron.

Loss allowed in iron	(given)	$P_I = 1000$ watts = 1.30 per cent.
„ „ copper	„	$P_C = 1000$ „ = 1.30 „
„ geom. mean		$\sqrt{P_I P_C} = 1000$ „ = 1.30 „
Output of secondary windings	(given)	$I_2 V_2 = 75$ kilo-volt-amperes.
Input to primary	„ (estimated)	$I_1 V_1 = 77$ „
Aggregate volt-amperage	„	$(I_1 V_1 + I_2 V_2) = 152$ „
Terminal P.D. per phase of primary (mesh connected)—		
		$V_1 = 6300$ volts.
„ „ secondary (star connected)—		$V_2 = 179$ „
Current in primary windings	(estimated)	$I_1 = 4.07$ amperes.
„ secondary „	(given)	$I_2 = 140$ „
Frequency	(given)	$f = 50$ cycles per second.
Loss-length for 15 mils Lohys	(from curve)	$L_L = 0.105$ mm.
Resistivity of copper (hot, A.C.)	(assumed)	$\rho_C = 23 \times 10^{-6}$ ohm-mm.
Space-factor for iron	(estimated)	$\sigma_I = 0.60$.
„ „ copper	„	$\sigma_C = 0.32$.
„ geom. mean	„	$\sqrt{\sigma_I \sigma_C} = 0.438$.
Specific cost ratio	„	$m = 3.5 \sigma_C / \sigma_I = 1.87$.
Fundamental length—		
$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{152 \text{ K.V.A.} \times 0.105 \text{ mm.}}{0.438 \times 1000 \text{ watts}} = 36.5 \text{ mm.}$		
Fundamental surface		$S_0 = L_0^2 = 1,330 \text{ mm.}^2$
„ volume		$V_0 = L_0^3 = 48,400 \text{ mm.}^3$

Conventional cooling surface of iron—

$$S_{CI} = (S_{CI}/S_0)S_0 = 1710 \times 1330 \quad \text{mm.}^2 = 2.28 \text{ metres}^2.$$

Conventional cooling surface of copper—

$$S_{CC} = (S_{CC}/S_0)S_0 = 2455 \times 1330 \quad \text{,,} = 3.26 \quad \text{,,}$$

Conventional dissipation intensity for iron—

$$P_I/S_{CI} = 1000 \text{ watts} \div 2.28 \text{ metres}^2 = 440 \text{ watts per metre}^2.$$

Conventional dissipation intensity for copper—

$$P_C/S_{CC} = 1000 \text{ watts} \div 3.26 \quad \text{,,} = 307 \quad \text{,,} \quad \text{,,}$$

Width of iron space—

$$l_{IS} = (l_{IS}/L_0)L_0 = 5.72 \times 36.5 \quad \text{mm.} = 209 \text{ mm.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_0)L_0 = 14.75 \times 36.5 \quad \text{,,} = 539 \quad \text{,,}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_0)L_0 = 2.02 \times 36.5 \quad \text{,,} = 73.6 \quad \text{,,}$$

Length of iron—

$$L_I = (6l_{IS} + 3L_{CS} + 8l_{CS}) = (1254 + 1617 + 589) \text{ mm.} = 3460 \text{ mm.}$$

Length of mean turn—

$$L_C = \pi(l_{IS} + l_{CS}) = \pi(209 + 74) \quad \text{,,} = 890 \quad \text{,,}$$

Section of iron space—

$$S_{IS} = \frac{\pi}{4} l_{IS}^2 = \frac{\pi}{4} \times 209^2 \quad \text{mm.}^2 = 34,300 \text{ mm.}^2$$

Section of copper space—

$$S_{CS} = 3L_{CS}l_{CS} = 1617 \text{ mm.} \times 73.6 \quad \text{mm.} = 119,000 \quad \text{,,}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.60 \times 34,300 \quad \text{mm.}^2 = 20,600 \quad \text{,,}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.32 \times 119,000 \quad \text{,,} = 38,100 \quad \text{,,}$$

Volume of iron—

$$V_I = L_I S_I = 3460 \text{ mm.} \times 20,600 \quad \text{mm.}^2 = 71.3 \times 10^6 \text{ mm.}^3$$

Volume of copper—

$$V_C = L_C S_C = 890 \quad \text{,,} \times 38,100 \quad \text{,,} = 33.9 \times 10^6 \quad \text{,,}$$

Mass of iron—

$$M_I = D_I V_I = \frac{7.80 \text{ kg.}}{10^{-6} \text{ mm.}^3} \times 71.3 \times 10^6 \text{ mm.}^3 = 556 \text{ kg.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{8.90 \text{ kg.}}{10^6 \text{ mm.}^3} \times 33.9 \times 10^6 \quad \text{,,} = 302 \quad \text{,,}$$

Mass of standard of equal cost—

$$M_S = M_I + 3M_C = (556 + 906) \text{ kg.} = 1462 \quad \text{,,} = 3230 \text{ lbs.}$$

Relative cost mass—

$$\begin{aligned} &= M_S \times \{ \sqrt{P_I P_C} / \frac{1}{2} (I_1 V_1 + I_2 V_2) \}^3 \times (f/50 \text{ C.P.S.})^2 \\ &= 1462 \text{ kg.} \times \{ 1000/76,000 \}^3 = 3.34 \text{ grams.} \\ &= 7.38 \times 10^{-3} \text{ lbs.} \end{aligned}$$

Turns in each phase of primary—

$$N_1 = \frac{1}{6I_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{24.4 \text{ amps.}} \left\{ \frac{1000 \text{ watts} \times 38,100 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 890 \text{ mm.}} \right\}^{\frac{1}{2}} = 1770 \text{ turns.}$$

Turns in each phase of secondary—

$$N_2 = \frac{1}{6I_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{840 \text{ amps.}} \left\{ \frac{1000 \text{ watts} \times 38,100 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 890 \text{ mm.}} \right\}^{\frac{1}{2}} = 51.5 \quad "$$

Section of wire for primary—

$$S_{W1} = S_C / 6N_1 = 38,100 \text{ mm.}^2 \div 10,620 = 3.59 \text{ mm.}^2$$

Section of wire for secondary—

$$S_{W2} = S_C / 6N_2 = 38,100 \quad " \quad \div 309 = 123 \quad "$$

Flux density (maximum)—

$$\mathcal{B} = \frac{V_1}{4.44 f N_1 S_1} = \frac{6300 \text{ volts}}{4.44 \times 50 \text{ per sec.} \times 1770 \times 20,600 \text{ mm.}^2} = 0.779 \times 10^{-6} \text{ V.S. per mm.}^2 = 7790 \text{ C.G.S.}$$

Current density (R.M.S.)—

$$I = I_1 / S_{W1} = I_2 / S_{W2} = 4.07 \text{ amps.} / 3.59 \text{ mm.}^2 = 1.14 \text{ amps. per mm.}^2$$

Magnetising field (maximum) required for iron (from curve)—

$$H = 0.20 \text{ amp.-turns per mm.}$$

Excitation (R.M.S.) per phase required for iron—

$$X_I = \frac{1}{\sqrt{2}} H L_F = H L_I / 3 \sqrt{2} = 163 \text{ amp.-turns.}$$

“ “ “ required for 2 lap joints—

$$X_J = 2 \times (20 \text{ amp.-turns per } 10^{-6} \text{ V.S./mm.}^2) / \mathcal{B} = 31 \quad "$$

“ (R.M.S.) per phase required for whole—

$$X_M = X_I + X_J = 194 \quad "$$

Exciting current per phase, idle component $I_{IO} = X_M / N_1 = 0.110$ amperes.

“ “ working component—

$$I_{WO} = P_I / 3V_1 = 0.0530 \quad "$$

“ “ total—

$$I_O = (I_{IO}^2 + I_{WO}^2)^{\frac{1}{2}} = (0.0121 + 0.0028)^{\frac{1}{2}} \text{ amps.} = 0.122 \quad " = 3.00 \text{ per cent.}$$

Total equivalent resistance per phase, referred to secondary—

$$R_{T2} = P_C / 3I_2^2 = 0.0170 \text{ ohms.}$$

“ “ leakage reactance per phase, referred to secondary—

$$\begin{aligned} \mathcal{L}_{TL2} \omega &= 2\pi \mu_a f N_2^2 (L_C / L_{CS}) \times l_{CS} / 3 \\ &= \frac{8 \times 10^{-9} \text{ henry}}{\text{mm.}} \times \frac{50}{\text{sec.}} \times 51.5^2 \times \frac{890}{539} \times \frac{73.6 \text{ mm.}}{3} = 0.0430 \text{ ohms.} \end{aligned}$$

Total equivalent impedance per phase, referred to secondary—

$$Z_{T2} = (R_{T2}^2 + \mathcal{L}_{TL2}^2 \omega^2)^{\frac{1}{2}} = (289 + 1850)^{\frac{1}{2}} \times 10^{-3} \text{ ohms} = 0.0461 \quad "$$

Internal lag

$$\cos \phi = R_{T2} / Z_{T2} = 0.37 \quad \phi = 68^\circ.$$

Voltage drop per phase at unity power-factor	$\equiv R_{T_2} I_2$	$= 2.38$ volts	$= 1.33$ per cent.
„ „ zero (lagging) „	$\equiv \mathcal{L}_{TL_2} \omega I_2$	$= 6.03$ „	$= 3.37$ „
„ „ 0.37 „ „	$\equiv Z_{T_2} I_2$	$= 6.46$ „	$= 3.61$ „

EFFICIENCY AT UNITY POWER-FACTOR.

Load . . .	18,750	37,500	56,250	75,000	93,750	watts.
Iron loss . . .	1,000	1,000	1,000	1,000	1,000	„
Copper loss . . .	63	250	563	1,000	1,560	„
Input . . .	19,813	38,750	57,813	77,000	96,310	„
Total loss . . .	5.37	3.23	2.71	2.60	2.66	per cent.
Efficiency . . .	94.63	96.77	97.29	97.40	97.34	„

The details of this design can be worked out in a similar manner to those of Plate 12, which is for the same output, but different voltages, and with which it should be compared. Lohys has been adopted for this design, as that in the plate was made before the general use of alloyed iron, in order to give an opportunity of comparing the results of this method of design with the practice of a first-class maker, without having to allow for great differences of material and change of efficiency, etc., as in the other examples given. It will be seen that the two designs do not differ much; the new one is slightly cheaper and has a somewhat greater efficiency. The relative cost mass for this example is higher than three times that of the single-phase designs already worked out, instead of lower, as might be expected. This is accounted for by the inferior quality of Lohys compared with Stalloy, and by the lower copper space-factor.

Preliminary Design for an Oil-Insulated Three-Phase Transformer of the Three-Limb Type with Rectangular Coils, to give 770 K.V.A. at 5000/20,000 Line Volts, 90.2/22.2 Line Amperes, and 50 Cycles per Second, using Stalloy Iron.

Loss allowed in iron	(given)	$P_I = 5000$ watts	$= 0.642$ per cent.
„ „ copper	„	$P_C = 5000$ „	$= 0.642$ „
„ geom. mean		$\sqrt{P_I P_C} = 5000$ „	$= 0.642$ „
Output of secondary windings	(given)	$I_2 V_2 = 770$ kilo-volt-amperes.	
Input to primary windings	(estimated)	$I_1 V_1 = 780$ „	
Aggregate volt-amperage	„	$(I_1 V_1 + I_2 V_2) = 1550$ „	
Terminal P.D. per phase of primary (star connected)—			
		$V_1 = 2,885$ volts.	
„ „ secondary (star connected)—			
		$V_2 = 11,530$ „	

Current in primary windings (estimated)	I_1	= 90.2 amperes.
„ secondary „ (given)	I_2	= 22.2 „
Frequency (given)	f	= 50 cycles per second.
Loss-length for 20 mils Stalloy (from curve)	L_L	= 0.080 mm.
Resistivity of copper (hot, A.C.) (assumed)	ρ_C	= 23×10^{-6} ohm-mm.
Space-factor for iron (estimated)	σ_I	= 0.75.
„ „ copper „	σ_C	= 0.25.
„ geom. mean „	$\sqrt{\sigma_I \sigma_C}$	= 0.433.
Specific cost ratio „	$m = 2\sigma_C/\sigma_I$	= 0.667.

Fundamental length—

$$L_0 = \frac{(I_1 V_1 + I_2 V_2) L_L}{\sqrt{\sigma_I \sigma_C} \sqrt{P_I P_C}} = \frac{1550 \text{ K.V.A.} \times 0.080 \text{ mm.}}{0.433 \times 5000 \text{ watts}} = 57.3 \text{ mm.}$$

Fundamental surface

$$S_0 = L_0^2 = 3270 \text{ mm.}^2$$

„ volume

$$V_0 = L_0^3 = 188,000 \text{ mm.}^3$$

Conventional cooling surface of iron—

$$S_{CI} = (S_{CI}/S_0) S_0 = 1560 \times 3270 \text{ mm.}^2 = 5.10 \text{ metres}^2$$

Conventional cooling surface of copper—

$$S_{CC} = (S_{CC}/S_0) S_0 = 2580 \times 3270 \text{ „} = 8.44 \text{ „}$$

Conventional dissipation intensity for iron—

$$P_I/S_{CI} = 5000 \text{ watts} \div 5.10 \text{ metres}^2 = 980 \text{ watts per metre.}^2$$

Conventional dissipation intensity for copper—

$$P_C/S_{CC} = 5000 \text{ watts} \div 8.44 \text{ „} = 593 \text{ „ „}$$

Width of iron space—

$$l_{IS} = (l_{IS}/L_0) L_0 = 3.34 \times 57.3 \text{ mm.} = 191.3 \text{ mm.}$$

Distance between yokes—

$$L_{CS} = (L_{CS}/L_0) L_0 = 13.2 \times 57.3 \text{ „} = 756 \text{ „}$$

Depth of copper space—

$$l_{CS} = (l_{CS}/L_0) L_0 = 2.40 \times 57.3 \text{ „} = 137.4 \text{ „}$$

Depth of iron space—

$$L_{IS} = (L_{IS}/L_0) L_0 = 6.90 \times 57.3 \text{ „} = 395 \text{ „}$$

Length of iron—

$$L_I = (6l_{IS} + 3L_{CS} + 8l_{CS}) = (1148 + 2268 + 1099) \text{ mm.} = 4515 \text{ mm.}$$

Length of mean turn—

$$L_C = (2l_{IS} + 3l_{CS} + 2L_{IS}) = (383 + 412 + 790) \text{ „} = 1585 \text{ „}$$

Section of iron space—

$$S_{IS} = l_{IS} L_{IS} = 191.3 \text{ mm.} \times 395 \text{ mm.} = 75,500 \text{ mm.}^2$$

Section of copper space—

$$S_{CS} = 3 L_{CS} l_{CS} = 2268 \text{ ,} \times 137.4 \text{ ,} = 312,000 \text{ ,}$$

Section of iron—

$$S_I = \sigma_I S_{IS} = 0.75 \times 75,500 \text{ mm.}^2 = 56,600 \text{ ,}$$

Section of copper—

$$S_C = \sigma_C S_{CS} = 0.25 \times 312,000 \text{ ,} = 78,000 \text{ ,}$$

Volume of iron—

$$V_I = L_I S_I = 4515 \text{ mm.} \times 56,600 \text{ mm.}^2 = 256 \times 10^6 \text{ mm.}^3$$

Volume of copper—

$$V_C = L_C S_C = 1585 \text{ ,} \times 78,000 \text{ ,} = 123.7 \times 10^6 \text{ ,}$$

Mass of iron—

$$M_I = D_I V_I = \frac{7.80 \text{ kg.}}{10^6 \text{ mm.}^3} \times 256 \times 10^6 \text{ mm.}^3 = 2000 \text{ kg.}$$

Mass of copper—

$$M_C = D_C V_C = \frac{8.90 \text{ kg.}}{10^6 \text{ mm.}^3} \times 123.7 \times 10^6 \text{ ,} = 1100 \text{ ,}$$

Mass of standard of equal cost—

$$M_S = 1.75 M_I + 3 M_C = (3500 + 3300) \text{ kg.} = 6800 \text{ ,} = 15,000 \text{ lbs.}$$

Relative cost mass—

$$\begin{aligned} &= M_S \times \left\{ \sqrt{\frac{P_I}{P_C}} \left(\frac{1}{2} (I_1 V_1 + I_2 V_2) \right) \right\}^3 \times (f/50 \text{ C.P.S.})^3 \\ &= 6800 \text{ kg.} \times \{5/775\}^3 = 1.83 \text{ grams} \\ &= 4.03 \times 10^{-3} \text{ lbs.} \end{aligned}$$

Turns in each phase of primary—

$$N_1 = \frac{1}{6 I_1} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{541 \text{ amps.}} \left\{ \frac{5000 \text{ watts} \times 78,000 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 1585 \text{ mm.}} \right\}^{\frac{1}{2}} = 191 \text{ turns.}$$

Turns in each phase of secondary—

$$N_2 = \frac{1}{6 I_2} \left\{ \frac{P_C S_C}{\rho_C L_C} \right\}^{\frac{1}{2}} = \frac{1}{133.2 \text{ amps.}} \left\{ \frac{5000 \text{ watts} \times 78,000 \text{ mm.}^2}{23 \times 10^{-6} \text{ ohm-mm.} \times 1585 \text{ mm.}} \right\}^{\frac{1}{2}} = 775 \text{ ,}$$

Section of wire for primary—

$$S_{W1} = S_C / 6 N_1 = 78,000 \text{ mm.}^2 \div 1146 = 68.1 \text{ mm.}^2$$

Section of wire for secondary—

$$S_{W2} = S_C / 6 N_2 = 78,000 \text{ ,} \div 4650 = 16.8 \text{ ,}$$

Flux density (maximum)—

$$\begin{aligned} \mathcal{B} &= \frac{V_1}{4 f f N_1 S_I} = \frac{2885 \text{ volts}}{4 \times 50 \text{ per sec.} \times 191 \times 56,600 \text{ mm.}^2} = 1.20 \times 10^{-6} \text{ V.S. per mm.}^2 \\ &= 12,000 \text{ C.G.S.} \end{aligned}$$

Current density (R.M.S.)—

$$I = I_1 / S_{W1} = I_2 / S_{W2} = 90.2 \text{ amps.} \div 68.1 \text{ mm.}^2 = 1.32 \text{ amps. per mm.}^2$$

Magnetising field (maximum) required for iron (from curve)—

$$H = 0.42 \text{ amp.-turns per mm.}$$

Excitation (R.M.S.) per phase required for iron—

$$X_I = \frac{1}{\sqrt{2}} H L_F = H L_I / 3\sqrt{2} = 447 \text{ amp.-turns.}$$

” ” ” required for 2 lap joints—
 $X_J = 2 \times (20 \text{ amp.-turns per } 10^{-6} \text{ V.S./mm.}) / 3 = 48$ ”

” (R.M.S.) per phase required for whole—
 $X_M = X_I + X_J = 495$ ”

Exciting current per phase, idle component—

$$I_{IO} = X_M / N_I = 2.59 \text{ amperes.}$$

” ” working component—

$$I_{WO} = P_I / 3 V_1 = 0.578$$
 ”

” ” total—

$$I_O = (I_{IO}^2 + I_{WO}^2)^{1/2} = (6.72 + 0.33)^{1/2} \text{ amps.} = 2.65$$
 ”
 $= 2.93 \text{ per cent.}$

Total equivalent resistance per phase, referred to secondary—

$$R_{T_2} = P_C / 3 I_2^2 = 3.38 \text{ ohms.}$$

” ” leakage reactance, referred to secondary—
 $\mathcal{L}_{TL_2} \omega = 2\pi \mu_a f N_2^2 (L_C / L_{CS}) \times l_{CS} / 3$

$$= \frac{8 \times 10^{-9} \text{ henry}}{\text{mm.}} \times \frac{50}{\text{sec.}} \times 775^2 \times \frac{1585}{756} \times \frac{137.4 \text{ mm.}}{3} = 23.1$$
 ”

Total equivalent impedance, referred to secondary—

$$Z_{T_2} = (R_{T_2}^2 + \mathcal{L}_{TL_2}^2 \omega^2)^{1/2} = (11.4 + 535)^{1/2} \text{ ohms} = 23.4$$
 ”

Internal lag $\cos \phi = R_{T_2} / Z_{T_2} = 0.145$ $\phi = 82^\circ$.

Voltage drop per phase at unity power-factor $= R_{T_2} I_2 = 75.0 \text{ volts} = 0.65 \text{ per cent.}$

” ” zero (lagging) ” $= \mathcal{L}_{TL_2} \omega I_2 = 513$ ” $= 4.45$ ”

” ” 0.145 ” ” $= Z_{T_2} I_2 = 520 \text{ volts} = 4.51$ ”

EFFICIENCY AT UNITY POWER-FACTOR.

Load . . .	193	385	578	770	963	kilowatts.
Iron loss . . .	5	5	5	5	5	”
Copper loss . . .	0.31	1.25	2.82	5	7.81	”
Input . . .	198.3	391.3	585.8	780	976	”
Total loss . . .	2.68	1.60	1.33	1.28	1.31	per cent.
Efficiency . . .	97.32	98.40	98.67	98.72	98.69	”

This design has been worked out for the same output as that of Plate 17, but using Stalloy instead of ordinary iron. By employing this material the iron losses have been reduced to half, while the cost of the active material is practically the same, and there will be an appreciable saving in the cost of the case and cooling apparatus. The relative cost mass has

been reduced to less than one-third. The complete design is not given, as it can be worked out similarly to that in the plate.

Remarks re Transformers illustrated in Plates.—Four of the designs, and a number of actual transformers of which drawings have been supplied by the makers, are fully illustrated in the plates at the end of the book. The plates are preceded by two tables, one for single-phase and the other for three-phase transformers, giving full data and values worked out for the various constants on the lines of the method here explained. In most cases, the leading particulars have also been supplied by the makers, but in others the authors have inserted suitable values to fit the drawings.

The mass of standard iron gives a means of approximately comparing the actual costs of the different transformers, while the relative cost mass enables a comparison to be made after roughly eliminating the effects of differences in efficiency and frequency. It will be seen that in all cases the new designs are cheaper than the others; but it must be remembered that the former employ Stalloy, whereas most of the others were made before alloyed iron had gained its present position. Also, it should not be forgotten that the relative cost mass is lower for a low-voltage transformer than for a high-voltage design of equal merit, owing to the larger copper space-factor which is permissible in the former, and that an extra low value for this mass may be due to cutting down the insulation space below a safe limit. The relative cost mass is especially low for the choking coil of Plate 3, because in that design no insulating space is required to separate primary and secondary coils, while the voltage is also low. It is most instructive to examine the tables, and study the causes for the differences which they show.

The symmetrical type of three-phase transformer comes out very expensive. As usually made, this type is severely handicapped by the fact that the flux in each half of the cores is practically identical with that in the corresponding yoke, owing to the way in which the halves are separated from one another. Since these fluxes are not in phase, the section of the core has to be $2/\sqrt{3}$ times what would otherwise be necessary. If provision were made for an exchange of flux between the two halves, this disadvantage would be overcome, but the losses at the corners would be increased, and difficulties of construction arise.

A bibliography on the subject of Transformer Design is given at the end of a paper on the subject by Low, *Journ. I.E.E.*, vol. 43, p. 232, 7th July 1909. Special reference may be made to a paper by Pohl and Bohle in the *Elektrotechnische Zeitschrift*, vol. 26, p. 897, 28th September 1905, and the *Electrical Engineer*, vol. 36, pp. 446 and 519, 29th September 1905.

CHAPTER XI.

APPLICATIONS OF TRANSFORMERS.

Uses of Transformers.—Transformers are employed for raising the voltage for transmission when the distance and power require a greater P.D. than 11,000 volts, which is about the highest that can be safely generated in an alternator, and for reducing it again for distribution, or to a value suitable for the apparatus in which the power is to be utilised.

The Board of Trade and Home Office Rules classify systems in which the R.M.S. P.D. between any two terminals, or between one and earth cannot exceed 250 volts as low voltage, from 250 to 650 volts as medium voltage, from 650 to 3000 volts as high voltage, and above that as extra high voltage. The low-voltage limit is practically that of incandescent lamp manufacture, and the medium-voltage is that for ordinary motor work. Except in special cases of large users, high voltage is only allowed on consumers' premises in lock-up chambers out of the consumer's control, and extra high voltage only in premises entirely under the control of the undertakers or authorised distributors. For many purposes, a still lower E.M.F. than that of the ordinary lighting supply is required ; for instance, alternating-current arc lamps require about 35 volts, and metallic-filament lamps of small candle-power cannot at present be obtained for more than 130 volts. It is therefore not uncommon to find transformers employed to reduce the voltage to a very low value, or to divide the supply P.D. into smaller parts.

Transformers are also employed for dividing the load between the two sides of a three-wire or other multiple-wire system ; for boosting up the E.M.F. in long feeders ; for giving a variable voltage ; and for regulating purposes in general. They may also be employed to connect two circuits which require to be insulated from one another ; for example, their use on high-voltage systems allows the indicating instruments to be kept at safe potentials.

Station Transformers.—Sub-Stations.—When step-up transformers

are employed, they are generally placed in separate chambers as close to the switchboard as convenient. Very often each, or each group, is placed in a separate brick cubicle closed by an iron door, in order to confine any fire

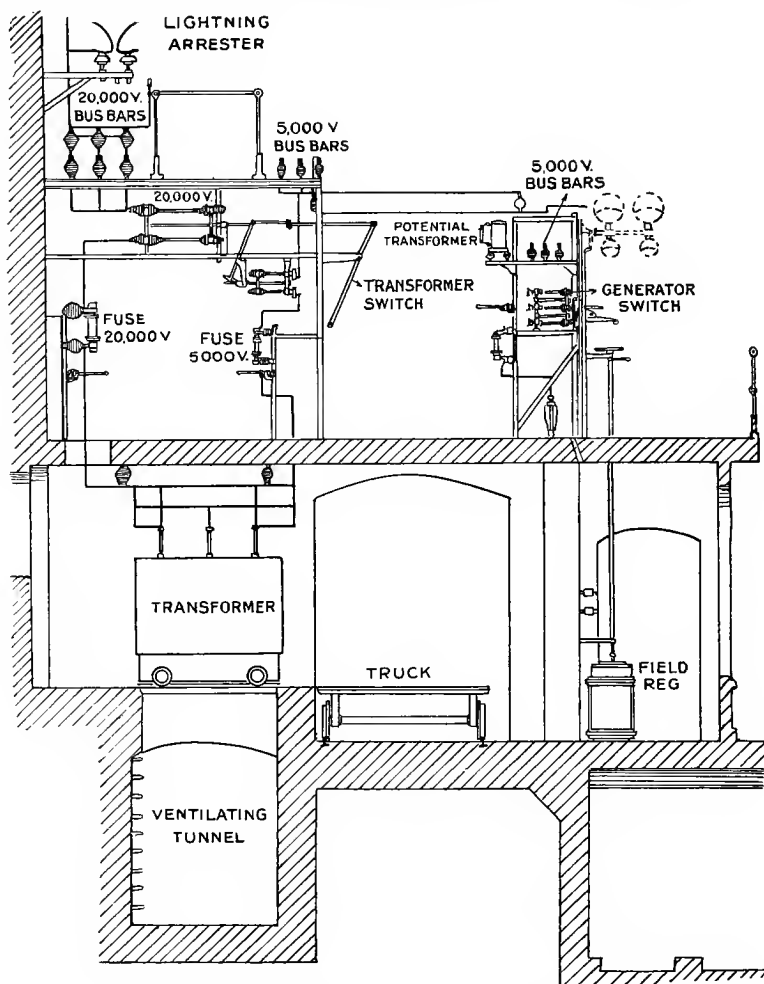


FIG. 11'01.—Transformer Chamber and Switchboard for Drammen Transmission (Oerlikon). 770 K. V. A. each; 5000 to 20,000 volts.

due to a breakdown to the faulty transformer. Fig. 11'01 shows a sectional view of the switchboard and transformer chamber of the generating station for supplying the town of Drammen in Scandinavia.* The equipment by the

* The Authors are indebted to the Oerlikon Company for particulars of this scheme. See also the *Elektrotechniker Zeitschrift*, vols. xxiii. and xxiv. (1905).

Oerlikon Company, comprises three transformers of 770 K.V.A. each, transforming from 5000 to 20,000 volts for transmission. One of the transformers

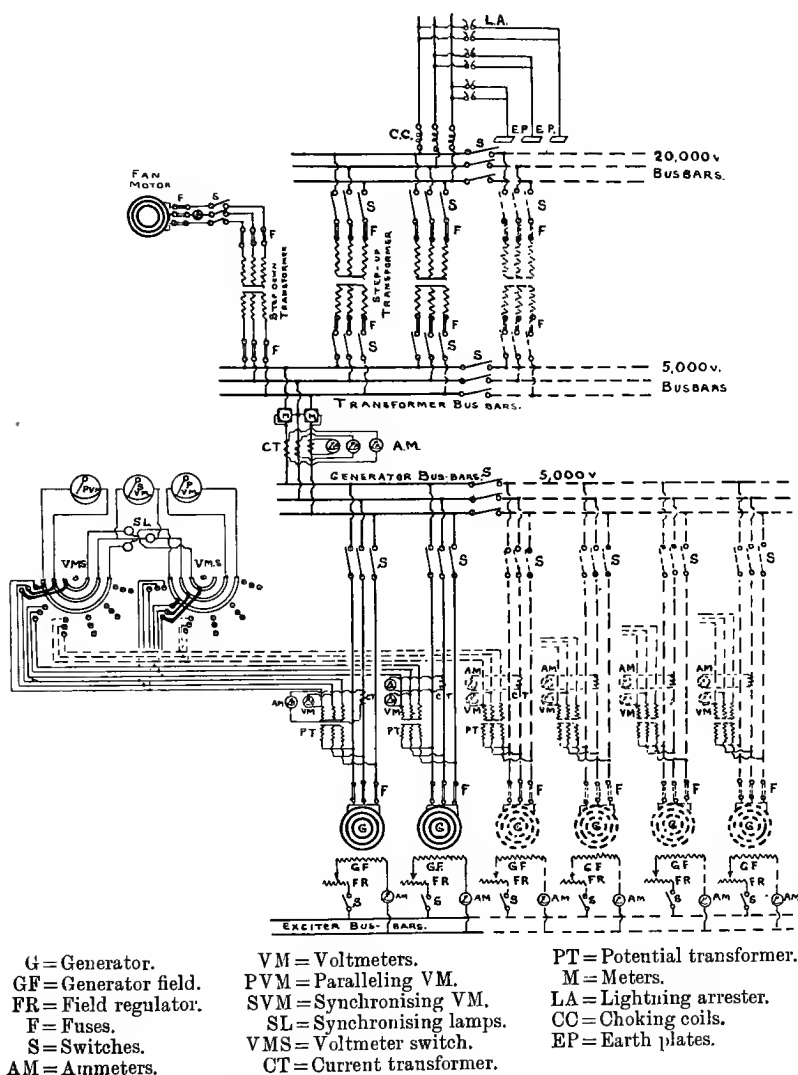


FIG. 11'02. —Diagram of Connections of Generating Station for supplying Drammen, Scandinavia. 5000 and 20,000 volts.

is shown in fig. 9'30 and Plate 17. Cooling air is supplied from underneath by one of the fans which are placed at each end of the tunnel. Only one fan is run at a time, the other acting as a spare.

The transformers rest on a foundation of concrete, and are spaced so as

to make inspection easy. If one should get damaged, it can easily be run on its own wheels to a truck and taken away for repair. The switching arrangement consists of three separate gears, two for 5000 and one for 20,000 volts, and is placed in a room directly above the transformers. The switches are of the air-brake type, manipulated by rod and link, while the fuses are placed in a tube partly filled with oil. The fuse wire is above the

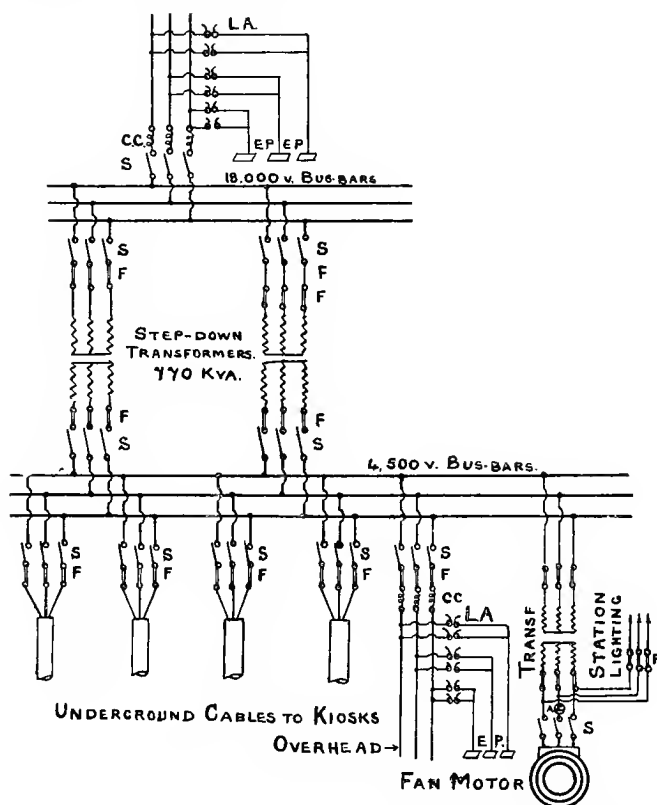


FIG. 11'03.—Diagram of Connections of Receiving Sub-Station at Drammen.

surface of the oil, but is drawn underneath it by a spring when it melts. The 20,000-volt transmission lines are connected to the bus bars, and are guarded inside and outside against abnormal potential rises by choking coils and lightning arresters.

A complete diagram of the connections is given in fig. 11'02, which also shows how the transformers for the measuring instruments and synchronisers are connected.

At the other end of the transmission line there is a sub-station where

the P.D. is transformed down from 18,000 volts (2000 volts having been lost in transmission) to 4500 volts, at which it is distributed to other sub-stations, in which a further reduction to 220 volts is made for distribution to the consumers.

The arrangement of the first receiving sub-station is similar to that of the transformer part of the generating station, while the others are simply iron kiosks containing two transformers of 50 K.V.A. each, and the necessary high- and low-voltage switchgear. The connections of the former are shown

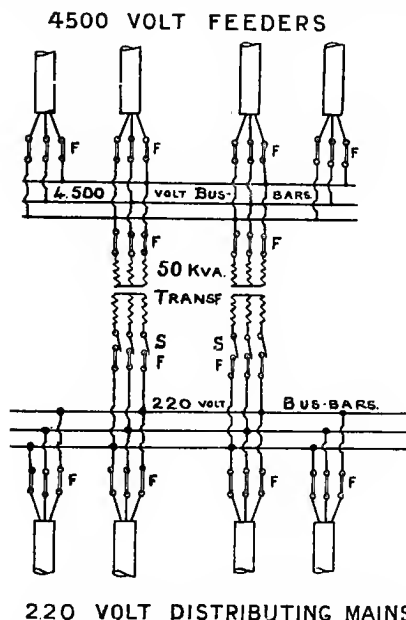


FIG. 11.04.—Diagram of Connections of Distributing Sub-Stations.

in fig. 11.03 and of the latter in fig. 11.04, while sections of the kiosks are given in fig. 11.05.

The iron structure rests on a concrete foundation, and is well earthed by connection to a copper plate sunk in the ground. The outer case has doors on opposite sides for access to the switches, and can be rotated about a vertical axis to facilitate any work being done inside. Openings are left at the bottom and underneath the protecting hood at the top, in order to allow a free circulation of cooling air. The high- and low-voltage switches are on opposite sides of the kiosk, which is lighted by lamps placed between them. The bus bars are arranged so that lighting and power circuits can be separated and one worked from each transformer. Fig. 11.06 is a view of a high-voltage sub-station erected by the same firm.

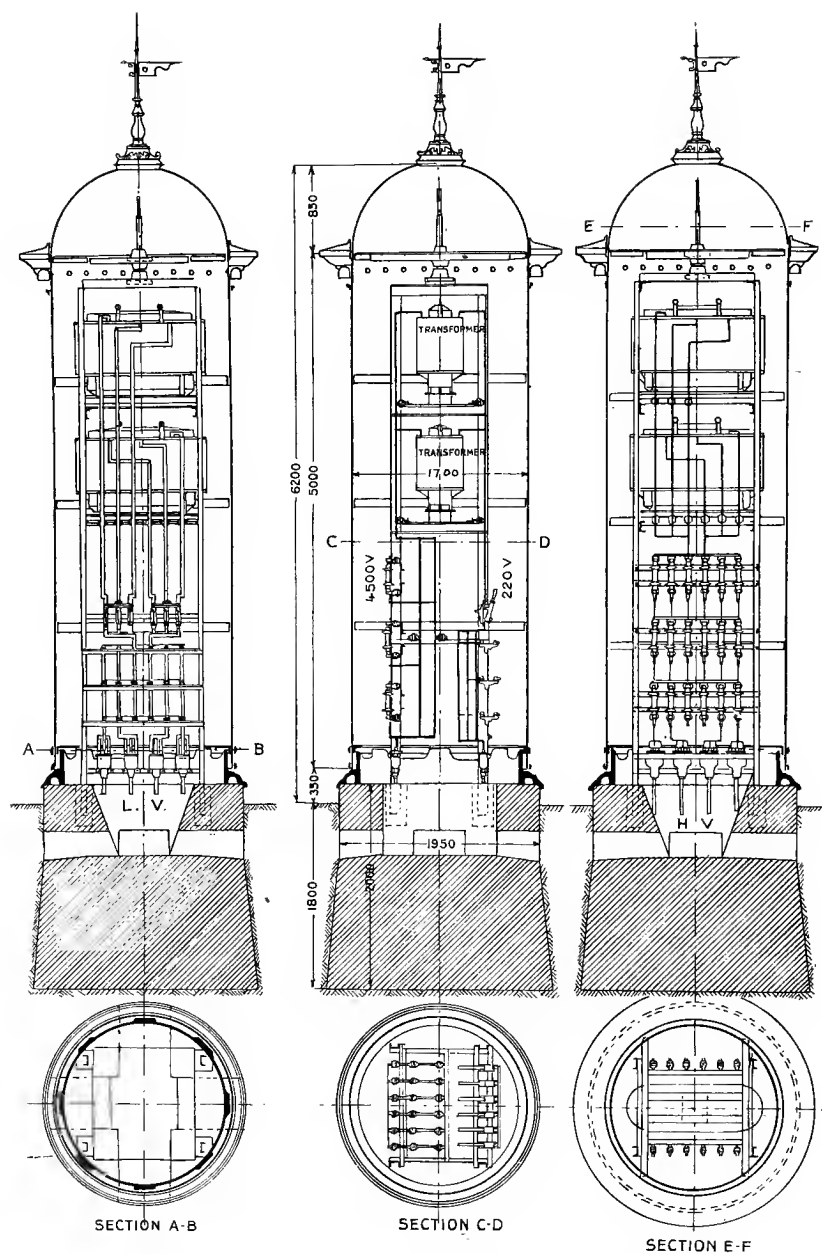


FIG. 11'05.—Transformer Kiosks used at Drammen (Oerlikon).

Where overground kiosks are not permissible, and a suitable room or cellar is not available at a moderate rental, sub-stations must be placed

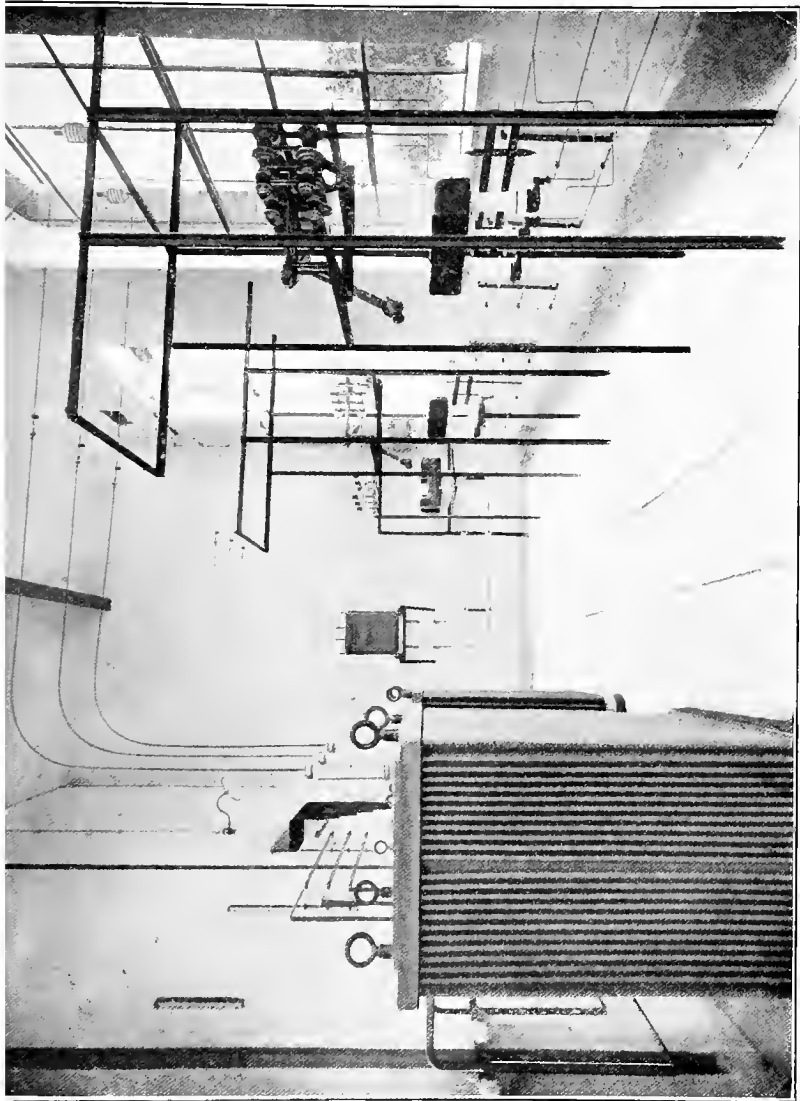
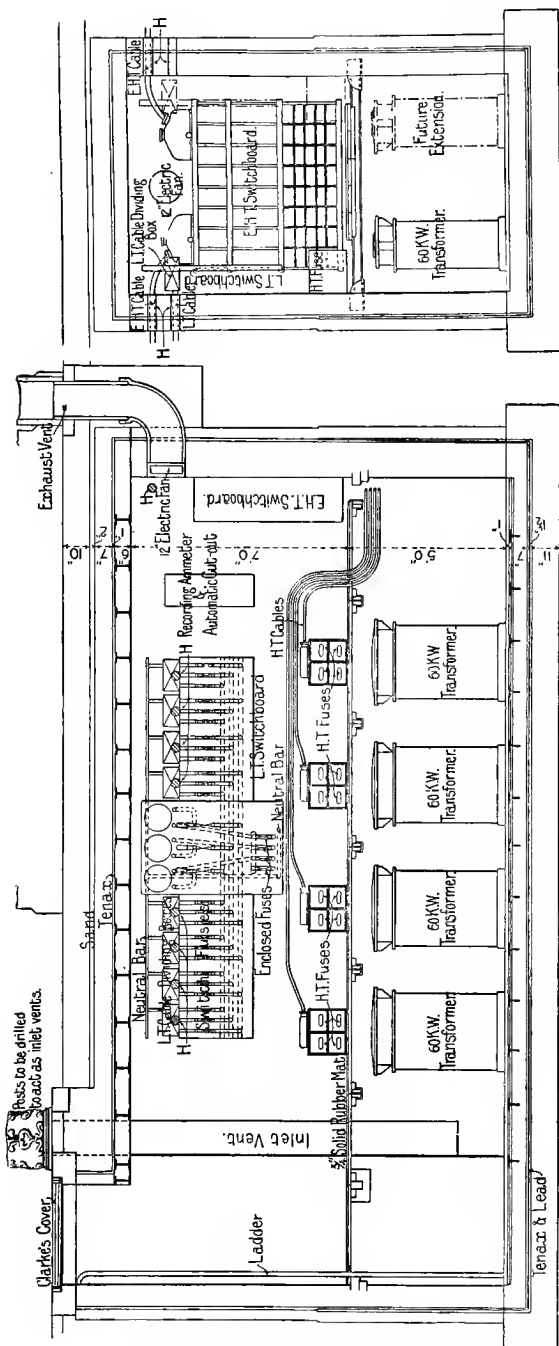


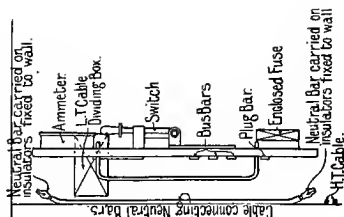
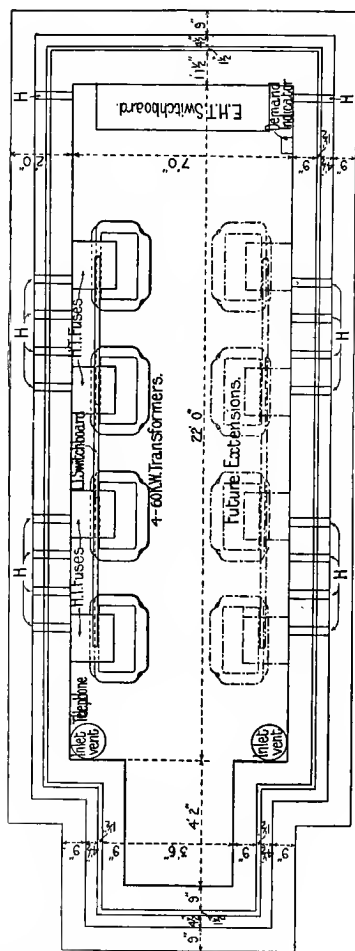
FIG. 11-06.—Interior of a Receiving Sub-Station (Oerlikon).

derground. In any case, they must be easily accessible to authorised rsons, but to no others. The transformers should have a clear space of least 300 mm. (12 in.) all round them to allow a free circulation of ; but more is preferable for the sake of easy inspection. The chamber



H = Position of holes in wall for cable

FIG. 11-07. — Sectional Elevations of Underground Sub-Stations at Dublin. 5000-200 volts.*



* Brew, "Three-Phase Working, with special reference to the Dublin System," *Journ. Inst. E.E.*, vol. xxxiii. p. 570, 14th Jan. 1904.

must be ventilated, and artificial cooling will be necessary for sizes above 50 to 100 K.V.A. All live metal-work should be suitably protected, and all uninsulated metal-work should be well earthed. Unless the transformers are oil-insulated and fitted with air-tight cases, water must not be allowed to drop on them, or rain blow in upon them through windows or ventilators.

Figs. 11·07 and 11·08 give sections and plan of one of the underground

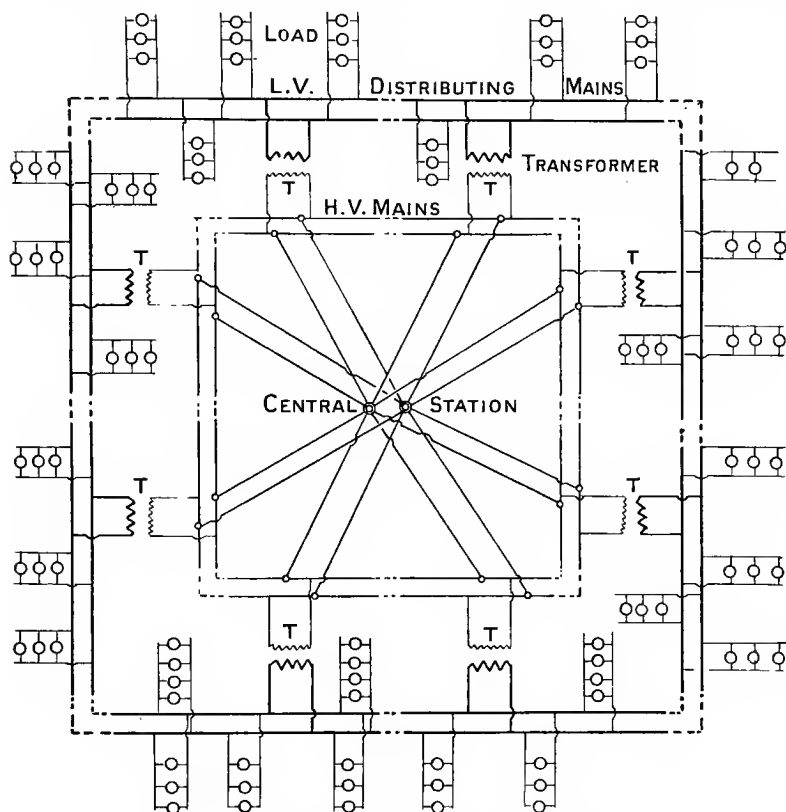


FIG. 11·09.—Diagram of Single-Phase Sub-Station System.

sub-stations for three-phase supply in Dublin. They are constructed large enough to contain twice the original equipment, and in such a manner as will prevent the entrance of water.

Fig. 11·09 shows diagrammatically the method of distributing single-phase currents by means of sub-stations placed at convenient points throughout the distributing area. The various sub-stations may either be entirely independent, or they may be linked up on the high-voltage or low-voltage sides, or on both, as shown by the dotted lines. Consumers

king very large powers are usually supplied from a special sub-station on their own premises, from which the power is distributed throughout the works as required.

In three-phase systems the connections may be "pure" (primary and secondary both star-connected, or both mesh) or "mixed" (one mesh and the other star), but all those which have to work in parallel on both sides must be similarly connected, owing to the phase difference introduced by mixed connections (see Chapter VI.).

House-to-House System.--When the distance between individual consumers is too great to permit of economical distribution at low voltage,

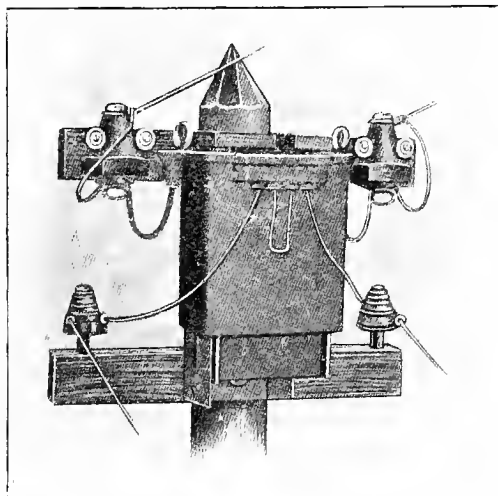


FIG. 11·10.—Outdoor Transformer Fixed on Top of Line Pole.

each may have a separate transformer placed in a convenient cellar or attic, as close as possible to the point of entry of the cables, or, with overhead distribution, the transformer may be placed on the top of a pole, as in fig. 11·10, provided the P.D. be not above 3000 volts. This is known as the house-to-house system, and is shown diagrammatically in fig. 11·11.

Whether the distribution should be carried out in this way or by sub-stations depends entirely on the power required, the load factor, and the distances. Against the considerable saving in copper which, especially with considerable power and long distances, the house-to-house system gives, must be placed the greater cost of insulation and of small transformers for a given total output, their smaller efficiency, and the fact that they have to be kept

alive all the time, while in a sub-station some can be shut down during times of light load.

Wherever the transformer is placed in the house-to-house system, all high-voltage wires, terminals, and apparatus must be enclosed in earthed metal cases, and locked up so that the consumer cannot tamper with them.

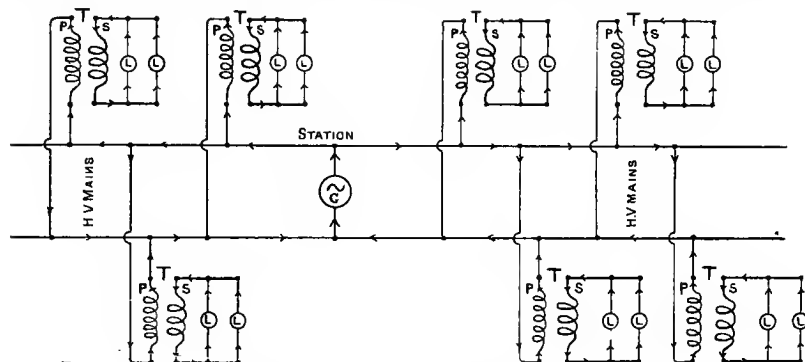


FIG. 11.11.—Diagram of House-to-House System of Distribution.

Multi-Voltage Transformers.—Standard types of transformers are generally divided so as to be suitable for several voltages, for the ability to vary the secondary P.D. in steps of a few per cent. has the great advantage of reducing the number of stock sizes required by making one standard type do for various services. For instance, in circuits of considerable length there is frequently a drop of several per cent.; transformers with slightly different

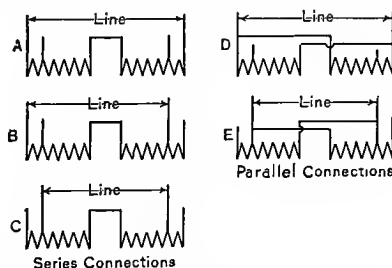


FIG. 11.12.—Different Primary Connections of Westinghouse O.D. Transformer.

voltage ratios may therefore be used with advantage at different parts of the circuit.

In the O.D. type Westinghouse transformer the primary winding is divided into two equal parts, which may be joined in series or parallel to suit different primary voltages. The secondary is divided into four sections, allowing series, series-parallel, or parallel connections. In order to provide

for a small variation of the secondary P.D., a tap is brought out from each half of the primary winding at such a position that 5 per cent. of the total

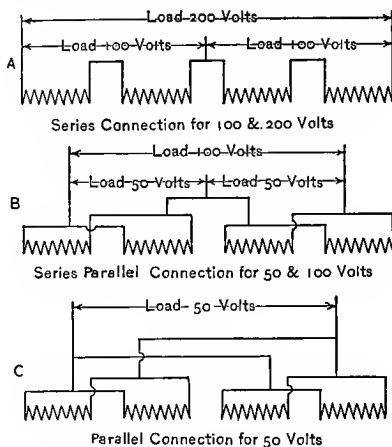


FIG. 11'13. —Different Secondary Connections of Westinghouse O.D. Transformer.

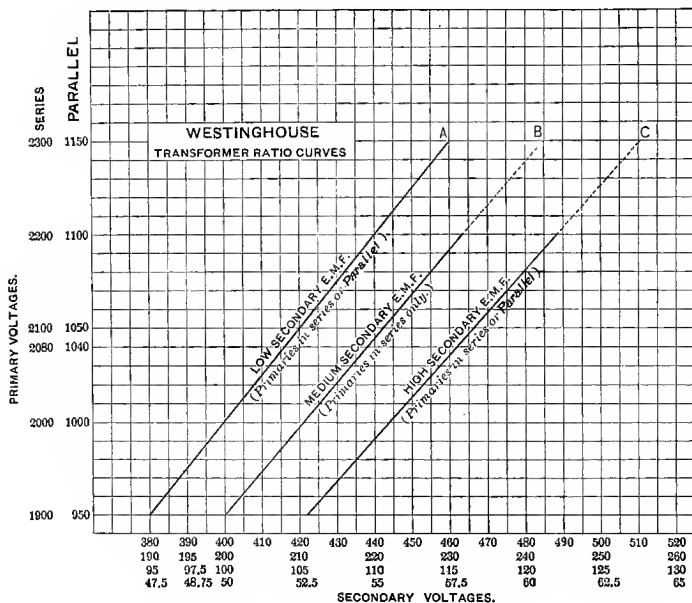


FIG. 11'14. —Voltage-Ratio Diagram for Westinghouse O.D. Transformer.

turns (10 per cent. of the turns in the half-section) lies between the tap and the outside terminal. With the connections shown diagrammatically in A (fig. 11'12) the whole of the primary is in series; in B, 5 per cent. of the

winding is cut out, and in C 10 per cent., raising the secondary voltage about these amounts. These are suitable, with the standard windings, for primary P.D.'s in the neighbourhood of 2000 volts. In D the two parts of the winding are in parallel, and in E 90 per cent. of the winding in each half

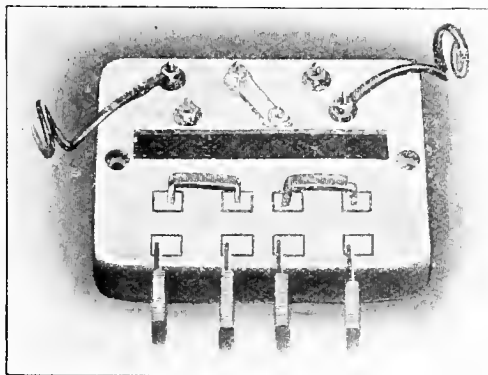


FIG. 11-15.—Terminal Block for Westinghouse O.D. Transformer.

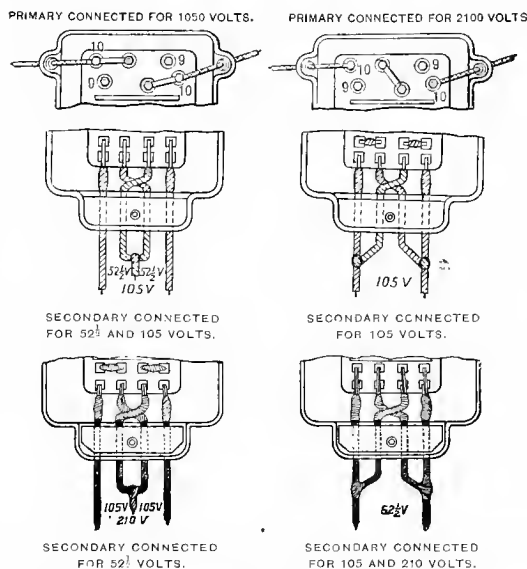


FIG. 11-16.—Connections of Terminal Board for Lowest Secondary Voltage (Westinghouse).

only is used. These are for use on primary circuits of about 1000 volts. If the voltage ratio be 20 to 1 with the connections as in A, it will be 19, 18, 10, and 9 respectively with the other arrangements. The secondaries are wound for 50 volts per quarter section up to 25 kw., and for 100 volts

from 2 to 50 kw. Fig. 11·13 shows the different connections of the secondary, and fig. 11·14 the different secondary E.M.F.'s obtainable with

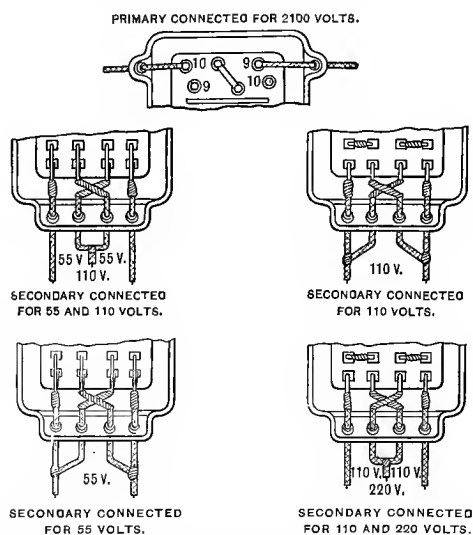


FIG. 11·17.—Connections of Terminal Board for Medium Secondary Voltage (Westinghouse).

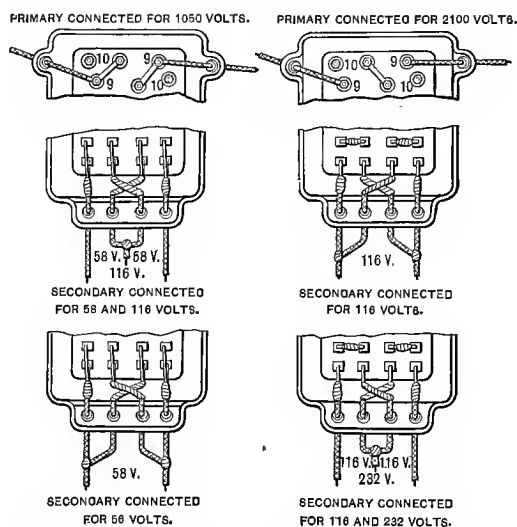


FIG. 11·18.—Connections of Terminal Board for Highest Secondary Voltage (Westinghouse).

various primary P.D.'s. One vertical scale corresponds to the series arrangements of the primary, and the other to the parallel arrangements. The

three upper horizontal scales refer to the three possible connections of the secondaries of the smaller class of transformer, and the lower three to those of the larger class. Line A gives the values when the primaries are connected as in A or D (fig. 11·12); line B when they are as in B; and line C applies to the connections of C or E.

The terminal block of the transformer is shown in fig. 11·15, and the actual arrangements for the various connections in figs. 11·16–11·18. The primary connections are made by means of screws and links, but the secondary ones have soldered joints, as they have to carry heavy currents.

Auto-Transformers.—When it is not necessary to insulate the two circuits from one another, the low-voltage winding may form a part of the high-voltage winding, as shown in fig. 11·19. It is then called an “auto-transformer.” This arrangement saves a considerable amount of copper when the voltage ratio does not differ very greatly from unity, for the low-

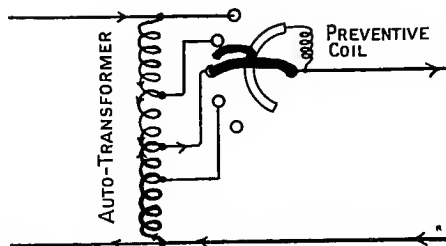


FIG. 11·19.—Auto-Transformer with several Taps and Preventive Coil.

voltage winding has only to carry the difference between the two currents. The saving consists, therefore, of a reduction of the turns in the high-voltage winding by that in the low-voltage winding, and of the reduction in the cross-section of the latter corresponding to the reduced current. It is small when the voltage ratio is large; auto-transformers are therefore only employed for moderate changes of voltage, such as in reducing the P.D. for arc lamps, or for starting induction motors. The two windings should be placed concentrically, or subdivided, just like ordinary transformers if a small voltage drop is required; but for some of the purposes for which they are employed the voltage drop is of no importance, and in others, *e.g.* for arc lamps, a large drop is necessary, and then the two parts need not be so placed.

When employed for starting induction motors, several taps are generally provided in order to raise the voltage by steps; the total number of turns from one end to the various taps should form a geometrical progression like the conductances of a direct-current motor-starter. If the circuit is not to be broken while changing, the switch must touch the new contact before

leaving the old one. To limit the current in the short-circuit so formed, the switch is divided into two parts insulated from one another but connected by a choking coil, which is then termed a "preventive coil," or simply by a resistance if the voltage is very low.

Three-Wire Single-Phase System.—Balancing Transformer.—

When the three-wire system is employed, the proper division of the P.D. may be ensured by dividing the secondary of the transformer into two equal portions and connecting its middle point to the middle wire. A good result will not, however, be obtained by this simple arrangement unless all the coils are wound concentrically on one limb. Suppose the two parts of the secondary to be wound on separate limbs, and that the two sections of the primary are joined in series. If only one of the secondaries be loaded, both primaries, being in series, must carry the same current, and consequently

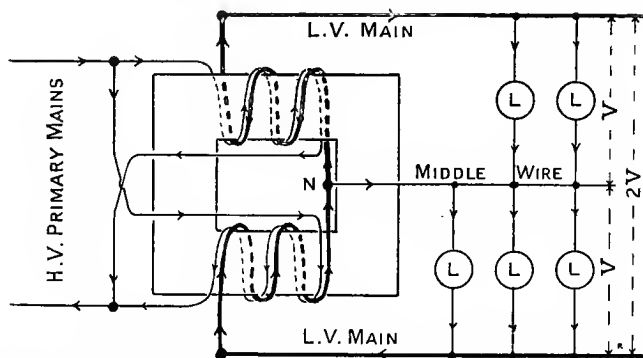


FIG. 11·20.—Three-Wire Single-Phase System. Balancing Transformer with Primaries in Parallel.

each does half the work. That half on the same limb as the loaded secondary is working normally, but that on the other half will be working with a large magnetic leakage, due to the fact that the free polarity between the ends of the limbs has to transmit the whole excitation of one primary section instead of only that to magnetise the yoke. The result is that there is a larger flux than normal in the unloaded limb and a smaller in the other, the difference returning between the limbs. The voltage on the unloaded side consequently rises while that on the other falls.

This may be obviated to a large extent by winding each half of the primary for the full voltage and joining them in parallel as in fig. 11·20. The primary current will now mostly flow round that limb which is loaded, and the voltage variation on the loaded side does not differ appreciably from the drop corresponding to the load, while that on the other side is unaffected.

It is rather a disadvantage to join the two halves of the primary in

parallel, as it means twice as many turns of smaller wire. A good result can be obtained without doing this if the secondary be divided into four equal sections and connected so that each side of the system has a coil on each limb, as in fig. 11·21.

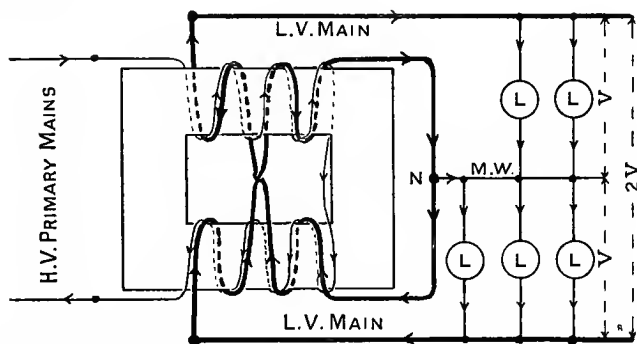


FIG. 11·21.—Three-Wire Single-Phase System. Balancing Transformer with Primaries in Series, and Cross-Divided Secondaries.

The three-wire system is especially suited for a mixed load, as the full P.D. can be used for motors and the half for lighting.

Auto-Balancer for Three-Wire Single-Phase System.—When the distance of distribution is considerable, the balancers may be placed elsewhere than at the sub-station in order to reduce the size of the necessary middle wire, or even to obviate bringing it to the sub-station altogether in ex-

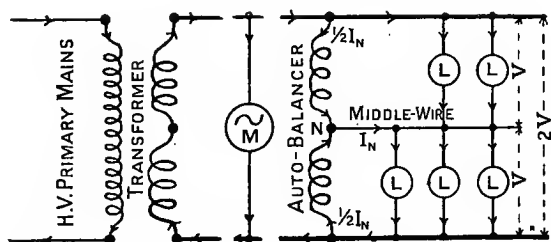


FIG. 11·22.—Auto-Balancer for Three-Wire Single-Phase System.

ceptional cases where there is no half-voltage load near the sub-station. The balancer may then take the form of an auto-balancer, which is simply a transformer of 1 : 1 ratio, with each coil wound for half the P.D. between the outers, the two connected in series between them, and their junction joined to the middle wire. This arrangement (see fig. 11·22) takes power from the side whose P.D. rises, and gives it to the other side. It thus obviously limits the difference between the voltages of the two sides, at the place where the

balancer is, to the voltage drop of the balancer produced by half the out-of-balance current in the middle wire. Assuming that the total P.D. is kept constant, the rise on one side and the drop on the other will thus be equal to one-half the voltage drop of the transformer with half the middle-wire current flowing in each winding. In the extreme case when one side of the system is fully loaded and the other idle, the currents in the windings of the balancer and that taken from the mains are each half the load current, neglecting the no-load current of the transformer. Under these conditions, the load on the balancer is half the full-load line current with half the line P.D., or half the load current with the load P.D., or half the load. When fully loaded, the system would have an equal load on the other side; consequently, the size of the balancer need only be one-fourth that of the main transformer supplying the system in order to be able to meet the worst conditions. In actual practice, the balancer need not be quite as large as this, and in many cases may be much smaller. If, however, the supply is to be maintained on one side while the other has been cut off by a fault, the balancer must be capable of working under these extreme conditions without overheating.

Auto-Balancer for Multi-Wire Single-Phase System.—The auto-balancer is convenient for use with metallic filament lamps and for arc lamps which are to be run independently, as by its means the P.D. can be divided into any number of equal or unequal parts by making the transformer with as many windings, each with the requisite numbers of turns, and all joined in series in such a way that the E.M.F.'s are added. If these are all arranged concentrically to make the magnetic leakage small, equal E.M.F.'s will be induced in every turn, and the variations in the division of the P.D. are due only to the voltage drop. If, however, the different sections of the winding are wound on different parts of the core, the magnetic leakage will be much more, the voltage drop correspondingly increased, and there will also be a voltage rise in the lightly loaded sections. Fig. 11·23 shows the connections of such an arrangement for dividing the P.D. into four parts, three of which are to be loaded. If we neglect the no-load current of the transformer, the total excitation of the coils must be zero; consequently, if there be N turns in any section, and N_T turns in all, a current I in the apparatus connected to that section causes a current to be taken from the mains equal to

$$I_M = \frac{N}{N_T} I \quad . \quad . \quad . \quad . \quad . \quad 11\cdot01.$$

This current flows through all the sections. The current in any section of the winding is equal to the current in its load minus the resultant of all

the currents taken from the mains; when it alone is loaded, the current in it is

$$I_W = \left(1 - \frac{N}{N_T}\right)I \quad . \quad . \quad . \quad . \quad 11.02.$$

In the case commonly met with, the full-load current is the same for all the parts, and then there is only the exciting current in the windings when the lights are all off or all on. The largest current in any section of the balancer occurs when that section only is loaded, or when that one only is unloaded. Thus, with equal sections, each turn must be able to carry the current $\left(1 - \frac{N}{N_T}\right)I$, equivalent to a total current-carrying capacity of $(N_T - N)I$ in the winding space. An ordinary transformer to supply all

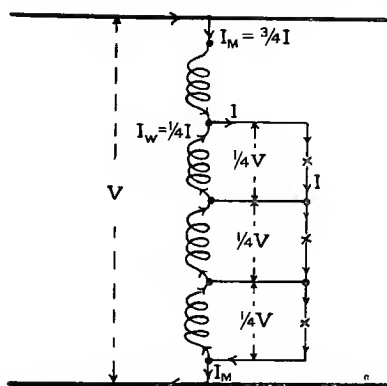


FIG. 11.23.—Auto-Balancer for Multi-Wire Single-Phase System for Independent Use of Arc Lamps in Series.

the loads in parallel would have a secondary of N turns to carry the current $\frac{N_T}{N}I$, or an excitation of $N_T I$, and a primary to give the same excitation, making a total of $2N_T I$. The ratio of the amounts of copper required in the two cases would thus be:—

$$\frac{\text{Rated output of auto-balancer}}{\text{Rated output of transformer for same duty}} = \frac{N_T - N}{2N_T} = \frac{1}{2} \left(1 - \frac{N}{N_T}\right) \quad 11.03.$$

The auto-balancer can really be made a little smaller than this, for all the sections cannot be fully loaded at the same time, whereas both the primary and secondary of the transformer are.

Suppose that there are three lamps, each taking 15 amperes and one-third the total voltage. If one lamp is on, its section carries 10 amperes, and the others 5 amperes. If two lamps are on, the remaining coil carries

10 amperes, and the loaded ones only 5 amperes, both in the opposite way to what they are when the conditions are reversed. The rating of the balancer need therefore only be one-third that of a transformer to supply the same load.

Three-Phase Four-Wire System.—Three-Phase Auto-Balancers.—When the load on a three-phase system is a mixed one, it is con-

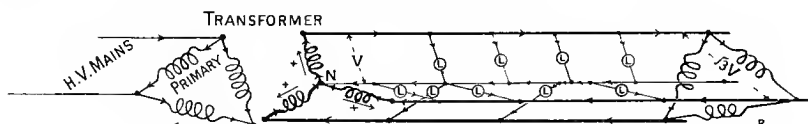


FIG. 11-24.—Four-Wire Three-Phase System.

venient to have the secondaries star-connected and to put the lighting load between the mains and a fourth wire taken from the neutral point (fig. 11-24), where they get $\frac{1}{\sqrt{3}}$ of the line P.D. only. The load should be distributed as evenly as possible between the three legs in order to disturb the potentials as

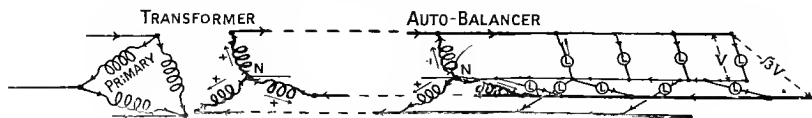


FIG. 11-25.—Simple Three-Phase Auto-Balancer.

little as possible. The star connection, in addition to the lower voltage available for lighting, has the advantage that the unbalanced load comes between one main and the neutral point, and so only causes the P.D. of that leg to fall without raising the P.D. of the others. As in the three-wire single-phase system, auto-balancers may be employed to reduce the cost of the neutral wire in extensive networks. Such a balancer (fig. 11-25) may consist of



FIG. 11-26.—Six-Ray Three-Phase Auto-Balancer. Six Different Phases.

three coils wound on three iron limbs like a three-phase transformer, and joined in a star to whose neutral point the fourth wire is connected. A balancer of this type is not very effective, for since the coils are on different limbs the leakage flux will materially reduce the effect of one on the other.

By a modification, however (see figs. 11-26 and 11-27), the inductive action of one phase upon the other may be intensified. Surround the star

by a mesh connected across its points, continue each coil of the star beyond the neutral point by adding half as many turns as it had before, and join the ends of these additions to the middle points of the sides of the mesh, as shown. The sides of the mesh should have $\sqrt{3}$ times as many turns as the legs of the original star. We have now a six-rayed star, and the neutral point will take current from all six, and thus load all the phases to some extent.

As the phases of the mesh differ from those of the star, six different limbs would be required with this arrangement.

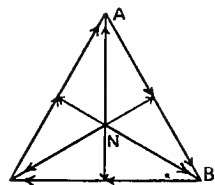


FIG. 11-27. —Vector Diagram for Six-Rayed Three-Phase Auto-Balancer. Six Different Phases.



FIG. 11-28. —Symmetrical Six-Rayed Three-Phase Auto-Balancer. Three Different Phases only.

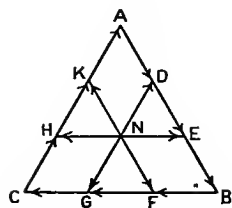


FIG. 11-29. —Vector Diagram for Symmetrical Six-Rayed Three-Phase Auto-Balancer.

By disconnecting the star from the corners of the mesh, and connecting it to points dividing the sides of the mesh into three equal parts, as shown in fig. 11-28, the load is more evenly divided between the three phases. At the same time there is the further great advantage that each side of the mesh is in phase with one pair of rays, so that the number of limbs is reduced three, and also that all the sections should now have the same number of turns.

With either arrangement, primary coils can be added to make it into a transformer, as shown in fig. 11-30 for the symmetrical case.

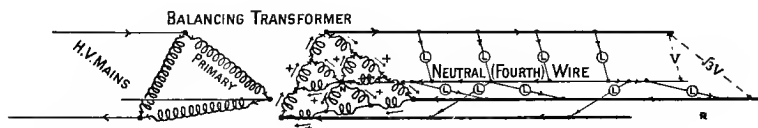


FIG. 11-30. —Symmetrical Six-Rayed Three-Phase Balancing Transformer.

Neutral-Point Balancer for Three-Wire Direct-Current System. — The same principles can be applied to maintain the correct potential of the middle wire of a direct-current three-wire system. The

generator is provided with two, three, or four slip rings for collecting alternating currents from its windings. These are joined to a two-, three-, or four-phase balancer whose neutral point is connected to the middle wire. Four rings in phase quadrature are generally preferred, and two single-phase auto-transformers are joined across rings in opposite phases with their middle points connected together and to the middle wire (see fig. 11'31). This is equivalent to a four-phase star connection. This arrangement is much smaller than a rotating balancer for the same service, has a larger reserve of power for dealing with a short-circuit on one side, and works more satisfactorily.

When rotary converters are employed, the secondaries of the transformers are generally connected star fashion, and the neutral point is connected to the middle wire of the D.C. system.

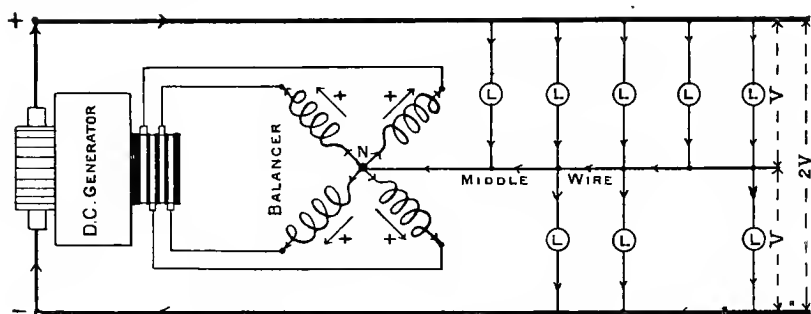


FIG. 11'31.—Neutral-Point Balancer for Three-Wire D.C. System.

Volt-Reducing Choking Coils.—When a single arc has to be run on an ordinary supply, instead of employing an auto-transformer, or “economy coil” as it is generally termed, a simple choking may be put in series with the lamp or other apparatus (see fig. 11'32). Such a choking coil may be constructed exactly like a transformer, except that it has only one winding, or it may have an air gap in its magnetic circuit with an adjustable bridge for regulating its effect. In either case it is to be noted that the whole current is available for magnetising, instead of only a small fraction, as in a transformer. If made with an air gap and a low-flux density in the iron, the choking coil will have an approximately constant impedance; if made without an air gap and with a high-flux density in the iron, the impedance will fall considerably as the current is increased, and consequently the impedance E.M.F. will rise much less rapidly than in proportion to the current.

The copper and iron losses in a choking coil are generally small; consequently the current and E.M.F. are nearly in quadrature, the choking coil

P.D., V_C , is considerably in excess of the arithmetical difference between the main P.D., V_M , and lamp P.D., V_L (see fig. 11·33), and the power taken from the mains is not much greater than that required by the lamp. But the current taken is the full lamp current, and the power factor is low. If an ordinary resistance be substituted for the choking coil, the current taken

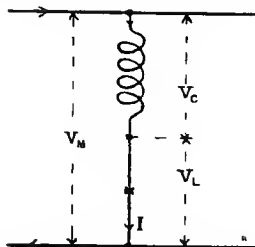


FIG. 11·32.—Choking Coil in Series with Arc.

remains the same, but the power taken is about as many times that used in the lamp as the mains P.D. contains that of the lamp. The remainder is spent in heating the resistance coils. There is thus a considerable reduction in the energy taken from the mains by using a choking coil instead of a resistance, which will correspondingly reduce the consumer's bill if an *energy* meter be employed, but not if an ampere-hour meter be installed.

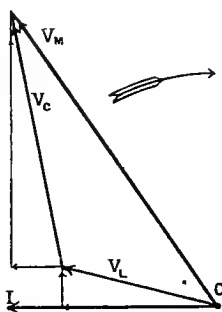


FIG. 11·33.—Vector Diagram for Choking Coil in Series with Arc.

The actual saving to the supply authorities is, however, very minute. The same mains and switchboard are required for a given current, and there are the same losses in them, whether the current is in phase with the P.D. or not; while the generators and transformers will actually have to be larger to supply the choking coils than the resistances with the same voltage drop, and the losses in them will be greater. The only saving is a small reduction in the coal consumption, because the engines and boilers will not be

working at full power, and these costs generally form only a small fraction of the total costs of supply, being as a rule considerably less than the capital charges, which would be practically the same in both cases, as the diminution of boiler power required could be set against the increased size of generators, transformers, and exciting plant.

Comparison of Choking Coil and Auto-Transformer.—An auto-transformer, on the other hand, reduces the current taken from the mains in nearly the same proportion as the power; its first cost is also a little less for the same losses, and it is therefore to be preferred to a choking coil.

With a non-reactive load—

$$\frac{\text{Equivalent transformer rating of choking coil}}{\text{Rating of ordinary transformer for same duty}} = \frac{\frac{1}{2}(V_M^2 - V_L^2)^{\frac{1}{2}}}{V_L} \quad 11.04.$$

$$\frac{\text{Equivalent transformer rating of auto-transformer}}{\text{Rating of ordinary transformer for same duty}} = \frac{(V_M - V_L)}{V_M} \quad 11.05.$$

(See equation 10.209, where $V_1 = V_M - V_L$ and $V_2 = V_L$.)

$$\begin{aligned} \frac{\text{Equivalent transformer rating of choking coil}}{\text{Ditto of auto-transformer for same duty}} &= \frac{V_M(V_M^2 - V_L^2)^{\frac{1}{2}}}{2V_L(V_M - V_L)} \\ &= \frac{V_M(V_M + V_L)^{\frac{1}{2}}}{2V_L(V_M - V_L)^{\frac{1}{2}}} \quad 11.06. \end{aligned}$$

Table 11.01 shows how these ratios change with the ratio of mains voltage to load voltage. It will be seen that with unity power-factor the choking coil is always larger than the auto-transformer. In the case of an arc lamp, however, a resistance will also be required with the auto-transformer, increasing the losses and the cost.

When the load is inductive, the conditions are much more favourable to the choking coil. In the extreme case when the load current lags nearly a quarter cycle, the coil voltage comes into phase with the load voltage, and

$$\frac{\text{Equivalent transformer rating of choking coil}}{\text{Rating of ordinary transformer for same duty}} = \frac{V_M - V_L}{2V_L} \quad 11.07,$$

and

$$\frac{\text{Equivalent transformer rating of choking coil}}{\text{Ditto of auto-transformer for same duty}} = \frac{V_M}{2V_L} \quad 11.08.$$

In this case the coil is the smaller so long as the mains voltage is less than twice that of the load.

TABLE 11.01.—COMPARISON OF EQUIVALENT TRANSFORMER RATINGS OF CHOKING COIL AND AUTO-TRANSFORMER FOR SAME DUTY.

Mains Voltage. Load Voltage. $\frac{V_M}{V_L}$	E.T.R. of Choking Coil. Rating of Ord. Trans. for same duty.		E.T.R. of Auto-Trans. E.T.R. of Ord. Trans.	E.T.R. of Choking Coil. E.T.R. of Auto-Trans. for same duty.	
	Non-reactive Load.	Inductive Load.		Non-reactive Load.	Highly Inductive Load.
1	0	0	0	∞	0.50
1.1	0.229	0.05	0.091	2.52	0.55
1.2	0.332	0.10	0.167	1.99	0.60
1.3	0.415	0.15	0.231	1.80	0.65
1.4	0.490	0.20	0.286	1.71	0.70
1.5	0.559	0.25	0.333	1.68	0.75
1.6	0.625	0.30	0.375	1.67	0.80
1.7	0.687	0.35	0.412	1.67	0.85
1.8	0.748	0.40	0.444	1.68	0.90
1.9	0.808	0.45	0.474	1.71	0.95
2.0	0.866	0.50	0.500	1.73	1.00
2.5	1.146	0.75	0.600	1.91	1.25
3	1.414	1.00	0.667	2.12	1.5
4	1.937	1.5	0.750	2.58	2.0
5	2.450	2.0	0.800	3.06	2.5
10	4.975	4.5	0.900	5.53	5
∞	∞	∞	1.000	∞	∞

Continuity Choking Coils for Series System.—When a number of lamps are in series and at too great a distance from one another to be

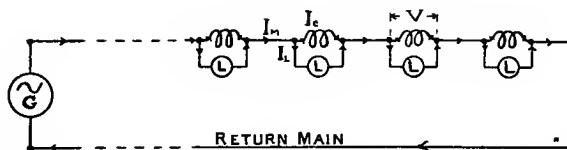


FIG. 11.34.—Choking Coils in Parallel with Lamps to Ensure Continuity in Series System.

economically connected to one balancer, the continuity of the circuit when one lamp is switched out, or fails, can be secured by putting a choking coil in parallel with each, as in fig. 11.34. The line current is then the resultant of the lamp and choking-coil currents. When the lamp circuit is broken, the choking coil will have to take the whole current instead of only a small part, and there must consequently be a rise of the P.D. between its terminals and a corresponding reduction of the total current if the generator E.M.F. be constant. This reduction of the current can be minimised by making the choking coil with an air gap of high reluctance, and by using a very high flux density. The former acts by making the no-load current in the choking

coil to be a large fraction of the total, and is objectionable because it increases the line current and losses; with the latter, the effective impedance gets less as the current is increased, due to the saturation of the iron, as already mentioned, and it has the disadvantage of causing large iron losses. In any case, the iron losses will be very great when the full current goes through the choking coil, and the design must be such as to permit the dissipation of the heat generated without an injurious rise of temperature.

Fig. 11·35 gives the vector diagram for this arrangement. I_L , I_C , and I_M are the currents in the lamp, coil, and mains respectively; V is the P.D.

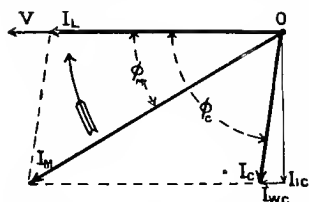


FIG. 11·35.—Vector Diagram for Choking Coils in Parallel with Lamps Run on Series System.

between the terminals of one lamp or coil, and I_{WC} and I_{IC} are the working and idle components of the current in the latter. When a lamp is out, I_C and I_M become coincident with one another by mutual approach, the proportion by which I_M changes being smaller the smaller the ratio of V to the total E.M.F. of the supply is. V will still be ahead of I_C by an angle approximately the same as ϕ_C , but a little less owing to the diminution of

the reactance by the reduced permeability of the iron. On the Baltic Canal 250 lamps taking 25 volts each are run in series with choking coils arranged in this way, and it is found that about one-third of all the lights may be out before the remaining ones are seriously affected. In a case of this kind where the lamps extend along a long line, the saving in cables by the use of the series system is very great, and more than enough to pay for the coils.

If a constant-current regulator be added to the circuit, the E.M.F. of the generator would have to be raised as lamps are put out of circuit. With such a regulator in use, however, it would be simpler to provide each lamp with a switch which automatically short-circuits the lamp when it fails.

Transformers in Series.—In the series system just described the potential of the lamps, except near one part of the circuit, differs considerably from that of the earth; consequently they must be very carefully insulated, and must not be handled except when disconnected at both poles. This disadvantage may be avoided by providing a separate transformer for each lamp and connecting all their primaries in series, as in fig. 11·36. The transformers are, however, more expensive, and the cases in which this arrangement can be profitably employed are few. When transformers are in series, the excitation of the primary is practically independent of that in the secondary. Consequently there is a large increase in the effective excitation when the opposing excitation of the secondary is removed by open-circuiting

it. The flux density is thus largely increased, and may heat the iron sufficiently to burn out the transformer. This may be avoided in the same way as in the choking coils of the other system, but it is better to provide an automatic switch to short-circuit the secondary when a lamp fails or is cut out, an equivalent impedance being put in circuit if the current is not kept constant by a regulator.

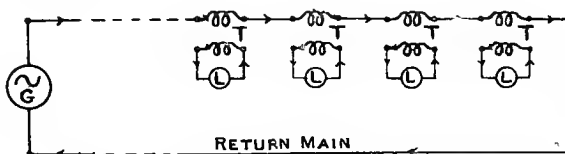


FIG. 11·36.—Transformers with their Primaries in Series.

Boosting Transformer.—In order to get the full advantage of parallel running, it is desirable to have all the generators and feeders in a station connected to the same bus bars; but the P.D. between the far ends of long, heavily loaded feeders will then be less than at the ends of the short, lightly loaded ones. This can be remedied by adding an extra E.M.F., or “boost,” to those feeders which require it. Boosting is generally done at the generating end, but it may be done at the other end, or at one or more intermediate

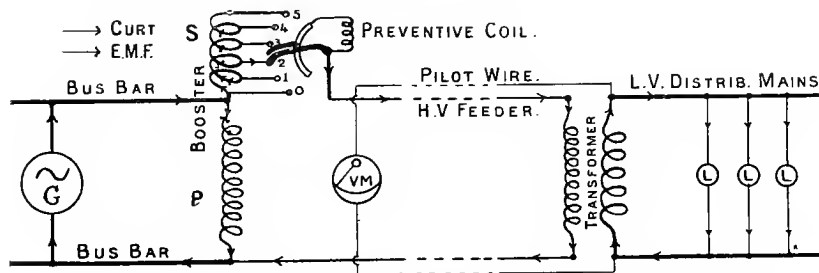


FIG. 11·37.—Boosting Auto-Transformer with Switch in Secondary (Kapp).

points if the circumstances make that procedure desirable. On A.C. systems the boosting is done by a transformer whose primary is connected across the mains, and whose secondary is put in series with one main of the feeder to be boosted. Several taps are taken from the secondary, as originally proposed by Kapp, or from the primary, in order to permit the amount of boost to be regulated. The former arrangement (fig. 11·37) has the disadvantage that the regulating switch is in the main circuit, where it has to carry the whole current and where its breakdown would cause an interruption of the supply. The latter (fig. 11·38) obviates this, but has the disadvantage that

the whole of the material is not in active use when the maximum boost is required, for then only a portion of the primary coils is in circuit. It has, however, the additional advantage that if one of the mains is at earth potential the switch can be connected to that side of the primary, which

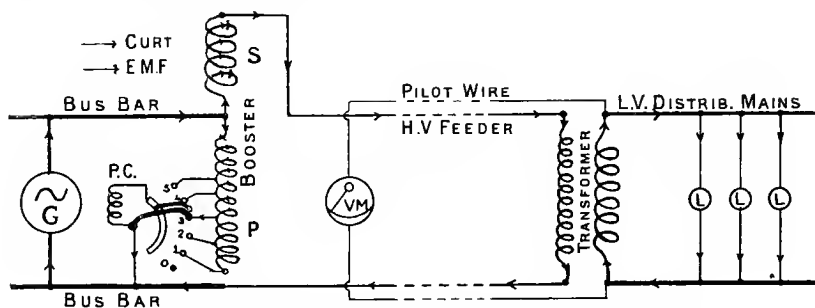


FIG. 11'38.—Boosting Auto-Transformer with Switch in Primary.

could not be done with the first arrangement when the return main is uninsulated.

High-Voltage Booster.—Greater safety of working with high voltages may be obtained with an arrangement similar to that shown in fig. 11'39, due to the Elektricitats-Gesellschaft, Alioth.* The arrangement consists of

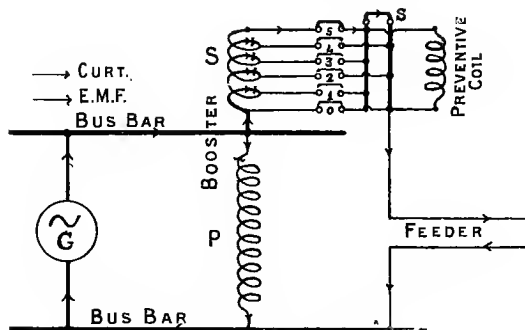


FIG. 11'39.—High-Voltage Single-Phase Booster (Elektricitats-Gesellschaft, Alioth).

a boosting transformer with a special high-voltage regulating switch and a preventive coil. The switch gear consists of several blades and two auxiliary bus bars, connected by a preventive coil, which can be short-circuited by a switch S. When No. 5 and S are on, all the turns of the transformer are in series with one main, and the boosting E.M.F. is a maximum. In order to reduce it by one step, open S, close switch 4, open 5. For the next step,

* See also *E.T.Z.*, 1906, p. 263.

close 3, open 4, and then close S; and so on for the others. The change-over thus takes place without interrupting the circuit or short-circuiting one section of the transformer.

For a two-phase circuit we would require double-pole switches and two regulating transformers. A three-phase circuit may be similarly regulated by employing triple-pole switches in connection with a three-phase boosting transformer.

Single-Phase Induction Regulator.—A continuous adjustment without switches is obtained by employing a booster arranged like fig. 11'40, where the primary is wound on a movable core and the secondary on a ring. The connections are made to the moving coil through flexible wires. This

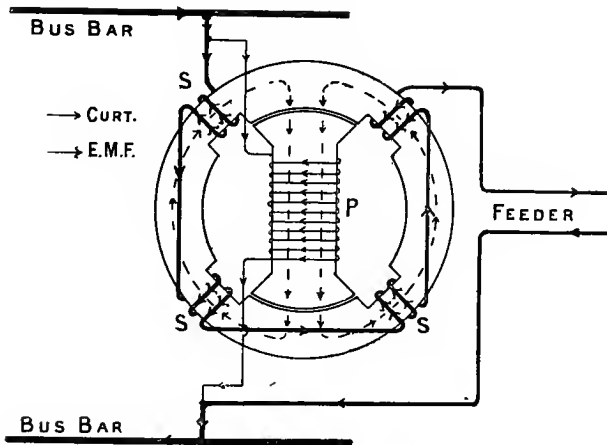


FIG. 11'40.—Single-Phase Induction Booster.

arrangement is generally termed an “induction regulator.” In the position shown, the E.M.F.’s in the four coils of the secondary are added, and the boosting effect is a maximum. When the primary is rotated through a right angle these E.M.F.’s are opposed in pairs, and the boost is zero if they are alike. If the core be further rotated, the booster will take energy from the current, and the boost be negative. By placing the primary in an intermediate position we may obtain any desired boost, from a maximum in one way, through zero, to a maximum in the other way. The changes take place gradually, without the jerks given by boosters with regulating switches. The adjustment may be done by hand, or automatically by a relay and motor.

When the booster is loaded there is a considerable torque acting which makes it difficult to move the core, but this can be overcome by dividing the booster into two parts producing torques in opposite ways.

Compensated Single-Phase Induction Regulator.*—The booster just described has considerable magnetic reluctance and leakage, and conse-

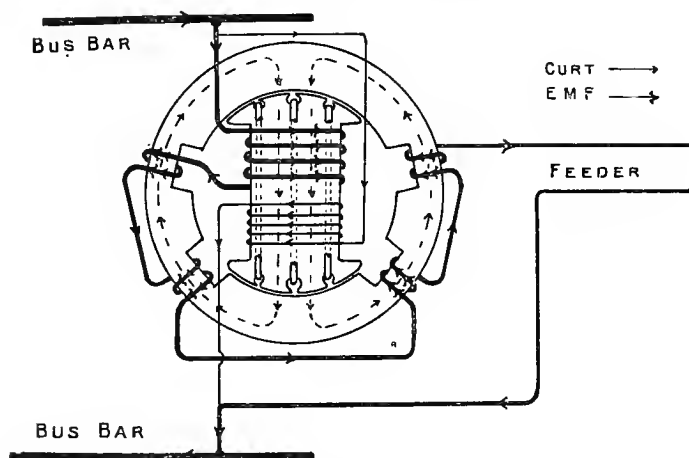


FIG. 11'41.—Cowans-Still Compensated Single-Phase Induction Regulator in Position of Maximum Boost.

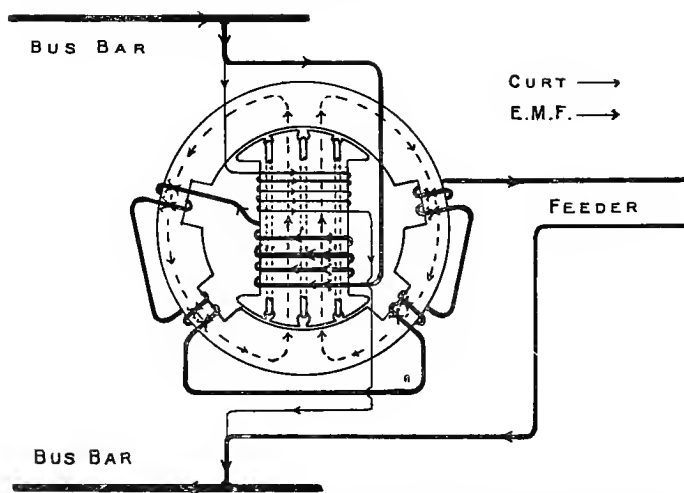


FIG. 11'42.—Cowans-Still Compensated Single-Phase Induction Regulator in Position of Minimum Boost.

quently takes a large exciting current and has an appreciable voltage drop. These disadvantages are lessened in the modification by Cowans and Still shown in the next two illustrations. The reluctance is reduced by increasing

* *Journ. I.E.E.*, vol. xxxii, p. 907, 3rd March 1903.

the polar arc, and the magnetic leakage by winding half of the secondary on the movable core. In every position of the core a boosting E.M.F. is induced in this half, and in the position of fig. 11.41 the E.M.F. in the other half also assists and the boost is a maximum. As the core is gradually turned through a right angle, the E.M.F. in the fixed coils falls to zero, when the boost is that of the moving coil only. A rotation through a further quarter turn causes the former E.M.F. to increase in the negative direction, until, finally, it cancels that of the moving coils, and the net boost is zero.

When the core is not vertical, the leakage flux of the fixed portion of the secondary will, in part, cut across the moving core at right angles to its length, and so induce currents in the short-circuited coils let into the polar faces of the moving core; these currents oppose the leakage flux, and consequently the short-circuited coils neutralise to a large extent the leakage inductance of the fixed secondary.

Three-Phase Induction Regulator. — The three-phase induction regulator is essentially the same as a three-phase induction motor with a wound rotor which is held fast in position either by hand gear or by a motor controlled by an automatic relay (see fig. 11.43). One winding, say the rotor, is connected across the mains, and the other, whose phases are independent, in series with the three mains of the feeder to be boosted. The primary winding produces a rotating field which induces E.M.F.'s in the secondary which are constant in amount but whose phase relation with the mains P.D. depends on the position of the rotor, and is therefore capable of adjustment by turning the rotor. The effect of this on the resultant E.M.F. is shown vectorially in fig. 11.44. V_s is the station bus-bar P.D., V_L the line P.D., and E_B the total E.M.F. in one phase of the booster. It is also evident from the vector diagram that the phase difference between the current and P.D. is altered by the booster. The phase of the current on the load side of the booster is fixed by the nature of the load, but the booster allows it to be adjusted on the generator side. The two adjustments, power-factor and P.D., are not, however, entirely independent of one another. Induction regulators are often employed in connection with rotary converters.

Boosting transformers are suitable for voltage drops which do not exceed 10 to 15 per cent. of the line P.D., so long as the load fluctuations are not too large and sudden, when, on account of the mechanical and magnetic inertia of the controlling apparatus, regulation would take place too late and be unsatisfactory. In America, the transmission of electrical energy over several hundred miles is not uncommon, and the expenditure in copper for mains to give only a drop of 10 per cent. in such a case would be enormous. A drop of 20 per cent. and more is then not out of place, and the reduction

in the price of the feeders easily counterbalances the cost of the regulating plant. A boosting transformer is, however, not suitable, and boosting generators with differential excitations are required. For a description of

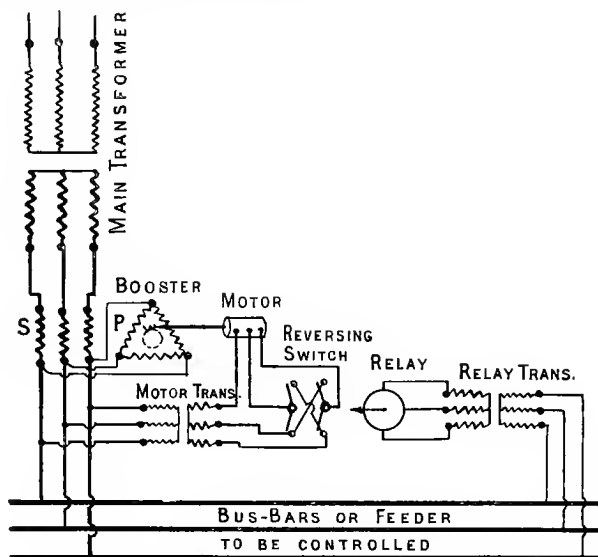


FIG. 11'43.—Three-Phase Automatic Booster System.

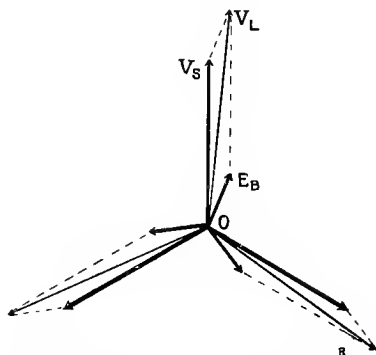


FIG. 11'44.—Vector Diagram for Three-Phase Induction Regulator.

such plant, the reader is referred to an article by Dr H. Hinden in the *E.T.Z.* on 26th April and 3rd May 1906.

Return - Current Booster.*—In a system of traction with track return, troubles from electrolysis of pipes and disturbance of telephones

* See also Behn-Eschenburg, *E.T.Z.*, 1904, p. 311.

and telegraphs arise unless the currents straying from the track are kept down to a very small value, and this can only be attained by limiting the greatest P.D. between one part of the track and another. With alternating systems the disturbances are much greater than with direct-current systems, while the electrolysis is much less, although not entirely absent. The higher the frequency, the less is the influence on telegraphic working, but the greater is that on the telephones.

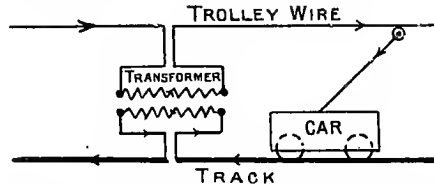


FIG. 11'45.—Return Boosting Transformer (Kapp).

Owing to the inductance, and to the fact that an alternating current flowing in a steel rail is practically confined to a thin skin about 3 mm. ($\frac{1}{8}$ in.) thick, the impedance E.M.F.'s in the rails and the corresponding differences between the potentials of one part of the track and another are much greater when carrying alternating current than with an equal direct current. Kapp * has proposed to transfer the impedance E.M.F.'s of the track to the trolley-wire by means of transformers whose primaries are in series with the trolley-

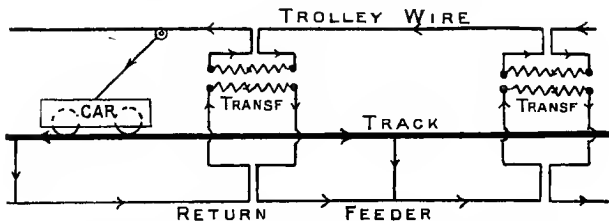


FIG. 11'46.—Return Boosting Transformer (Oerlikon).

wire and secondaries with the rails, as shown in fig. 11'45. The division of the rails by the transformer winding is avoided by the modification employed by the Oerlikon Company shown in fig. 11'46. Here the track is connected to a return feeder at suitable intervals, and each section of the feeder contains a return booster of the Kapp type. In either case, if the no-load current of the transformer can be neglected, the excitation of the secondary will be the same as that of the primary. Consequently, with equal numbers of turns in each winding, the transformer will transfer sufficient E.M.F. from the

* *E. T. Z.* 1902, p. 19.

outgoing to the return main to make the currents be the same in both. With the Kapp system, this would mean that there is no current left to stray past the division, although some may leave the track at one place and enter it again at another. In the Oerlikon system, equal currents in the two windings means that none is left for the rails to carry, and consequently the two ends of that section of the track will be at the same potential if there be no car on it. By making the section sufficiently short, the greatest P.D. between two points on the track may be made as small as desired. The effect of the transformer is thus virtually to transfer the impedance of the track and return main to the trolley-wire or outgoing main. The P.D. between the two ends of the outgoing main or trolley-wire is increased by that transferred from the return main or track, the drop of P.D. being concentrated on that main.

Actually, the no-load current of the transformer will not be quite negligible, and so the primary excitation must exceed the secondary. The primary ought therefore to have a slightly larger number of turns than the secondary. In an experiment made by the Oerlikon Company, a booster with a primary of 23 turns and a secondary of 20 turns reduced the track current from 100 amperes to 4 amperes.

A second advantage of the Oerlikon arrangement is that the effect of the inexactness of the adjustment of the transformer is less than in the Kapp system, owing to the points of lowest and highest potentials in the track being widely separated instead of close together.

Constant-Current Transformers.—If a transformer be built with a very large magnetic leakage, its impedance will be excessive, and the secondary current will vary comparatively little with a considerable change in the load. The greater its impedance is compared with that of the load—in other words, the more nearly it is run under short-circuit conditions—the smaller will be the change of current produced by a given change in the impedance of the load. Thus, by doubling the external impedance from $\frac{1}{5}$ to $\frac{2}{5}$ of the internal impedance to current is only reduced from A B to C D, fig. 11·47. This principle is employed in constant-current transformers used with the series system which is useful in certain special cases. Fig. 11·48 shows diagrammatically a transformer in which the leakage is made very great by putting the coils on separate limbs and providing a special path for the leakage flux. The regulating effect can be enhanced by allowing the coils, as the current increases, to separate under the repulsive action of their oppositely directed currents. Fig. 11·49 shows a transformer in which the secondary is divided into two portions suspended by chains and partly balanced by weights. An increase in the secondary currents causes them

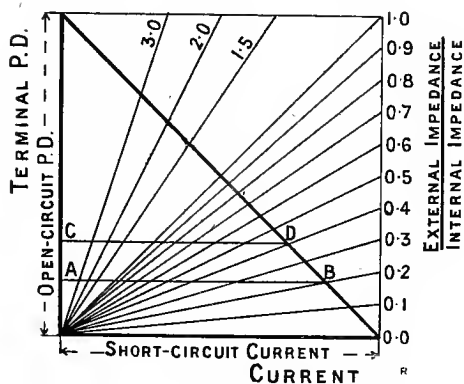


FIG. 11'47. — Variation of P.D. of Transformer with Large Magnetic Leakage.

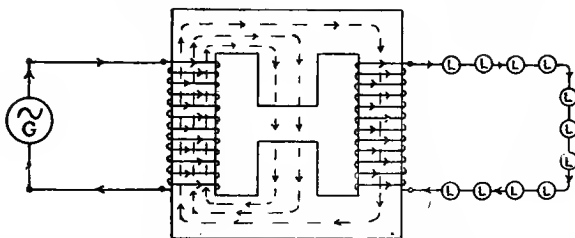


FIG. 11'48. — Constant-Current Transformer.

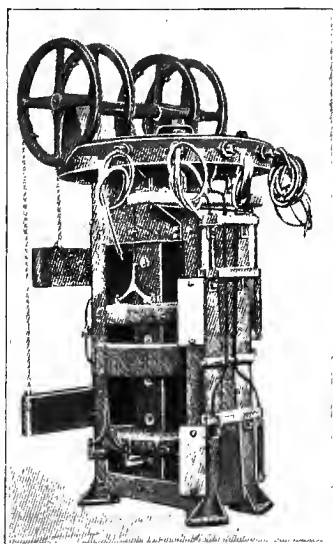


FIG. 11'49. — Constant-Current Transformer with Sliding Coils (Westinghouse).

to move farther apart and augments the primary leakage flux to the side yokes.

Potential Transformers.—For safety in high-voltage stations the instruments are worked at low voltage and coupled to the circuit inductively by transformers. For the voltmeters (see fig. 11·50) it is sufficient if the transformation ratio be fairly constant, but for station working, where the P.D. to be measured is always about the same, even this is not absolutely necessary, as the instrument and transformer may be calibrated together. When, however, the transformer is to be used with a wattmeter or power-factor indicator, it is of the utmost importance that the secondary current be in phase with the primary P.D. To secure this end, the voltage drop of the transformer must be as small as possible. The coils should therefore be wound concentrically and run at a low-current density. In other words, the

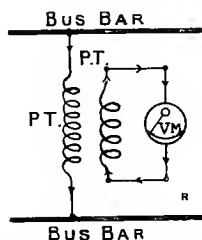


FIG. 11·50.—Connections of Voltmeter Transformer.

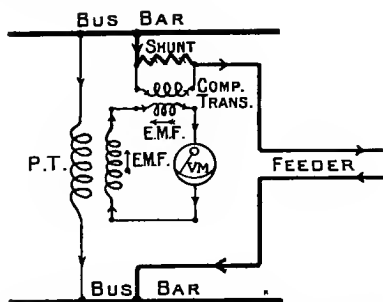


FIG. 11·51 —Compensating Transformer to make Voltmeter indicate P.D. between Far Ends of Feeder.

transformer is to be used under light-load conditions. Potential transformers are generally wound to give about 100 volts for the voltmeters and wattmeters, but the instruments are generally marked so as to indicate the values in the H.V. circuit.

The voltmeter can be made to indicate the P.D. at the far end of a feeder without using pilot wires by connecting a compensating transformer (see fig. 11·51) to subtract an E.M.F. proportional to the impedance E.M.F. of the feeder. The P.D. between the ends of the primary of this transformer is made proportional to the current flowing by shunting it by an impedance through which the feeder current passes. The impedance of the shunt and the ratio of the compensating transformer must be such as to make the ratio of the secondary E.M.F. to the voltage drop in the feeder the same as the ratio of transformation of the potential transformer.

Current Transformers.—The action of a current transformer depends on the fact that the secondary excitation of a transformer on closed circuit

is nearly equal, but opposite, to that of the primary, and that consequently the currents are in the inverse ratio of the numbers of turns. For this to be true, the exciting current must be negligible; the secondary should therefore be run as nearly under short-circuit conditions as practicable, and the iron losses and reluctance must be kept very low. The greater the resistance of the ammeter circuit, the larger will be the flux required to induce the necessary E.M.F., and the greater will be the magnetising and iron-loss currents. Should the secondary become open-circuited, the flux will become excessive, since there will be no back excitation due to the secondary; there will be a large voltage rise in the secondary, and the iron losses due to the excessive flux density may cause a burn-out of the transformer.

The number of turns is generally such as will make the secondary

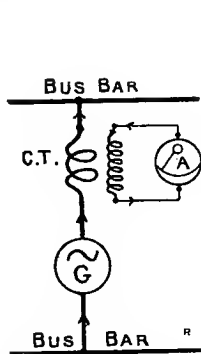


FIG. 11:52.—Connections of Current Transformer.

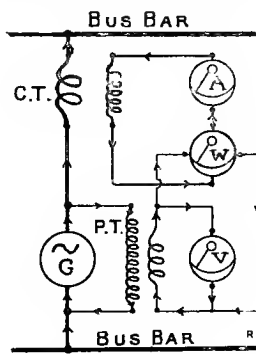


FIG. 11:53.—Connections of Transformers for Ammeter, Voltmeter, and Wattmeter in Single-Phase Circuit.

current 1–10 amperes at full load, depending on the indicating instruments selected. If, with a given winding space, the number of secondary turns be increased m times, its resistance will be m^2 times and the current $\frac{1}{m}$ times what they were before. If, at the same time, the instrument be altered in the same way, the effect on the moving system will be the same as before. The E.M.F. required in each turn is changed in the ratio $\left(m^2 \times \frac{1}{m}\right) \div m$, and is thus, provided the leads have a negligible resistance, exactly what it was before. In other words, the same flux is required in both cases. Thus, the actual number of turns on the secondary is of little moment when suitable instruments are chosen. On the one hand, the resistance of the connecting wires produces less effect with the larger number of turns; but, on the other, the resistances of the coils will increase rather more rapidly than

in proportion to the square of the number of turns, owing to the lower space factor with smaller wire.

Fig. 11·53 shows the connections for measuring the current P.D. and power of a single-phase circuit.

Schüler's Method of Connecting Instruments.—Another way of connecting the instruments has been devised by Schüler and used by the Lahmeyer Company. One coil in each phase of the generator is not connected conductively to the main circuit, but is coupled to it through a transformer of unity ratio. Neglecting the exciting currents and the losses of these transformers, the currents in their primaries are the same as in the main circuit, and their P.D.'s a definite fraction of the bus-bar P.D. Further, one point of the primary circuit can be earthed without affecting the main circuit. Consequently the ammeters, voltmeters, and wattmeters may be

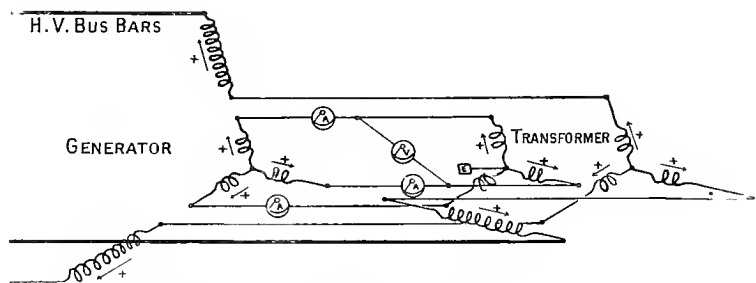


FIG. 11·54.—Schüler's Method of Connecting the Instruments to a Three-Phase Generator.

connected to the auxiliary circuit. It is shown applied to a three-phase system in fig. 11·54.

Sumpner's Quadrature Current-Transformer.—We have seen that a current transformer must have a small reluctance, and be used under nearly short-circuit conditions, so as to make the magnetising current negligible. Going to the other extreme, if there be a high resistance in the secondary circuit, and a high reluctance requiring a large magnetising current, the load excitation will be negligible compared with the primary excitation. If, further, the reluctance be constant, the flux will be at every instant proportional to the current, and the induced E.M.F. in the secondary proportional to the rate of change of current; with a non-inductive secondary circuit, the secondary current will thus be proportional to the primary current, but in quadrature with it. The reluctance of the transformer may be made sensibly constant by having an air gap in the path of the flux sufficiently long to make the variation of that of the iron path of no importance.

This is the principle of the "quadrature transformer" invented by Sumpner* for use with an iron-cored wattmeter whose moving coil is supplied by the quadrature transformer, and whose magnet coils are connected as a shunt to the mains, either directly or through a potential transformer. Neglecting the resistance E.M.F., the P.D. has to be balanced by the change of flux; the flux is therefore in quadrature with it, and consequently the current in the moving coil must also be in quadrature with the main current if the torque is to be a measure of the power. Hence the necessity for the quadrature transformer.

The fact that the excitation of the secondary is negligible makes the rise in flux and voltage on open-circuiting it quite inappreciable, and therefore

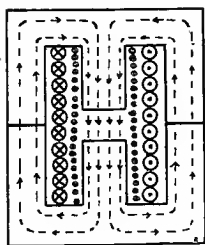


FIG. 11'55.—Quadrature Current-Transformer (Sumpner).

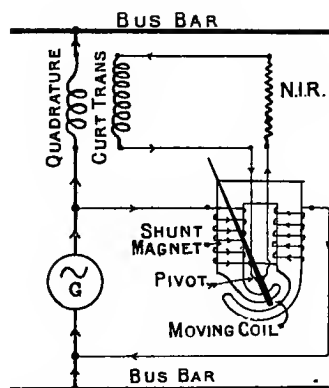


FIG. 11'56.—Connections of Shunt-Magnet Wattmeter and Quadrature Transformer (Sumpner).

avoids a danger of burn-out which is present with an ordinary current transformer. It is therefore possible to employ one indicating instrument for a number of different circuits, by giving each its own quadrature transformer and connecting the desired one to the moving coil of the wattmeter by means of a multiple-way two-pole switch. As the resistance of the moving coil circuit is high and the current small, troubles do not readily arise from the resistance of the switch contacts.

Scott's Two-Phase to Three-Phase Connection.—A D, D B are the two halves of the secondary of a transformer whose primary is excited from one phase of a two-phase system, while D C is the secondary of another excited from the other phase. The transformation ratios are such as to

* *Journ. I.E.E.*, vol. xli. p. 231, 19th March 1908; *Electrician*, vol. lx. p. 875, 20th March 1908.

make V_{AB} equal to the three-phase P.D. required, and

$$V_{DC} = \sqrt{3}V_{DA} = \frac{\sqrt{3}}{2}V_{AB} \quad . \quad . \quad . \quad 11\cdot09.$$

The P.D.'s between the terminals A, B, C, taken in pairs, are

$$\left. \begin{aligned} V_{AB} &= E_{DA} - E_{DB} \\ V_{BC} &= E_{DB} - E_{DC} \\ V_{CA} &= E_{DC} - E_{DA} \end{aligned} \right\} \quad . \quad . \quad . \quad 11\cdot10.$$

These are shown vectorially in fig. 11·58, from which it may be seen that

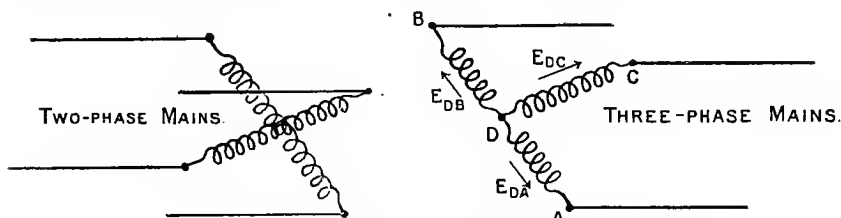


FIG. 11·57.—Scott's Two-Phase to Three-Phase Connection.

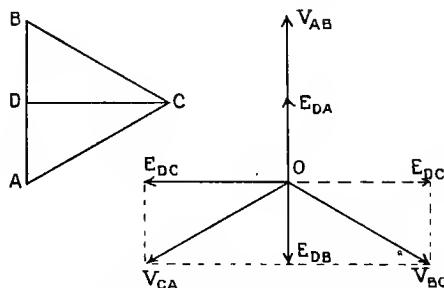


FIG. 11·58.—Vector Diagrams for Scott's Two-Phase to Three-Phase Connection.

these voltages are all equal, but differ in phase by $\frac{1}{3}$ period. This arrangement, which is due to F. C. Scott, therefore enables us to obtain three-phase currents from two-phase, or *vice versa*, or to link together a two-phase and a three-phase system of the same frequency. It is only in exceptional circumstances that a practical use for this arrangement arises.

This connection can also be made by means of a three-phase transformer instead of a two-phase one. The three-phase side may be a symmetrical star or mesh; the other coil on one limb is wound for the full two-phase voltage, while those on the two remaining limbs are designed for $\frac{1}{\sqrt{3}}$ times that

voltage. The last two are joined in opposition to give the other phase of the two-phase side. It is easily seen from fig. 11'58 that $V_{CA} - V_{BC}$ is along E_{DC} , which is at right angles to V_{AB} , and that with the ratio of turns mentioned it will also be equal to V_{AB} .

Steinmetz' Monocyclic System.—The Steinmetz monocyclic system is essentially the same in principle as Scott's, the secondary connections and voltages being identical. Instead, however, of having a regular two-phase generator, the second phase is wound for only one-fourth the voltage of the other, and it is connected between the middle point of the other and a third main. The primaries of the transformers are connected in the same way as the windings of the generators. The idea was to use both phases when starting induction motors, and then to switch off the auxiliary one and leave the motor running as a single-phase machine. The system has no advantages over the ordinary three-phase system, while the generators and motors have most of the disadvantages of single-phase machines.

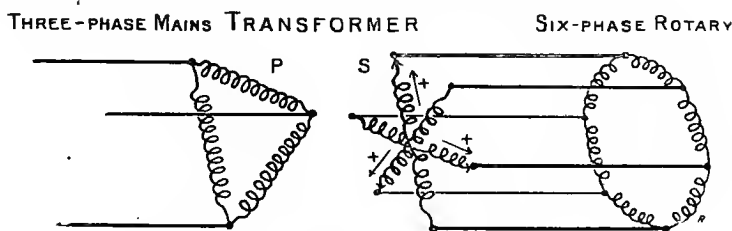


FIG. 11'59.—Three-Phase to Six-Phase Connection.

Three-Phase to Six- and Twelve-Phase Transformations.—

The more phases employed in a rotary converter, the smaller will be the heating for a given output, or the greater the output for a given heating. In order to take advantage of this fact, the number of phases is often raised from three to six, or even twelve, before supplying the current to the converters. Six phases may be obtained from three by simply winding the secondaries of the transformers for about $\frac{1}{\sqrt{2}}$ of the D.C. voltage, and

joining the ends of each to a pair of rings of opposite phase, as in fig. 11'59. The middle points of the secondaries may be brought together to form a neutral point for connection to the middle wire of the D.C. system when there is one. The arrangement then forms a six-rayed star, in which form it is shown in fig. 11'60, and vectorially in 11'61. The circle in the latter represents the vector polygon for the different elements of the rotor winding. Three single-phase transformers may be employed, or one three-phase transformer.

Mesh connections may be used if the two halves of the secondary be separated and connected to symmetrical pairs of tappings, which may be either $\frac{1}{3}$ cycle or $\frac{1}{6}$ cycle apart. The voltage of each coil of the latter will be the same as that of each ray of the six-phase star ($\frac{1}{2\sqrt{2}}$ times the D.C. voltage), while that of the former will be $\sqrt{3}$ times as much ($\frac{\sqrt{3}}{2\sqrt{2}}$ times the D.C. voltage.) These are shown in figs. 11·62–11·65.

THREE PHASE MAINS. TRANSFORMER. SIX-PHASE ROTARY.

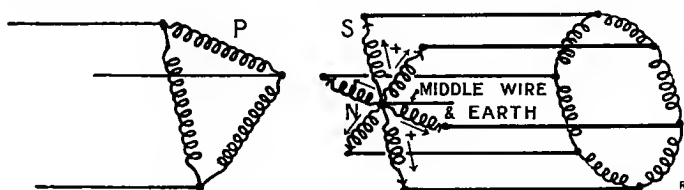


FIG. 11·60.—Connections.

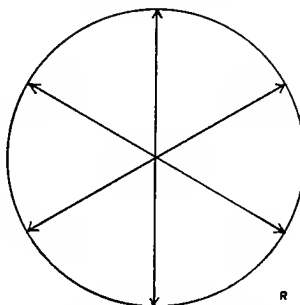


FIG. 11·61.—Vector Diagram.

Three-Phase to Six-Phase Star Connection.

The $\frac{1}{6}$ -phase mesh gives the same points as the six-rayed star, but the $\frac{1}{3}$ -phase mesh gives points midway between. Hence twelve phases may be obtained by superposing the double $\frac{1}{3}$ -phase mesh on the six-phase star, as in figs. 11·66 and 11·67. This requires three secondaries on each limb, or on each transformer when separate ones are employed, two being for $\frac{\sqrt{3}}{2\sqrt{2}}$ ($=0·612$) and the other for $\frac{1}{\sqrt{2}}$ ($=0·707$) times the D.C. voltage required. The middle points of the latter may be joined to form a neutral point for the middle wire if required. Coils in the same limb are shown parallel to one another in the diagrams. The $\frac{1}{6}$ -phase mesh could be used instead of the

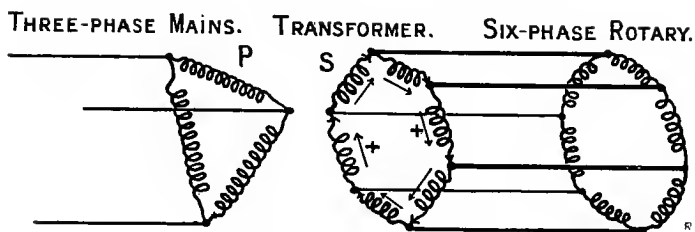


FIG. 11'62.—Connections.

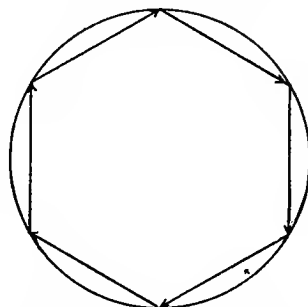


FIG. 11'63.—Vector Diagram.

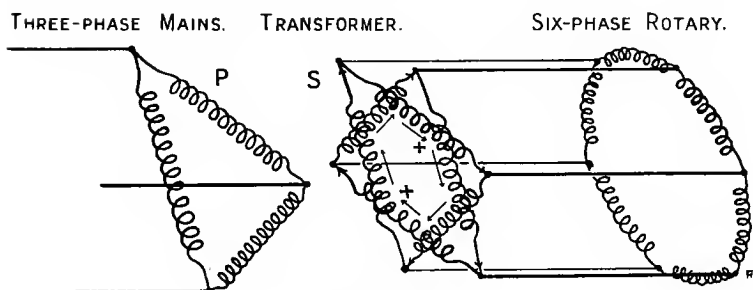
Three-Phase to Six-Phase Mesh Connection. $\frac{1}{6}$ -Phase.

FIG. 11'64.—Connections.

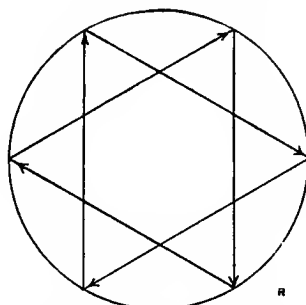


FIG. 11'65.—Vector Diagram.

Three-Phase to Six-Phase Mesh Connection. $\frac{1}{3}$ Phase.

six-rayed star, and star-connected primaries might in all cases be used instead of mesh.

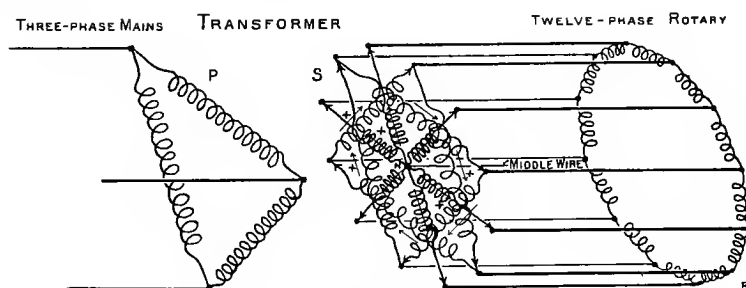


FIG. 11'66.—Connections.

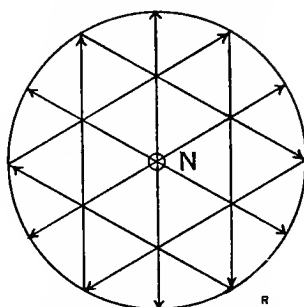


FIG. 11'67.—Vector Diagram.

Three-Phase to Twelve-Phase Connection.

Polyphase to Single-Phase Connection.—Any pair of terminals of a polyphase system will, of course, give an alternating P.D. to which single-phase apparatus may be connected ; but if it is required to distribute the load between the phases, so as not to throw the system out of balance, all the secondaries of the polyphase transformer, or equivalent bank of transformers, must be joined in series in such a way as to give a resultant E.M.F. With a two-phase supply, as may readily be seen from the vector diagram fig. 11'69, the resultant P.D. is $\sqrt{2}$ times that of one phase, whichever way the connection is made, and its phase differs by $\frac{1}{8}$ period from either. For the same output the current will only be $\frac{1}{\sqrt{2}}$ times that with one phase only loaded ; it goes through twice the resistance, and consequently the resistance loss is the same.

In the case of a three-phase supply, one phase must be reversed as compared with an ordinary mesh connection ; the resultant P.D. will be double that of one phase, and in the same phase as one of them. Comparing

together one phase, two in series, and three in series, we have

Ratio of voltages and outputs with same current	$1 : \sqrt{3} : 2$	} 11·11.
Ratio of copper losses with same current	$1 : 2 : 3$	
Ratio of output	$1 : \frac{2}{3} : \frac{3}{4}$	

The converse transformation, obtaining polyphase currents from single-

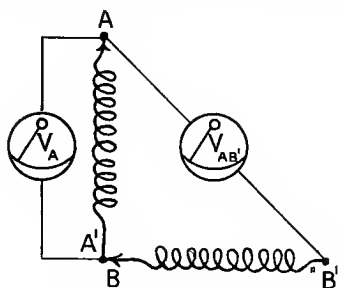


FIG. 11·68.—Connections.

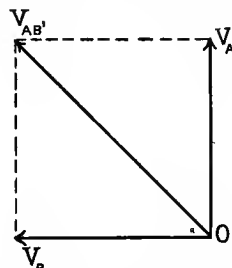


FIG. 11·69.—Vector Diagram.

Two Phases in Series for Single Phase.

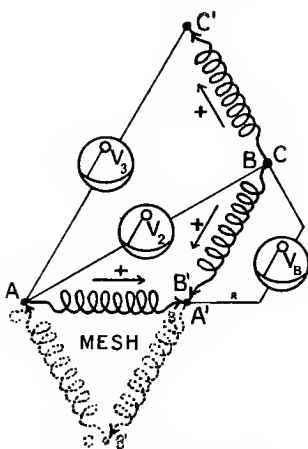


FIG. 11·70.—Connections.

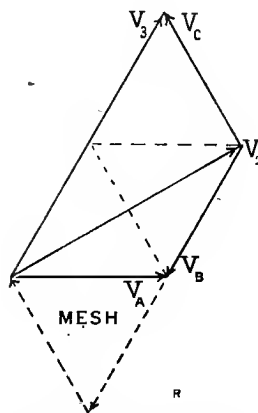


FIG. 11·71.—Vector Diagram.

Obtaining One Phase from Three.

phase, cannot be done by static transformers, but requires a polyphase motor wound for the desired number of phases. One of these phases receives current from the single-phase mains, and the others give out the necessary currents for the other phases. The motor may either be of the synchronous or of the asynchronous type, but is generally the latter, owing to its easy starting properties. It is usually a squirrel-cage induction motor, as originally proposed by Ferraris and Arno. Although not self-starting, unless

made into a two-phaser by having two circuits of different time constants, it only requires to be started by hand in either direction to give a torque in that direction which will run it up to speed if it is not loaded. So long as any such motor of sufficient power is connected to the mains, two- (or three-) phase currents are available for starting other motors under load. A full study of the action of such machines is, however, beyond the scope of the present work.

This arrangement compares unfavourably with a two- or three-phase system, but is sometimes of service when power is required for small motors at a place where only a single-phase supply is available.

Berry Series System.—Since the iron losses are the same whether the transformer is loaded or not, any arrangement which enables these to be reduced during times of light load possesses the obvious advantage of saving some energy, and also the further, and really much more important, one, that the transformer is able to cool down during the light-load period, and is consequently able to take a much greater load for a short time during the peak than it otherwise could, besides having a lower resistance due to its lower temperature.

A very important method of doing this, invented by Berry, is to run a large and a small transformer in series on both sides, the ratings of the transformers being in the ratio of, say, 10:1, and to provide an automatic switch to short-circuit the smaller one on both sides when the load gets too great for it. The system is shown diagrammatically in fig. 11·72, and a view of the switch and relay are given in fig. 11·73. The large transformer requires a larger exciting current than the small one, but when in series the same current has to do for both. Consequently the small transformer takes the greater part of the total voltage and does most of the work. The actual division between them depends on the design of the transformers, but is roughly about 20:1. Since the losses are proportional to the square of the current and approximately to the square of the voltage, those in the large transformer will be practically negligible, and it is able to cool down. But before the load gets too large for it the small one is short-circuited by the switches, and then the big one has to take the whole of the load.

The switch is controlled by a relay A, whose coil is in the primary circuit. When the current is insufficient to hold down its lever against the counterpoise, the relay makes connection through the upper contact to the pull-off coil of the switch, which is thus pulled off if it be on. In its motion it breaks the circuit of the pull-off coil, but makes that of the pull-on coil ready for the next action of the relay. When the current exceeds the predetermined limit the relay lever moves down and touches the lower contact, when the

switch is pulled on. In its motion, it now breaks the pull-on circuit and

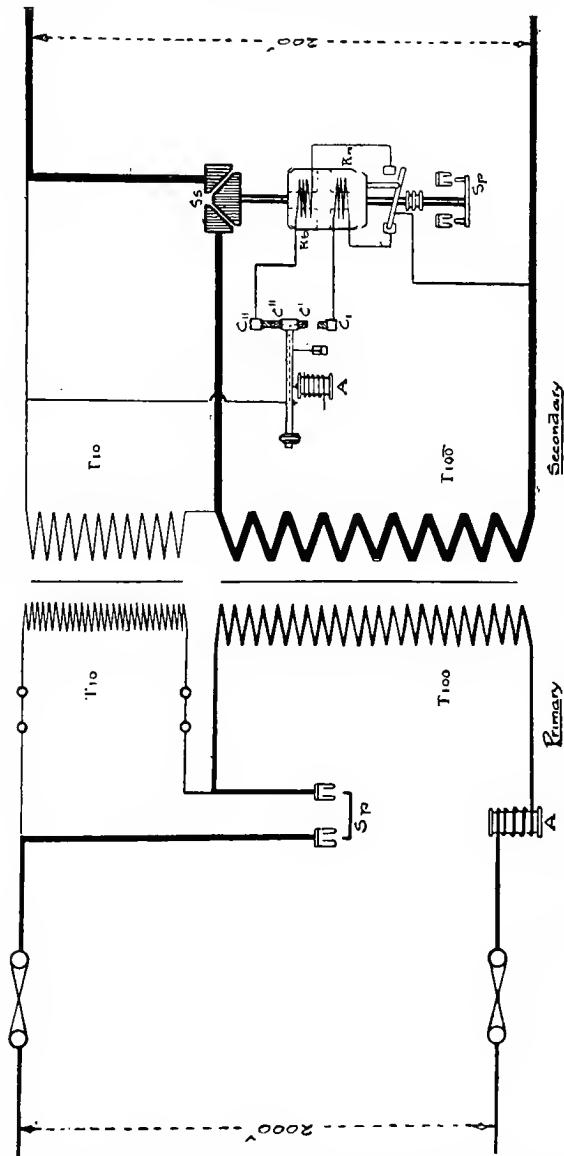


Fig. 11-72.—Berry Series System. (British Electric Transformer Company, Ltd.)

makes the pull-off one. The switch is held in either position by a spring toggle, and a dash-pot is attached to the relay lever to prevent hunting.

In the diagram the primary switch S_p and the relay coil A are shown twice: once beside the mechanism they control, and again where their

electrical connections can be more easily followed. In the photograph the relay and transformer fuses are shown to the left, and the switch to the right, the high-voltage part of the latter being inside the small inner compartment marked "Danger."

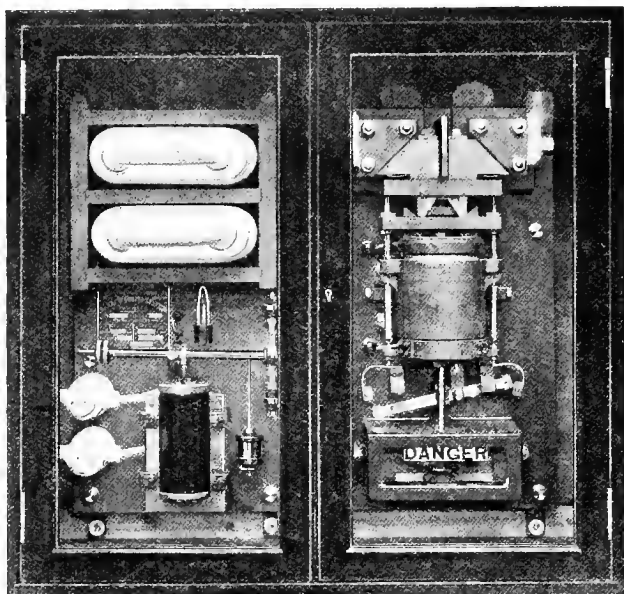


FIG. 11-73.—Relay and Switch for Berry Series System.
(British Electric Transformer Company, Ltd.)

Brockie-Pell Auto-Transformer Switch.—Small auto-transformers have been installed to a considerable extent of late with the object of allowing the use of very low voltages for tungsten lamps, and these are of course out of use for a large part of the total time. Quite a number of switches have been devised for cutting them out when the last lamp is put out, and for cutting them in again when one is put on. Most of them are worked in conjunction with a battery and relay; and this is, indeed, necessary when the two windings of the transformer are not connected electrically. The one shown in figs. 11-74 and 11-75 uses the lamp current itself to work the switch, and can therefore only be used with auto-transformers.

Suppose one lamp to be switched on when the auto-switch is off. Current flows as shown by the arrows through the pull-on and hold-on coils, the lamp, part of the transformer winding, and a resistance sufficient to prevent the current, which has the full voltage behind it, from being large enough to damage the lamp. The switch is pulled on against the counterpoise weights,

and completes the primary circuit of the transformer. At the same time it short-circuits the pull-on coil, but the lever is still held down by the hold-on coils. When the lamp is switched off the weights pull the lever back to the off position, and so cut off the exciting current. In the larger sizes the hold-on coil is divided, and one winding is wound with fewer turns of thicker

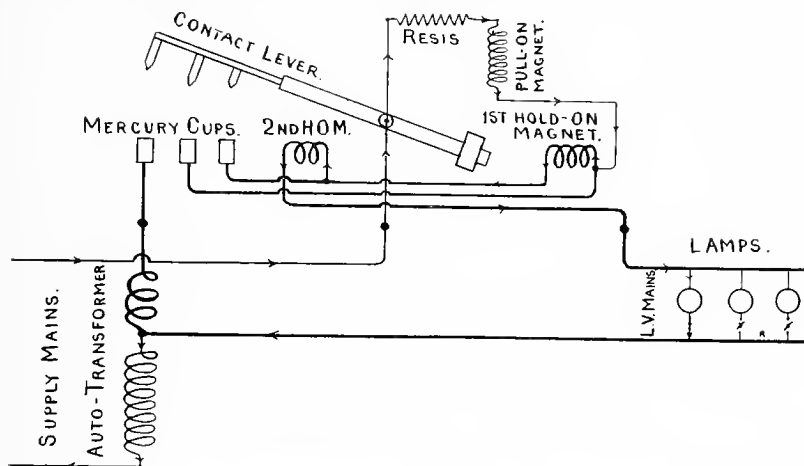


FIG. 11-74.—Connections of Brockie-Pell Automatic Switch for Auto-Transformers.

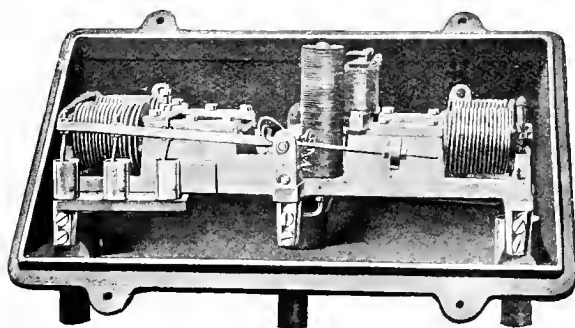


FIG. 11-75.—Automatic Switch for Auto-Transformers.
(Brockie-Pell Arc Lamp Company.)

wire than the other. Above a certain load that other is short-circuited, and the thick wire one holds down the lever. The reason for having so many coils is the difficulty of making one which has enough turns to work with the current of a single lamp and still run cool enough with full load. The pull-on coil has to operate the switch with the current taken by the smallest lamp on the system, and must therefore have a considerable number of turns. The first hold-on coil has to hold the lever down with the same current, but

does not have to do the work of moving it nor to act through a considerable air-gap. It does not therefore need anything like as many turns as the pull-on coil, and so may be made of thick enough wire to carry the current of several lamps. But when its thermal limit is reached, it has also to be short-circuited while the second hold-on coil takes up the duty of keeping the switch on. The voltage absorbed by the induced E.M.F. in the coils is also an important factor, as it sets a limit to the number of turns which is absent in direct current magnets.

CHAPTER XII.

POLYCYCLIC SYSTEMS OF CURRENT DISTRIBUTION.

Lighting and Power Loads.—If the energy is to be used solely for power purposes a low frequency is preferable, since the construction of motors and rotary converters is more rational for frequencies under than for those over 40 cycles per second. For lighting, however, the frequency should be at least 50 per second ; stroboscopic effects are annoying with lower frequencies, especially with arc lamps, which also burn unsatisfactorily. For mixed loads a compromise is usually made by choosing a frequency of about 50 cycles per second, which is ideal neither for lighting nor for power.

Polyphase generators, motors, and rotaries are superior in starting and running qualities to single-phase machines, and they are less expensive to manufacture. On the other hand, the regulation of the voltage is easier and simpler for single-phase than for polyphase currents, but a greater voltage drop is permissible in a circuit supplying motors only than in one which does lighting in addition or alone, owing to the comparatively great sensibility of lamps to fluctuations of P.D. With a mixed lighting and power load, the allowable drop is soon reached, and the size of mains required is greater than for an equal power load.

It is, of course, possible to have entirely separate plants for light and power, each with the most suitable frequency and voltage ; but the advantage so gained would nothing like counterbalance the extra capital cost and complication caused by the necessary duplication of the plant, feeders, transformers, and distributing mains, especially as each is incapable of assisting the other.

Another solution of this problem is found in a system which transmits currents of different frequencies and phases through the same network at the same time. In such a system, which is known as a "polycyclic" system, each current will, on the whole, only perform work with the E.M.F. of its own frequency, and not with the E.M.F. of the other frequency. The total

power absorbed by the circuit is the sum of the powers of the two currents, and the copper loss is also the sum of the losses caused by each separately.

Bedell's Polycyclic System.—The authors believe that the first polycyclic system was invented by Dr F. Bedell, whose connection for a three-

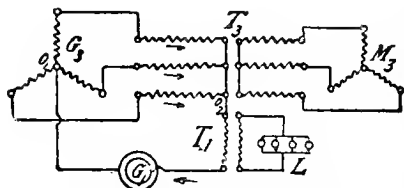


FIG. 12'01.—Bedell's Polycyclic System.

phase single-phase system is shown in fig. 12'01. Here G_3 and G_1 represent respectively three-phase and single-phase generators; T_3 and T_1 three- and single-phase transformers; M_3 a three-phase motor; and L a lighting circuit. The single-phase currents, which alone are marked on the diagram, enter the three-phase system at the neutral point of the generator, flow through the three-phase mains in parallel, leave at the neutral point of the power transformer, go through the lighting transformer, and then return by a fourth wire, which must be large enough to carry the whole single-phase current. The same principle is applicable to a star connection of any number of rays from two (the ordinary single-phase divided into two parts) upwards. Bedell's system has the disadvantage that the total, and not merely the leakage, inductance of the main generator and transformers is effective for the superposed single-phase currents, and that consequently the voltage drop is enormous.

Arnold, Bragstad, and La Cour's Systems.—The disadvantage just mentioned may be obviated by winding the circuits non-inductively for the superposed current without interfering with the inductive effect of the original current. Consider the choking coil shown in fig. 12'02, and assume that the original current enters at b_1 and leaves at b_2 , while the superposed current enters at these two points and leaves at e . The superposed current will give no inductive action, for the excitation caused by one coil is neutralised by that of the other on the same limb, but the inductive actions of the original current are added. Such bifilar windings may be given to all kinds of polyphase transformers and generators; fig. 12'03 shows one for a three-phase transformer, and fig. 12'04 for a three-phase generator.

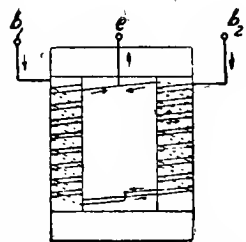


FIG. 12'02.—Bifilar Choking Coil (Arnold, Bragstad, and La Cour).

Arnold, Bragstad, and La Cour, the inventors of this system, have further shown that the simultaneous supply of alternating currents of different frequencies and phases may be accomplished by employing transformers with

two kinds of primary and one kind of secondary winding, and they may be separated again by means of transformers with one primary and two secondaries. In fig. 12·05, A represents the sending and B the receiving station. In the former, two kinds of currents are generated and transformed, and then transmitted by the same mains to the latter, from which two net-

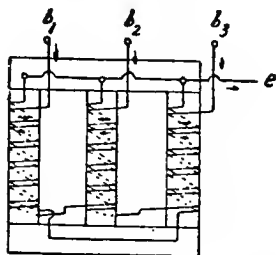


FIG. 12·03. —Bifilar Three-Phase Transformer Winding (Arnold, Bragstad, and La Cour).

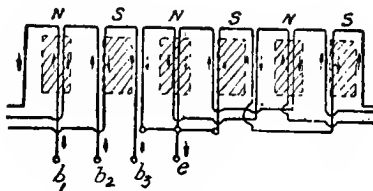


FIG. 12·04. —Bifilar Three-Phase Generator Winding (Arnold, Bragstad, and La Cour).

works are supplied. The sending transformers have each one primary for the corresponding phase of the polyphase system, one for the single-phase system, and one secondary. The former primaries are connected in star form, and the latter all in series. In this way, both kinds of current are induced in the secondary winding, and the single-phase current is carried along with the three-phase currents, but returns by a fourth wire. The

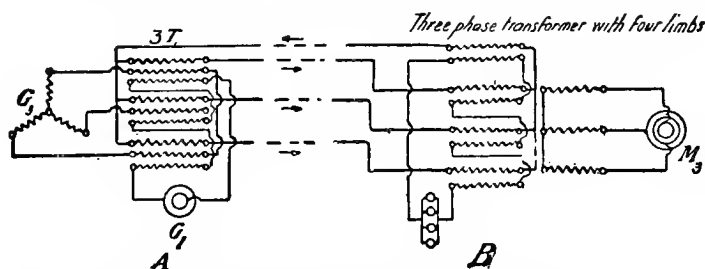


FIG. 12·05. —Three-Phase Polycyclic System (Arnold, Bragstad, and La Cour)

E.M.F.'s induced in one primary by the currents in the other cancel out if the three-phase system be balanced, which will generally be the case, as only three-phase apparatus would be connected to it. The receiving transformers are arranged in a similar manner, but with one primary and two secondaries.

Instead of a bank of single-phase transformers, we may employ one three-phase transformer, but it must have a common magnetic return, like fig. 1·09, or be one of the tandem types of figs. 10·55, 10·57, 10·59, or 10·61,

to provide for the flux produced by the single-phase current, as in the three-limb type that flux passes through the three columns in the same direction.

The system may be modified to that of fig. 12·06, so as to make all the mains be used by both systems, instead of having a special return wire for

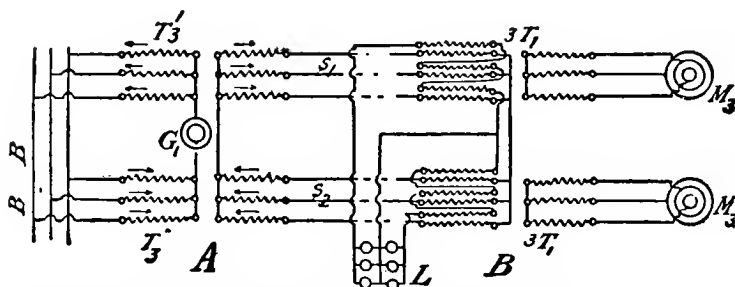


FIG. 12·06.—Three-Phase Polycyclic System without Special Return (Arnold, Bragstad, and La Cour).

the single-phase current. This advantage is, however, gained at the expense of greater complexity.

Instead of keeping them quite separate, one wire may be used in common for both distribution systems, as in fig. 12·07. With this arrangement, the P.D. of one system is not quite independent of the load on the other, as the voltage drop on the common wire affects them both, no matter which is loaded. This effect is, however, small.

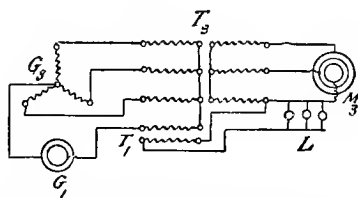


FIG. 12·07.—Three-Phase Polycyclic System with Common Distributing Main (Arnold, Bragstad, and La Cour).

Four-Phase Polycyclic Systems.—Figs. 12·08 to 12·10 show three different ways of superposing single-phase currents on a four-phase (commonly called a two-phase) system. In the first, what would be the neutral point of the four-phase transformer is split, and the single-phase generator is connected between the two parts; thus the polyphase generator does not send any current through the single-phase machine so long as the system is balanced. The primary superposed current flows through the transformer coils in parallel, then through the two opposite phases of the generator to the

neutral point, and back in a similar manner. In order that the four-phase generator may have little effective inductance for the superposed single-phase current, these coils of opposite phases should be wound in the same slots and connected so that the superposed current flows through them in opposite ways. They then form the bifilar arrangement previously mentioned. Since the motion E.M.F. is in the same way in two conductors in one slot, the superposed current at any instant is flowing in the same way as the original current in one phase and in the opposite way in the other. Consequently the transformers for each of the four phases must be quite separate, or at least have separate limbs and a common return; if wound on one limb, the inductive effect of either the original current or of the superposed one would cancel out.

The separation is done by four transformers, each with two secondary

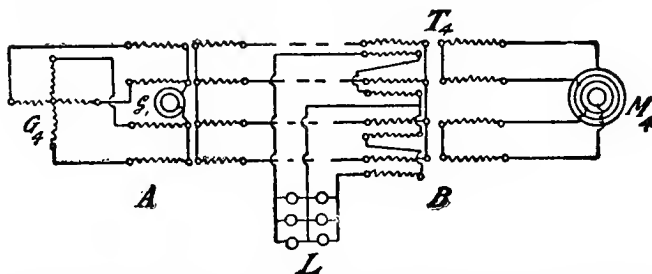


FIG. 12·08.—Two-Phase Polycyclic System (Arnold, Bragstad, and La Cour).

windings, in the way already explained. The lighting load is shown divided to form a three-wire system; an auto-balancer is then necessary in addition, for, owing to the fact that the primaries of the top pair are in series with those of the lower pair, the transformer will supply equal currents to the two sides. This could be improved by interchanging the two middle secondaries so that each side of the lighting system includes a secondary in one of the “go” and in one of the “return” transformers. Even then the balance would be bad without an external balancer, for the inductance of the polyphase generator is only neutralised with equal currents in all the coils.

In fig. 12·09 there are no step-up transformers, but the neutral point of the polyphase generator is divided for the introduction of the single-phase current; further, two secondaries and the corresponding mains on the distributing system are made common to both currents. In this case, if the middle wire of the lighting system is to take the difference of the currents in the two sides like an ordinary three-wire system and not their sum, the two phases of all the polyphase apparatus must be kept quite separate from

one another. Consequently four-phase mesh connections cannot be employed, but only four-phase star with the neutral point divided as in the generator. Otherwise, they would receive the single-phase current as well as their own. In this case also, the lighting system would not balance satisfactorily without an external balancer.

In fig. 12·10 the two currents are separately transformed, and the lighting E.M.F. introduced between two mains of the two-phase system, thus doing away with the necessity of putting down extra wires. The arrangement can be made symmetrical by adding another single-phase secondary and connecting it between the other pair of mains, between which lamps could also be connected. This system may be regarded as a two-phase one in which the two phases are kept entirely separate throughout the polyphase apparatus. The single-phase apparatus is then joined between the two phases in such a way that the polyphase windings are bifilar for the single-

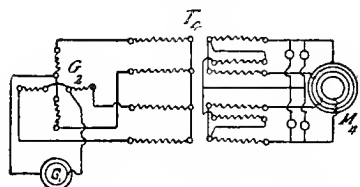


FIG. 12·09.—Two-Phase Polycyclic System with Two Common Distributing Mains (Arnold, Bragstad, and La Cour).

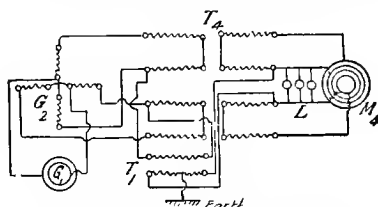


FIG. 12·10.—Two-Phase Polycyclic System without Special Mains.

phase current which flows through the two wires of one phase, in parallel. There are various modifications, which are shown in the diagram. Both windings of the single-phase transformer may be connected between the divided neutral point in the way shown for the generator, or the generator might be connected between two mains of unconnected phases as is shown for the transformer. To make the latter way symmetrical so as to use all the wires for both currents a second generator or winding on the same generator would have to be added between the other pair of mains, consequently the neutral point connection is simpler and is preferable for the generator. The primary of the transformer must be connected in the same way as the generator, and not in the other way as in the figure. The second connection with two rather than one secondary connected to the divided neutral secondaries is best for the distributing system if all the mains are to be used for lighting (in two pairs), for it makes the two sides of the lighting independent of one another. The two coils carrying the current for one phase of the two-phase system must be wound on the same limb, or

limbs, and not on separate transformers, in order that they may be bifilar for the single-phase current.

Simultaneous Generation of Two Currents of Different Frequencies.—In order to generate independent E.M.F.'s of different frequencies in the same generator, we require a machine with two independent field systems having numbers of poles in the ratio of the desired frequencies. If these are in the ratio 3 : 1, and are arranged as in fig. 12·11, the E.M.F. wave is peaky, like fig. 3·20 ; but if the polarity of one magnet system is reversed, as in fig. 12·12, the E.M.F. wave is flattened, as in fig. 3·21. The relative merits of these different wave forms have already been discussed in Chapter III. in connection with these figures.



FIG. 12·11.—Polycyclic Generator giving Peaky Wave.

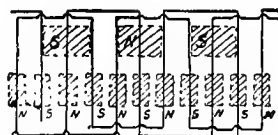


FIG. 12·12.—Polycyclic Generator giving Flattened Wave.

Potentials of Polycyclic Mains.—The mean square of two superposed sine quantities of different frequencies taken over a whole number of both cycles is simply the sum of their separate mean squares ; for

$$\begin{aligned} & \{ \bar{V}_1 \sin(\omega_1 T - \theta_1) + \bar{V}_2 \sin(\omega_2 T - \theta_2) + \bar{V}_3 \sin(\omega_3 T - \theta_3) + \text{etc.} \}^2 \\ &= \bar{V}_1^2 \sin^2(\omega_1 T - \theta_1) + \bar{V}_2^2 \sin^2(\omega_2 T - \theta_2) + \bar{V}_3 \sin(\omega_3 T - \theta_3) + \text{etc.} + \text{a number} \\ & \quad \text{of terms, each of which contains the product of two sines of different} \\ & \quad \text{frequencies, a typical one being} \\ & 2\bar{V}_1 \bar{V}_2 \sin(\omega_1 T - \theta_1) \sin(\omega_2 T - \theta_2) \\ &= \bar{V}_1 \bar{V}_2 [\cos\{(\omega_1 - \omega_2)T - (\theta_1 - \theta_2)\} - \cos\{(\omega_1 + \omega_2)T - (\theta_1 + \theta_2)\}]. \end{aligned}$$

Each of these is a sine function, and therefore its mean value over a complete multiple of its period is zero. Consequently, in getting the mean square of the original expression all the product terms vanish, and only the means of the square ones remain. Or, in R.M.S. values,

$$V^2 = V_1^2 + V_2^2 + V_3^2 + \text{etc.} \quad . \quad . \quad . \quad 12\cdot01,$$

no matter what is the phase relation of the different frequencies.

Since the superposed E.M.F. must be in series with the original one in order that one system may not form a short-circuit on the other, it follows from this principle that the mean square of the P.D. between any two points

on the polycyclic system is the sum of the mean squares of the P.D.'s produced by each separately. Or, in symbols,

$$V = \sqrt{V_1^2 + V_2^2}. \quad . \quad . \quad . \quad . \quad 12\cdot02 ;$$

where V_1 , V_2 , and V are the R.M.S. values of the P.D. between two given points, due to each system separately and both together. In figs. 12·01 and 12·07 it is the P.D. between the neutral point and the fourth wire that is altered, while in fig. 12·05 the neutral points at the sending and receiving ends are at different potentials. In fig. 12·06 the P.D. between a main in one group and one in another group, and also that between one or both groups and earth, are raised, while that between members of the same group is not affected. In the two-phase arrangements of figs. 12·08, 12·09, and 12·10 the voltage of each phase is unchanged, but the P.D. between the mains of different phases, and between some or all of them and earth, is raised.

Losses in Polycyclic Mains.—All cases can be reduced to that in which the currents are supplied over one wire and return by an earth of negligible impedence. For it will always be possible to replace any given system of transformers or loads by a star having one ray to each main and its neutral point at earth potential ; the star may not be symmetrical. The neutral point can be earthed without altering the current in any main or its potential. The whole system may then be treated as a number of independent ones, each working between one main and earth.

Consider, then, a main of resistance R carrying currents I_1 and I_2 of different frequencies and with potentials to earth V_1 and V_2 . The power transmitted at any instant is

$$\begin{aligned} V\dot{T} &= \{V_1 \sin \omega_1 T + V_2 \sin (\omega_2 T - \theta)\} \{I_1 \sin (\omega_1 T - \theta_1) + I_2 \sin (\omega_2 T - \theta - \theta_2)\} \\ &= V_1 I_1 \sin \omega_1 T \sin (\omega_1 T - \theta_1) + V_2 I_2 \sin (\omega_2 T - \theta) \sin (\omega_2 T - \theta - \theta_2) \\ &\quad + V_1 I_2 \sin \omega_1 T \sin (\omega_2 T - \theta - \theta_2) + V_2 I_1 \sin (\omega_2 T - \theta) \sin (\omega_1 T - \theta_1). \end{aligned}$$

$$\text{Now,} \quad \sin \omega_1 T \sin (\omega_1 T - \theta) = \frac{1}{2} \{ \cos \theta_1 - \cos (2\omega_1 T - \theta_1) \},$$

and its mean value taken over a complete number of periods is $\frac{1}{2} \cos \theta_1$. Similarly, the mean product of the two sines of the second frequency is $\frac{1}{2} \cos \theta_2$. But

$$\sin \omega_1 T \sin (\omega_2 T - \theta - \theta_2) = \frac{1}{2} \{ \cos (\omega_1 - \omega_2) T + \theta + \theta_2 \} - \cos (\omega_1 + \omega_2) T - \theta - \theta_2 \}.$$

These terms are both periodic, one having the difference and the other the sum of the given frequencies. Consequently the mean value over a complete number of their periods is zero. Hence the products involving

small in the particular case in which $V_1 I_2 = V_2 I_1$, which is that in which the impedances are the same for both circuits of the polycyclic system.

The advocates of the polycyclic system claim that it does reduce the transmission loss, but the above investigation shows that this is not so, even under the most favourable circumstances, when equal voltages are compared. The actual saving obtained with polycyclic systems, when any, is due to the increased R.M.S. value of the voltage between some of the wires and earth caused by the extra E.M.F., and not to the use of two frequencies. As far as the transmission losses are concerned, as good or better results could be obtained by simply raising the voltage of the original system to that which obtains with the polycyclic current.

The best use is made of the mains when the R.M.S. P.D. to earth is the same for all. In so far as the polycyclic system destroys the symmetry of the potentials to earth it is at a disadvantage compared with the simple sine current.

APPENDIX.

TABLE 1.—NUMERICAL COEFFICIENTS, ETC., AND THEIR SYMBOLS.

Name or Description.	Symbol.	Definition.	Units or Value.
Known numbers in equations	a, b, c, d, e		
Unknown numbers in equations	x, y, z		
Efficiency . .	η	Output \div input.	
Form-factor .	f, f	R.M.S. value \div mean value.	
Transformation ratio	q	Primary P.D. \div secondary P.D.	
Space-factor .	σ	Net section \div gross section.	
Number of active conductors	Z		
Base of Napierian logarithms	e	...	2.71828183.
Pi . . .	π	Circumference of circle \div diameter.	3.14159265.
Angle . . .	θ, ϕ	Arc \div radius.	Radian.
Rotation . .	N	...	Turn.
Solid angle .	Ω	Surface \div distance ² .	Radian ² .
Cost . . .	\mathcal{L}	...	(Various).
Cost per unit volume	c	Cost \div volume.	„
Specific cost ratio	m	$\frac{\text{Cost of copper space}}{\text{Cost of same volume of iron space}}$	
Leakage coefficient	ν	$\nu = N_2 \mathcal{L}_{T1} \div N_1 \mathcal{L}_M$.	

TABLE 2.—MECHANICAL QUANTITIES, THEIR SYMBOLS (GROTESQUE TYPE) AND UNITS.

Name.	Symbol.	Defining Equation.	Dimensions in Terms of L M T.	M.K.S. Units.	Value of M.K.S. in C.G.S. Units.
Length	L, l	Fundamental	$[L]$	Metre.	10^2 Centimetre.
Surface	S	$S = L_1 L_2$	$[L^2]$	Metre ² .	10^4 Centimetre ² .
Volume	V	$V = L_1 L_2 L_3$	$[L^3]$	Metre ³ .	10^6 Centimetre ³ .
Mass	M	Fundamental	$[M]$	Kilogram.	10^3 Gram.
Moment of inertia, or angular mass	K	$K = \Sigma M L^2$	$[L^2 M]$	Kg.-metre ² .	10^7 Gram.-cm. ²
Linear density	\dot{M}	$\dot{M} = M \div L$	$[L^{-1} M]$	Kg. per metre.	10^1 Gram per cm.
Surface density	\ddot{M}	$\ddot{M} = M \div S$	$[L^{-2} M]$	Kg. per metre ² .	10^{-1} Gram per cm. ²
Density	D	$D = M \div V = \ddot{M}$	$[L^{-3} M]$	Kg. per metre ³ .	10^{-3} Gram per cm. ³
Time	T	Fundamental	$[T]$	Second.	10^0 Second.
Frequency	f	$f = 1 \div T$	$[T^{-1}]$	Cycle per sec.	10^0 Cycle per sec.
Angular velocity	ω	$\omega = \theta \div T = \dot{\theta}$	$[T^{-1}]$	Radian per sec.	10^0 Radian per sec.
Rotational speed	n	$n = N \div T = \dot{N}$	$[T^{-1}]$	Revoln. per sec.	10^0 Revoln. per sec.
Angular acceleration	α	$\alpha = \omega \div T = \dot{\omega} = \ddot{\theta}$	$[T^{-2}]$	Radian per sec. ²	10^0 Radian per sec. ²
Rotational "	\dot{n}	$\dot{n} = n \div T = \dot{N}$	$[T^{-2}]$	Revoln. per sec. ²	10^0 Revoln. per sec. ²
Velocity	v	$v = L \div T = \dot{L}$	$[L T^{-1}]$	Metre per sec.	10^2 Cm. per sec.
Acceleration	a, g	$a = v \div T = \dot{v} = \ddot{L}$	$[L T^{-2}]$	Metre per sec. ²	10^2 Cm. per sec. ²
Force	F	$F = Ma = M\dot{v}$	$[L M T^{-2}]$	Joule per metre.	10^5 Dyne.
Pressure	p	$p = F \div S = \dot{F}$	$[L^{-1} M T^{-2}]$	Joule per metre ³ .	10^1 Dyne per cm. ²
Work—energy	W	$W = FL = \frac{1}{2} M v^2$	$[L^2 M T^{-2}]$	Joule.	10^7 Erg.
Power	P	$P = W \div t = \dot{W}$	$[L^2 M T^{-3}]$	Watt = joule per sec.	10^7 Erg per sec.
I over per unit surface	\dot{P}	$\dot{P} = P \div S$	$[M T^{-3}]$	Watt per metre ² .	10^3 Erg per sec.-cm. ²
Power per unit volume	\ddot{P}	$\ddot{P} = P \div V$	$[L^{-1} M T^{-3}]$	Watt per metre ³ .	10^1 Erg per sec.-cm. ³
Torque.	t	$t = W \div \theta$	$[L^2 M T^{-2}]$	Joule per radian.	10^7 Erg per radian.
Couple.	C	$C = FL$	$[L^2 M T^{-2}]$	"	10^7 Dyne-cm.

TABLE 3.—THERMAL QUANTITIES, THEIR SYMBOLS (ITALIC TYPE) AND UNITS.

Name.	Symbol.	Defining Equation.	Dimensions in Terms of $L M T$.	M.K.S.Cg. Units.	Value of M.K.S.Cg. in C.G.S.Cg. Units.
Temperature	t T (abs.)	Fundamental	$[T]$	Centigrade ($^{\circ}C.$).	10° Centigrade.
Temperature gradient	\dot{T}	$\dot{T} = (t_2 - t_1) \div L$	$[L^{-1} T]$	$^{\circ}C.$ per metre.	10^{-2} $^{\circ}C.$ per cm.
Heat	H	$H = W$	$[L^2 M T^{-2}]$	Joule.	10^7 Erg.
Thermal capacity	C	$H = C(t_2 - t_1)$	$[L^2 M T^{-2} T^{-1}]$	Joule per centigrade.	10^7 Erg per $^{\circ}C.$
Specific heat	S	$C = MS$	$[L^2 T^{-2} T^{-1}]$	Joule per kg. $^{\circ}C.$	10^4 Erg per gm. $^{\circ}C.$
Latent heat	L	$H = ML$	$[L^2 T^{-2}]$	Joule per kg.	10^4 Erg per gm.
Rate of gen. of heat	P	$P = H \div T = \dot{H}$	$[L^2 M T^{-3}]$	Watt.	10^7 Erg per sec.
Dissipation intensity	\dot{P}	$\dot{P} = P \div S$	$[M T^{-3}]$	Watt per metre ² .	10^3 (Erg p. sec.) p. cm ² .
Emissivity	A	$P = AS(t - t_c)$	$[M T^{-3} T^{-1}]$	Watt per metre ² . $^{\circ}C.$	10^3 $\frac{\text{Erg per sec.}}{\text{Cm.}^2 \times ^{\circ}C.}$
Thermal conductivity	K	$P = KTS$	$[L M T^{-3} T^{-1}]$	Watt per metre ² $^{\circ}C.$ per metre	10^5 $\frac{\text{Erg per sec.-cm.}^2}{^{\circ}C. \text{ per cm.}}$

TABLE 4.—ELECTRICAL SYSTEM OF DEFINITIONS AND UNITS.

Name.	Symbol.	Defining Equation.	Dimensions in Terms of $L M T Q$.	M.K.S.C. Units.	Value of M.K.S.C. in μ C.G.S. Units.	Value of M.K.S.C. in κ C.G.S. Units.
Electricity .	Q	Fundamental				
Electro-chemical equivalent	z	$M = zQ$	$[M]$	Coulomb.	10^{-1}	3×10^9
E.M.F. . . .	E	$W = EQ$	$[Q^{-1}]$	Kilogram per coulomb.	10^4	$(3 \times 10^9)^{-1}$
Potential difference	V	$W = VQ$	$[L^2 M T^{-2} Q^{-1}]$	Volt.	10^8	$(300)^{-1}$
Electric field or potential gradient	$H = \dot{E}$	$H = F \div Q = V \div L = \dot{V}$	$[L M T^{-2} Q^{-1}]$	Volt per metre.	10^6	$(3 \times 10^4)^{-1}$
Capacity . . .	C	$Q = CV$	$[L^{-2} M^{-1} T^2 Q^2]$	Farad.	10^{-9}	0.9×10^{12}
Permittivity . .	κ	$C = \kappa S \div l$	$[L^{-3} M^{-1} T^2 Q^2]$	Farad per metre.	10^{-11}	$36\pi \times 10^9$
Electric current .	I	$I = Q \div T = \dot{Q}$	$[T^{-1} Q]$	Ampere = cmb. per sec.	10^{-1}	3×10^9
Current density .	\dot{I}	$\dot{I} = l \div S$	$[L^{-2} T^{-1} Q]$	Ampere per metre ² .	10^{-5}	3×10^5
Impedance . . .	Z	$V = ZI$	$[L^2 M T^{-1} Q^{-2}]$	Ohm = volt per ampere.	10^9	$(0.9 \times 10^{12})^{-1}$
Resistance . . .	R	$P = RI^2$				
Resistivity . . .	ρ	$R = \rho L \div S$	$[L^3 M T^{-1} Q^{-2}]$	Ohm-metre.	10^{11}	

Mass resistivity	ρ_M	$R = \rho_M L^2 \div M$	$[M^2 \quad T^{-1} \quad Q^{-2}]$	Ohm-kg. per metre ² .	10^8
Conductance	G	$G = 1 \div R$	$[L^{-2} M^{-1} \quad T \quad Q^2]$	Mho = $1 \div$ ohm.	10^{-9}
Conductivity	γ	$G = \gamma S \div L$	$[L^{-3} M^{-1} \quad T \quad Q^2]$	Mho per metre.	10^{-11}
Excitation	$\chi \quad \mathcal{X}$	$\chi = NI$	$[T^{-1} \quad Q]$	Ampere-turn.	$4\pi \times 10^{-1}$
Magnetising field	$\mathcal{H} \quad \mathcal{H}$	$\mathcal{H} = \chi \div L = \dot{\chi}$	$[L^{-1} \quad T^{-1} \quad Q]$	Ampere-turn per metre.	$4\pi \times 10^{-3}$
Magnetic flux	$\varphi \quad \Phi \quad \varphi$	$\varphi = \int E \, dT$	$[L^2 \quad M \quad T^{-1} \quad Q^{-1}]$	Volt-second.	10^8
Reluctance	\mathcal{R}	$\chi = \mathcal{R} \varphi$	$[L^{-2} M^{-1} \quad Q^2]$	Amp.-turn per volt-sec.	$4\pi \times 10^{-9}$
Inductance	\mathcal{L}	$\varphi = \mathcal{L} I$	$[L^2 \quad M \quad Q^{-2}]$	Henry = sec. \times ohm.	10^9
Flux density	$\beta \quad \mathcal{B}$	$\beta = \varphi \div S$	$[M \quad T^{-1} \quad Q^{-1}]$	V.S. per metre ² .	10^4
Permeability	$\mu \quad \mu$	$\mu = \beta \div \mathcal{H}$	$[L \quad M \quad Q^{-2}]$	Henry per metre, or (V.S. per m. ²) \div (A.T. per m.)	$\frac{1}{4\pi} \times 10^7$
Hysteresis coefficient	\mathcal{S}	$P_H = \mathcal{S} \beta^{1.6} V_I$	$[L^{-1} M^{-0.6} T^{-1.4} Q^{1.6}]$	(Watts per metre ³) (V.S. per metre ³) ^{1/6} .	
Eddy current coefficient	e	$P_E = e f^{3/2} \beta^{3/2} V_I$	$[L^{-3} M^{-1} \quad T \quad Q^2]$	(Watts per metre ³) (C.P.S. \times V.S. per metre) ³ .	
Iron loss coefficient	K	$P_I = K f \beta^2 V_I$	$[L^{-1} M^{-1} \quad T^{-1} \quad Q^2]$	(Watts per metre ³) (Volt-sec. per metre ²) ² .	

Ordinary type is employed for the electrical and script for the magnetic symbols.

$$[CR] = [\mathcal{L} \div R] = [T]$$

$$[R] = [\mu v] = [\kappa^{-1} v^{-1}] = [G^{-1}]$$

$$[\mathcal{L}C] = [T^2]$$

$$[C] = [\kappa L]$$

$$[\kappa \mu] = [v^{-2}]$$

$$[\mathcal{L}] = [\mu L] = [R^{-1}]$$

TABLE 5.—CONVERSION CONSTANTS.

Angle .	. 1 radian = $180^\circ \div \pi = 57.296^\circ$.
Length .	. 1 metre = 10^6 microns = 39.370 inches = 3.2808 feet = 1.0936 yards = 0.62138×10^{-3} miles. 1 inch = 1000 mils = 25.400 millimetres.
Surface .	. 1 metre ² = 1550.0 inches ² = 10.764 feet ² = 1.196 yards ² . 1 inch ² = 645.16 millimetres ² .
Volume .	. 1 litre = 10^{-3} metres ³ = 10^6 mm. ³ = 61.023 inches ³ = 0.22010 gallons. 1 inch ³ = 16,387 mm. ³ = 0.016387 litres = 3.6041×10^{-3} gallons. 1 gallon = 277.46 inches ³ = 4.5434 litres.
Mass .	. 1 kilogram = 2.2046 lbs. = 15,432 grains = 35.274 ozs. = 0.98421×10^{-3} tons. 1 pound (av.) = 7000 grains = 453.59 grams.
Density .	. 1 kilogram per litre = 10^{-3} kg. per metre ³ = 0.036128 lbs. per inch ³ . 1 lb. per inch ³ = 27.680 kgs. per litre = 27,680 kgs. per metre ³ .
Velocity	. 1 metre per second = 196.85 feet per minute = 2.2370 miles per hour. 1 foot per minute = 5.0800 mm. per second.
Force .	. 1 joule per metre = 0.10192 kilogram wt. = 0.22469 lbs. wt. 1 kilogram wt. = 9.8117 joules per metre = 2.2046 lbs. wt. 1 lb. wt. = 4.4505 joules per metre = 0.45359 kg. wt.
Pressure	. 1 joule per metre ³ = 0.14496×10^{-3} lbs. wt. per inch ² = 7.5019×10^{-3} mm. of Hg. 1 lb. wt. per inch ² = 6898.3 joules per metre ³ = 51.750 mm. of Hg = 2.0359 inches of Hg. 1 mm. of Hg = 133.400 joules per metre ³ . 1 atmosphere = 760 mm. of Hg = 101,382 joules per metre ³ = 14.697 lbs. wt. per inch ² .
Energy .	. 1 joule = 0.10192 metre-kg. wt. = 0.73717 ft.-lbs. wt. 1 ft.-lb. wt. = 1.3562 joules = 0.13826 metre-kg. wt. 1 Board of Trade unit = 1 kilowatt-hour = 3.6×10^6 joules = 2.6538×10^6 ft.-lbs. wt. 1 kg. water °C. = 1000 therms = 4184 joules = 3084.3 ft.-lbs. wt. 1 British thermal unit = 1 lb. water °F. = 252.0 therms = 1054 joules = 777.2 ft.-lbs. wt.
Power .	. 1 kilowatt = 1000 watts = 10^3 joules per sec. = 1.3403 horse-power. 1 horse-power = 746.09 watts. 1 watt per metre ² = 0.64516×10^{-3} watts per inch ² . 1 watt per inch ² = 1550.0 watts per metre ² .

Power . . .	1 watt per mm. ³ = 16,387 watts per inch ³ .
	1 watt per inch ³ = 61·023 watts per litre.
Flux . . .	1 volt-second = 10 ⁸ lines = 100 megalines.
Flux density .	1 volt-second per metre ² = 1 microvolt-second per mm. ² = 10 kilolines per cm. ² = 0·64516 millivolt-seconds per inch ² .
	1 millivolt-second per inch ² = 1·5500 V.S. per metre ² = 15,500 lines per cm. ²
Mag. field . .	1 ampere-turn per mm. = 12·566 C.G.S. units = 25·400 amp.-turns per inch.
	1 ampere-turn per inch = 39·370 amp.-turns per metre = 0·49474 C.G.S. units.

Prefixed.

Kilo	= 10 ³ .	Milli	= 10 ⁻³ .
Mega	= 10 ⁶ .	Micro	= 10 ⁻⁶ .
Kilo-mega	= 10 ⁹ .	Micro-milli	= 10 ⁻⁹ .
Mega-mega	= 10 ¹² .	Micro-micro	= 10 ⁻¹² .

TABLE 6.—USEFUL CONSTANTS.

Density of water at 4° C. = 0·99996 kg. per litre = 0·036126 lbs. per inch³
= 62·426 lbs. per ft.³

iron	= 7·80	„	„	= 0·2818	lbs. per inch ³ .
copper	= 8·90	„	„	= 0·3215	„
mercury at 0°C.	= 13·5958	„	„	= 0·49119	„
air at N.T.P.	= 1·2932 gm. per litre			= 46·721 × 10 ⁻⁶	„

Acceleration due to gravity at London—

$$g = 9·8117 \text{ metres per sec.}^2 = 32·190 \text{ feet per sec.}^2$$

(This value is assumed in the conversion constants for the gravitational units.)

For iron, 1 watt per litre = 0·1282 watts per kg.

$$1 \text{ watt per inch}^3 = 3·549 \text{ watts per lb.} = 7·82 \text{ watts per kg.}$$

Specific heat of water (mean)

$$\begin{aligned} &= 4184 \text{ joules per kg. } ^\circ\text{C.} = 1·1622 \times 10^{-3} \text{ kw.-hrs.} \\ &\quad \text{per kg. } ^\circ\text{C.} = 1 \text{ therm per gn. } ^\circ\text{C.} \\ &= 1898 \text{ joules per lb. } ^\circ\text{C.} = 0·5272 \times 10^{-3} \text{ kw.-hrs.} \\ &\quad \text{per lb. } ^\circ\text{C.} = 1399·0 \text{ ft.-lbs. per lb. } ^\circ\text{C.} \\ &= 1054 \text{ joules per lb. } ^\circ\text{F.} = 0·2729 \times 10^{-3} \text{ kw.-hrs.} \\ &\quad \text{per lb. } ^\circ\text{F.} = 777·2 \text{ ft.-lbs. per lb. } ^\circ\text{F.} \end{aligned}$$

of air at constant pressure and N.T.P.

$$\begin{aligned} &= 1000 \text{ joules per kg. } ^\circ\text{C.} = 450 \text{ joules per lb. } ^\circ\text{C.} \\ &= 250 \text{ joules per lb. } ^\circ\text{F.} \end{aligned}$$

Latent heat of steam at 100° C.

$$= 2.242 \times 10^6 \text{ joules per kg.} = 0.6228 \text{ kw.-hrs. per kg.}$$

$$= 1.0170 \times 10^6 \text{ joules per lb.} = 0.2825 \text{ kw.-hrs. per lb.}$$

Reciprocal emissivity of coils in air

$$= 0.15 \text{ RC.}^\circ \text{ per (watt per metre}^2\text{)} = 240 \text{ RC.}^\circ \text{ per (watt per inch}^2\text{)}.$$

$$t_R = (238^\circ \text{ C.} + t_C)(R_H - R_C) \div R_C.$$

Resistivity of soft copper (Mathieson's standard)—

$$\text{at } 0^\circ \text{ C. } 16.00 \times 10^{-6} \text{ ohm-mms.} = 0.6300 \times 10^{-6} \text{ ohm-ins.}$$

$$50^\circ \text{ C. } 19.43 \quad \text{,,} \quad \text{,,} \quad = 0.7649 \quad \text{,,} \quad \text{,,}$$

$$100^\circ \text{ C. } 22.85 \quad \text{,,} \quad \text{,,} \quad = 0.8998 \quad \text{,,} \quad \text{,,}$$

(These values are used in the Wire Table ; see p. 47 for complete table.)

Resistance temperature coefficient for soft copper—

$$4.284 \times 10^{-3} \text{ per } ^\circ \text{C.} = 2.38 \times 10^{-3} \text{ per } ^\circ \text{F.}$$

Permittivity of vacuum

$$\kappa_V = \frac{1}{36\pi \times 10^9} \frac{\text{farads}}{\text{metre}} = 8.842 \times 10^{-15} \text{ farads per mm.}$$

$$= 0.2246 \times 10^{-12} \text{ farads per inch.}$$

Permeability of vacuum

$$\mu_V = 4\pi \times 10^{-7} \text{ henry per metre} = 1.2566 \times 10^{-9} \text{ henry per mm.}$$

$$= \frac{10^{-6} \text{ V.S. per mm.}^2}{795.77 \text{ amp.-turns per mm.}}$$

$$= 31.919 \times 10^{-9} \text{ henry per inch} = \frac{1 \text{ M.V.S. per inch}^2}{31,330 \text{ amp.-turns per inch}}$$

$$2\pi\mu_V = 7.8957 \times 10^{-9} \text{ henry per mm.} = 0.20055 \times 10^{-6} \text{ henry per inch.}$$

$$(\kappa_V \mu_V)^{-1} = 300 \times 10^6 \text{ metres per second.}$$

TABLE 7.—ANNEALED COPPER WIRE TABLE.

Gauge.	Diameter.			Section.		Mass÷Length.		Resistance÷Length.					
	Bare.		Covered,* (D.C.C.)	10 ⁻³ ins. ²	Mm. ²	10 ⁻⁶ lbs. per inch.	Grams per metre.	10 ⁻⁸ ohms per inch.			10 ⁻⁸ ohms per mm.		
	Mils.	Mm.						0° C.	50° C.	100° C.			
	Mils.	Mm.	Mm.	0° C.	50° C.	100° C.							
S.W.G.	Mils.	Mm.	Mm.										
50	1.0	0.0254	..	0.785 × 10 ⁻³	0.507 × 10 ⁻³	0.2525	0.00451	802,100	974,100	1,146,000	31,580	38,350	45,110
49	1.2	0.0305	..	1.131 "	0.730 "	0.3626	0.00649	557,000	676,300	795,600	21,930	26,700	31,470
48	1.6	0.0406	..	2.011 "	1.297 "	0.6464	0.01154	313,300	380,400	447,500	12,340	14,980	17,620
47	2.0	0.0508	..	3.142 "	2.027 "	1.010	0.01804	200,600	243,500	286,400	7,895	9,588	11,280
46	2.4	0.0610	..	4.524 "	2.919 "	1.454	0.02597	130,300	169,100	198,900	5,433	6,657	7,831
45	2.8	0.0711	..	6.158 "	3.973 "	1.980	0.03535	102,300	124,200	146,100	4,028	4,891	5,753
44	3.2	0.0813	..	8.043 "	5.189 "	2.586	0.04618	78,330	95,120	111,900	3,084	3,745	4,405
43	3.6	0.0914	..	10.18 "	6.567 "	3.272	0.05844	61,890	75,150	88,400	2,437	2,959	3,480
42	4.0	0.1016	10.0	12.57 "	8.107 "	4.040	0.07215	40,130	60,870	71,600	1,974	2,397	2,819
41	4.4	0.1118	10.4	15.21 "	9.810 "	4.888	0.08730	41,430	50,310	59,180	1,631	1,981	2,330
40	4.8	0.1219	10.8	18.10 "	11.67 "	5.818	0.1039	34,820	42,280	49,730	1,371	1,665	1,958
39	5.2	0.1321	11.2	21.24 "	13.70 "	6.828	0.1219	29,670	36,020	42,370	1,168	1,418	1,668
38	6.0	0.1524	12.0	28.27 "	18.24 "	9.090	0.1623	22,280	27,050	31,820	877.2	1,065	1,253
37	6.8	0.1727	12.8	36.32 "	23.43 "	11.68	0.2085	17,350	21,070	24,780	683.0	829.3	976.5
36	7.6	0.1930	13.6	45.37 "	29.27 "	14.58	0.2605	13,890	16,870	19,840	546.8	663.9	780.9
35	8.4	0.2134	14.4	56.42 "	35.75 "	17.82	0.3182	11,370	13,810	16,240	447.6	543.5	639.3
34	9.2	0.2337	15.2	66.48 "	42.89 "	21.37	0.3817	9,477	11,510	13,540	373.1	453.0	532.9
33	10.0	0.2540	16.0	78.64 "	50.67 "	25.25	0.4509	8,021	9,741	11,460	315.8	383.5	451.1
32	10.8	0.2743	16.8	91.61 "	59.10 "	29.45	0.5280	6,877	8,350	9,822	270.8	328.8	386.7
31	11.6	0.2946	17.6	105.7 "	68.18 "	33.98	0.6068	5,961	7,238	8,514	234.7	285.0	336.2
30	12.4	0.3150	20.4	0.1208	79.10 "	38.83	0.6923	5,217	6,334	7,451	205.4	249.4	293.4
29	13.6	0.3454	21.6	0.1453	93.72 "	46.70	0.8340	4,337	5,266	6,194	170.7	207.3	243.9
28	14.8	0.3759	22.8	0.1720	111.0 "	55.31	0.9877	3,662	4,446	5,220	144.2	175.1	205.9
27	16.4	0.4166	24.4	0.2112	136.3	67.91	1.213	2,982	3,621	4,260	117.4	142.6	167.7
26	18	0.4572	26	0.2545	164.2	81.81	1.461	2,476	3,006	3,536	97.47	118.4	139.2
25	20	0.5080	30	0.3142	202.7	101.0	1.804	2,005	2,435	2,864	78.95	95.88	112.3
24	22	0.5588	32	0.3801	245.2	122.2	2.182	1,657	2,012	2,367	65.25	79.22	93.19

* The thickness of covering assumed in these columns may be described as "ordinary." Wires can be obtained with considerably thicker or thinner coverings if desired.

TABLE 7.—ANNEALED COPPER WIRE TABLE—continued.

Gauge.	Diameter.			Section.		Mass + Length.		Resistance ÷ Length.			
	Bare.		Covered,* (D.C.C.)	10 ⁻³ Ins. ²	Mm. ²	10 ⁻⁶ lbs. per inch.	Grams per metre.	10 ⁻⁶ ohms per inch.			
	Mils.	Mm.						0° C.	50° C.	100° C.	100° C.
S.W.G.	Mils.	Mm.	Mm.								
23	24	0.6066	34	0.4524	0.2919	145.4	2.597	1.303	1.691	1.989	78.31
22	28	0.7112	38	0.6158	0.3973	158.0	3.535	1.023	1.242	1.461	57.53
21	32	0.8123	42	0.804	0.5130	258.0	4.512	1.033	951.2	1.119	44.05
20	36	0.9144	46	1.018	0.6567	327.2	5.844	618.9	751.5	884.0	31.80
19	40	1.016	50	1.267	0.8107	404.0	7.215	501.3	608.7	716.1	28.10
18	48	1.219	58	1.589	1.018	581.8	10.39	348.2	422.8	497.3	19.58
17	56	1.422	68	2.463	1.589	751.9	14.14	255.8	310.6	365.3	14.38
16	64	1.626	76	3.217	2.076	1.034	18.47	195.8	237.8	279.7	11.01
15	72	1.829	84	4.071	2.627	1.309	23.38	147.7	187.9	221.0	8.701
14	80	2.032	92	5.026	3.243	1.616	28.86	125.3	162.2	179.0	7.048
13	92	2.337	104	6.648	4.289	2.137	38.17	94.77	115.6	136.3	5.329
12	104	2.642	118	8.495	5.480	2.731	48.77	74.16	90.03	105.9	4.170
11	116	2.946	130	10.57	6.813	3.398	60.65	59.61	72.38	85.14	3.352
10	128	3.251	142	12.87	8.302	4.137	73.88	48.96	59.45	69.93	2.753
9	144	3.658	158	16.29	10.51	5.236	93.50	38.68	46.97	55.25	2.175
8	160	4.064	174	20.11	12.97	6.404	113.4	31.33	38.04	44.75	1.762
7	176	4.470	190	24.35	15.70	7.822	139.7	25.90	31.46	36.99	1.456
6	192	4.877	206	28.95	18.68	9.308	166.2	21.76	26.42	31.08	1.224
5	212	5.385	226	33.90	22.77	11.349	202.7	17.85	21.67	25.49	1.084
4	232	5.893	246	42.27	27.27	13.591	242.7	14.90	18.10	21.29	0.8380
3	252	6.401	266	49.88	32.13	16.035	286.4	12.63	15.34	18.04	0.7103
2	276	7.010	300	59.83	38.60	19.235	343.5	10.53	12.70	15.04	0.5921
1	300	7.620	314	70.68	45.60	22.725	405.8	8.913	10.82	12.73	0.5012
0	324	8.230	338	82.45	53.19	26.508	473.4	7.641	9.275	10.91	0.4297
02	348	8.839	362	95.12	61.36	30.579	546.1	6.624	8.043	9.460	0.3725
03	372	9.448	386	108.7	70.12	34.947	624.0	5.797	7.038	8.270	0.3260
04	400	10.160	414	126.7	81.07	40.401	721.5	5.013	6.087	7.160	0.2819
05	432	10.773	446	146.6	94.56	47.123	841.5	4.298	5.218	6.189	0.2417
06	464	11.752	478	169.1	109.1	59.609	970.8	3.726	4.594	5.321	0.2095
07	500	12.700	514	196.4	126.7	63.126	1127	3.260	3.890	4.553	0.1804

* The thickness of covering assumed in these columns may be described as "ordinary." Wires can be obtained with considerably thicker or thinner coverings if desired.

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TABLE A.—PARTICULARS OF SIN

1	Plate No.			1.	2.	3.
2	Maker.		UNITS.	Hartmann & Braun, Frankfurt-a.-M.	Hartmann & Braun, Frankfurt-a.-M.	Design.
3	Type—	Iron, magnetic circuit	..	Core.	Shell.	Core.
4		„ shape of section of core	...	Square.	Rectangular.	Rectangular.
5		„ number of ducts	...	None.	None.	None.
6		„ kind	...	Ordinary.	Ordinary.	Stalloy.
7		Coils, shape	...	Circular.	Rectangular.	Rectangular.
8		„ arrangement	...	On separate cores.	Sandwiched.	...
9		Filling	...	Air.	Air.	Air.
10		Cooling	..	Natural.	Natural.	Natural.
11		Remarks	...	Current transf.	Voltage transf.	Choking coil
12	Units.		Mm. System. Inch System.	Millimetre.	Millimetre.	Incl.
13	Rating—	Output, ordinary and special rating	Kilovolt-amperes.	0·015	0·08	0·5 as T.
14		Frequency	Cycles per second.	100	100	100
15				P.	S.	
16		Voltage, line	Volts.	0·75	15	M 2500
17		„ winding	„	0·75	15	M 2500
18		Current, line	Amperes	M 20	M 1	0·062
19		„ winding	„	M 20	M 1	0·062
20	Winding—	Number of turns	...	M 15	M 258	M 1628
21		„ coils	...	M 1	M 1	M 2
22		Turns per coil	...	M 15	M 258	M 814
23		Layers per coil	...	3	11	43
24		Turns per layer	...	6, 5, 4	25 and 8	19 and 16
25	Conductor—	Gauge, if round	Millimetres.
26		Diameter (ϕ), or dimensions, bare	Mils.	4·5 × 3·0	0·80ϕ	0·35ϕ
27		Ditto, covered	„	5·0 × 3·5	1·00ϕ	0·58ϕ
28		Section	10 ⁻³ mms. ²	13·5	0·502	0·096
29		Total section of copper	Inches ²	0·202	0·129	0·156
30		Current density	Amps. per mm ² .	1·48	1·99	0·646
31	Former—	Diameter (ϕ), or dimensions	Millimetres.	35ϕ	35ϕ	70 × 175
32		Perimeter of section	„	110	110	490
33		Allowance for corners	„	75
34		Mean length of one turn	„	141	141	565
35		Length of wire	Metres.	2·12	36·4	920
36	Coils—	Depth of winding	Millimetres.	10·5	11·0	25
37		Axial length of each	„	25	25	11
38		Exposed perimeter of section, total	„	71	72	72
39		Cooling surface, total	Metres ² .	0·0100	0·0102	0·0249
40		Volume of copper	Litres.	0·0285	0·0182	0·0883
41		Mass of copper	Kilograms.	0·254	0·162	0·785
42	Resistance—	Effective resistance, hot	Ohms.	0·0036	1·67	221
43		Copper loss	Watts.	1·44	1·67	0·850
44		Dissipation intensity	Watts per metre ² .	144	164	34·1
45		Temperature rise	° C.	22	25	6
46	Dimensions—	Diameter (ϕ), or width of iron space	Millimetres.	28ϕ	...	M 50
47		Distance between yokes	„	M 35	x = 1·25	M 50
48		Depth of copper space	„	M 21	y = 0·75	M 50
49		Depth of iron space	„	M 145
50		Net depth of iron	„	17	...	123
51	Length of	iron	Millimetres.	250	...	300
52		copper	„	141	...	574
53		flux	„	230	...	275
54	Section of	iron space	10 ³ mms. ² .	0·616	...	7·25
55		iron, net	„	0·340	...	6·16
56		copper space	„	1·470	...	2·50
57		copper, net	„	0·331	...	0·284
58	Space factor for	iron space	...	0·552	...	0·850
59		copper space	...	0·225	...	0·114
60		geometrical mean	...	0·352	...	0·311
61	Cooling surface of	iron space, conventional	Metres ² .	0·022	...	0·117
62		iron, actual	„	0·020	...	0·117
63		copper space, conventional	„	0·034	...	0·115
64		coils, actual	„	0·020	...	0·033
65		case	„
66		water pipes	„
67	Volume of	iron	Litres and	0·0850	5·18	1·85
68		copper	(10 ⁶ mms. ³)	0·0467	2·85	0·163
69		oil	„ Gallons.
70	Mass of	iron (7·80 kgs. per litre ; 0·282 lbs. per in. ³)	Kilograms and	0·664	1·46	14·4
71		copper (8·90 kgs. per litre ; 0·322 lbs. per in. ³)	„	0·416	0·92	1·45
		total active material	„	1·080	2·38	15·9
		standard iron of same cost	„	1·91	4·22	18·8
		ditto × {√F ₁ P _C /2(I ₁ V ₁ + I ₂ V ₂)} ³ × {f ÷ 50 C.P.S.} ²	Grams and	4·35	9·60	5·80
			10 ⁻³ lbs.			12·8
						0·716

OF SINGLE-PHASE TRANSFORMERS.

3.		4.		5 and fig. 9-10.		6.		7.		8.		9 and fig. 9-23.		10.	
Design.		Design.		Elektricitäts Allgemeine Gesellschaft (E. H. Geist), Cölne.		Design.		Gesellschaft für Elektrische Industrie, Karlsruhe.		W. E. Burnand & Co., Sheffield.		Electric Construction Co., Ltd., Wolverhampton.		Design.	
Core. Rectangular. One. Stalloy. Rectangular. ... Air. Natural. Hoking coil.		Core. Rectangular. None. Stalloy. Rectangular. Concentric. Air. Natural. Auto-transf.		Shell. Rectangular. None. Ordinary. Rectangular. Sandwiched. Oil. Natural. ...		Hex. ring. Rectangular. One. Stalloy. Rectangular. Concentric. Oil. Natural. ...		Core. Square. None. C. and K.'s alloy. Square. Concentric. Oil. Natural. ...		Square ring. Rectangular. None. Ordinary. Rectangular. Divided concentric. Oil. Natural. ...		Shell (☉ iron). Rectangular. None. Ordinary. Rectangular. Sandwiched. Oil. Natural. Circular flux path.		Core. Rectangular. Three. Stalloy. Rectangular. Concentric. Oil. Natural.	
Inch.		Inch.		Millimetre.		Inch.		Millimetre.		Inch.		Inch.		Inch.	
5 as T. 1 as C.C. 100		1 as T. 2 as A.T. 50		3 50		5 80		10 50		50 50		60 50		100 50	
		P.	S.	P.	S.	P.	S.	P.	S.	P.	S.	P.	S.	P.	S.
...	100	210	105	M 2000	M 220	2000	200	6350	M 230	2000	200	M 2000	M 404	6,000	2200
...	100	105	105	M 2000	M 220	2000	200	6350	M 230	2000	200	M 2000	M 404	6,000	2200
...	10	9·80	19·0	1·56	13·6	2·59	25·0	1·63	43·5	25·7	250	30·6	149	17·0	45·5
...	10	9·80	9·24	1·56	13·6	2·59	25·0	1·63	43·5	25·7	250	30·6	149	17·0	45·5
...	184	178	186	M 1320	M 148	1224	126	M 3360	M 124	324	33	224	46	864	320
...	2	2	2	M 2	M 4	6	6	M 10	M 2 in II	4 × 2	4 (in II) × 3	M 4	M 3	8	2
...	92	89	93	M 660	M 37	204	21	M 336	M 124	54 × 27	18, 11, 4	70 and 42	18 and 14	108	160
...	5	3	3	17	2	12	M 2	2	1	$\left\{ \begin{smallmatrix} 23 & 17 \\ 24 & 14 \\ 23 & 11 \end{smallmatrix} \right\}$	9 and 7	9	5
...	19 and 16	31, 31, 27	31	20·4	11·10	28	62	$\left\{ \begin{smallmatrix} 26, 13 \\ 28, 14 \end{smallmatrix} \right\}$	18, 11, 4	3	2	12	32
...	10	11	11	17	7	7	2	4
...	128φ	116φ	116φ	M 150φ	M 450φ	56φ	176φ	1·20φ	M 4·0φ	176φ	276φ	232φ	650 × 300	200 × 100	360 × 150
...	142φ	130φ	130φ	M 190φ	M 485φ	68φ	190φ	1·60φ	M 4·5φ	190φ	290φ	246φ	670 × 320	230 × 130	390 × 180
...	12·87	10·57	10·57	1·77	15·9	2·463	24·33	1·13	2 × 12·56	24·33	4 × 59·83	42·27	195	20·0	54·0
...	2·37	1·88	1·97	2340	2355	3·02	3·07	3790	3110	7·88	7·89	9·46	8·97	17·3	17·3
...	778	927	874	0·88	0·86	1050	1028	1·44	1·73	1060	1040	724	765	850	843
...	$1\frac{3}{16} \times 2\frac{9}{16}$	$4\frac{1}{2} \times 2\frac{5}{8}$	$3\frac{3}{8} \times 1\frac{1}{2}$	85 × 205	85 × 205	$6\frac{1}{4} \times 3\frac{1}{2}$	$5\frac{1}{8} \times 2\frac{3}{8}$	150 × 150	110 × 110	$\left\{ \begin{smallmatrix} 6\frac{3}{4} \times 18\frac{1}{4} \\ 9\frac{1}{4} \times 20\frac{3}{4} \end{smallmatrix} \right\}$	$\left\{ \begin{smallmatrix} 5\frac{1}{2} \times 17 \\ 18 \times 19\frac{1}{2} \\ 10\frac{1}{2} \times 22 \end{smallmatrix} \right\}$	$\left\{ \begin{smallmatrix} 6\frac{1}{2} \times 25 \\ (7\frac{1}{2} - 8\frac{1}{4}) \times 25 \end{smallmatrix} \right\}$	$\left\{ \begin{smallmatrix} 6\frac{1}{2} \times 25 \\ 7\frac{1}{4} \times 25 \end{smallmatrix} \right\}$	$8\frac{1}{4} \times 13\frac{3}{4}$	$5\frac{1}{4} \times 10\frac{3}{4}$
...	7·50	14·25	9·75	580	580	19·5	15·0	600	440	50, 60	45, 55, 65	63·0, 65·8	63·0, 64·5	44·0	32·0
...	2·13	1·20	1·20	145	145	2·3	1·2	30	30	1·1	0·9	17·2, 10·2	18·1, 14·1	3·5	2·7
...	9·63	15·45	10·95	725	725	21·8	16·2	M 630	M 470	54·4	51·7	80·2, 76·0 & 78·5	81·1, 78·6, & 79·6	47·5	34·7
...	1770	2750	2040	956	107	26,700	2040	2120	2 × 58·3	17,600	4 × 1705	17,600	3660	41,000	11,100
...	0·71	0·390	0·390	50	50	1·156	0·380	19·2	9·0	0·38	0·29	$\left\{ \begin{smallmatrix} 5·90, 4·18 \\ 5·66, 2·71 \end{smallmatrix} \right\}$	6·03, 4·69	1·17	0·90
...	2·84	4·16	4·16	28	10	$\left\{ \begin{smallmatrix} 0·34 & 2·09 \\ 1·43 & 2·28 \end{smallmatrix} \right\}$...	46·5	284	$\left\{ \begin{smallmatrix} 5·13, 2·66 & 5·51 \\ 5·51, 2·85 & 3·48, 1·45 \end{smallmatrix} \right\}$...	0·74	0·64	2·99	12·9
...	2 × 72·6	2 × 9·30	2 × 9·30	2 × 148	4 × 116	6 × 4·4	6 × 5·2	10 × 131	2 × 5·86	4 × 14·9	3160	13·0 + 9·0	13·3 + 10·7	8 × 8·52	2 × 27·8
...	140	287	204	0·214	0·336	396	504	0·83	0·552	1040 + 680	1080 + 840	3240	1930
...	22·8	29·1	21·5	1·70	1·71	65·8	49·7	2·39	1·47	428	408	1720	1920	820	600
...	7·34	9·36	6·94	15·1	15·2	21·2	16·0	21·3	13·1	138	131	745	714	264	193
...	0·124	0·235	0·174	12·4	0·155	9·74	0·0755	43·1	0·0534	0·651	0·00643	0·375	0·0169	1·84	0·185
...	12·4	22·5	14·8	30·3	28·7	65·3	47·2	114	101	430	400	350	375	534	383
...	0·088	0·079	0·073	142	86	0·114	0·094	138	183	...	0·263	0·204	0·195	0·165	0·198
...	21	19	17	18	11	23	19	17	23	53	...	41	39	33	40
...	1	...	$1\frac{3}{8}$	M 70	2·00	M 100	M 5	...	M 5 $\frac{5}{8}$...	$4\frac{1}{2}$
= 3	3	x = 3·18	$4\frac{3}{8}$	M 135	x = 1·93	M 310	x = 3·10	x = 3·33	15
= 1	1	y = 0·86	$1\frac{3}{16}$	M 64	y = 0·92	y = 1·22	2·44	M 57·5	y = 0·58	y = 0·75	M 3 $\frac{3}{4}$	y = 1·28	M 7 $\frac{1}{2}$	y = 0·75	$3\frac{3}{8}$
= 2·35	2·35	z = 2·31	3·18	M 200	z = 2·86	z = 2·38	4·75	M 100	z = 1·00	z = 3·20	M 16	z = 4·09	M 24	z = 2·22	10·0
...	2·00	...	2·70	150	4·00	M 89	13·6	...	20·4	...	8·0
...	14·0	...	19·0	528	23·8	1250	50	...	32·6	...	61·5
...	9·63	...	13·15	730	19·0	560	49·5	...	79·1	...	41·1
...	13·0	...	17·6	500	23·8	1150	45	...	32·6	...	57·0
...	2·35	...	4·37	14·0	9·50	10·00	80	...	141	...	45·0
...	2·00	...	3·71	10·5	8·00	8·90	68	...	117·2	...	36·0
...	6·00	...	10·38	8·64	20·60	35·7	56·2	...	44·2	...	101·3
...	2·37	...	3·85	4·695	6·09	6·90	15·77	...	18·43	...	34·6
...	0·852	...	0·849	0·750	0·843	0·890	0·850	...	0·832	...	0·800
...	0·395	...	0·371	0·544	0·295	0·193	0·281	...	0·417	...	0·341
...	0·580	...	0·560	0·639	0·500	0·415	0·489	...	0·590	...	0·523
...	93·8	...	174	0·358	322	0·500	2100	...	3500	...	1780
...	93·8	...	174	0·394	405	0·500	2100	...	3500	...	3320
...	155	...	282	0·291	935	0·824	3520	...	1870	...	2950
...	140	...	491	0·235	1080	1·38	3160	...	3640	...	5170
...	1070	M 1·18	3900	...	6300	...	5000
...
459	28·0	1·16	70·5	5·55	338	3·12	190	11·25	686	55·8	3400	62·6	3820	36·3	2214
374	22·8	0·83	50·6	3·41	208	1·90	115·5	3·86	236	13·7	836	23·9	1459	23·3	1420
...	21	4·7	44	9·7	150	33	132	29	202	44·5
58	7·90	9·01	19·9	43·3	95·4	24·4	53·7	87·8	194	434	956	489	1076	284	625
32	7·34	7·39	16·3	30·3	66·7	16·9	37·2	34·4	76	122	269	213	470	207	457
90	15·24	16·40	36·2	73·6	162·1	41·3	90·9	122	270	556	1225	702	1546	491	1082
2	35·8	38·0	83·7	134	295	93·4	206	257	567	800	1763	1128	2486	1120	2463
716	1·58	0·798	1·76	0·892	1·97	0·780	1·72	1·36	2·98	2·31	5·10	1·18	2·61	0·785	1·73
...	19	41	39	85	139	300	116	255

38		Exposed perimeter of section, total	"	"	71	72	72	70	
39		Cooling surface, total	Metres ² .	Inches ² .	0·0100	0·0102	0·0249	0·0082	...
40		Volume of copper	Litres.	Inches ³ .	0·0285	0·0182	0·0883	0·0745	...
41		Mass of copper	Kilograms.	Pounds.	0·254	0·162	0·785	0·663	...
42	Resistance—	Effective resistance, hot	Ohm ϕ	0·0036	1·67	221	0·547	...
43		Copper loss	Watts.	...	1·44	1·67	0·850	0·291	...
44		Dissipation intensity	Watts per metre ² .	Watts per inch ² .	144	164	34·1	35·5	...
45		Temperature rise	° C.	...	22	25	6	6	...
46	Dimensions—	Diameter (ϕ), or width of iron space	Millimetres.	Inches.	28 ϕ	...	M 50
47		Distance between yokes	"	"	M 35	x = 1·25	M 50	x = 1·00	x = 3
48		Depth of copper space	"	"	M 21	y = 0·75	M 50	y = 1·00	y = 1
49		Depth of iron space	"	"	M 145	z = 2·90	z = 2·35
50		Net depth of iron	"	"	17	...	123
51	Length of	iron	Millimetres.	Inches.	250	...	300
52		copper	"	"	141	...	574
53		flux	"	"	230	...	275
54	Section of	iron space	10 ³ mms ² .	Inches ² .	0·616	...	7·25
55		iron, net	"	"	0·340	...	6·16
56		copper space	"	"	1·470	...	2·50
57		copper, net	"	"	0·331	...	0·284
58	Space factor for	iron space	0·552	...	0·850
59		copper space	0·225	...	0·114
60		geometrical mean	0·352	...	0·311
61	Cooling surface of	iron space, conventional	Metres ² .	Inches ² .	0·022	...	0·117
62		iron, actual	"	"	0·020	...	0·117
63		copper space, conventional	"	"	0·034	...	0·115
64		coils, actual	"	"	0·020	...	0·033
65		case	"	"
66		water pipes	"	"
67	Volume of	iron	Litres and	Inches ³ .	0·0850	5·18	1·85	113	0·459
68		copper	(10 ⁶ mms. ³)	"	0·0467	2·85	0·163	99·5	0·374
69		oil	"	Gallons.
70	Mass of	iron (7·80 kgs. per litre ; 0·282 lbs. per in. ³)	Kilograms and	Pounds.	0·664	1·46	14·4	31·8	3·58
71		copper (8·90 kgs. per litre ; 0·322 lbs. per in. ³)	"	"	0·416	0·92	1·45	3·20	3·32
		total active material	"	"	1·080	2·38	15·9	35·0	6·90
		standard iron of same cost	"	"	1·91	4·22	18·8	41·5	16·2
		ditto $\times \{ \sqrt{P_1 P_2} / \frac{1}{2} (I_1 V_1 + I_2 V_2) \}^3 \times \{ f \div 50 \text{ C.P.S.} \}^2$	Grams and	10 ⁻³ lbs.	4·35	9·60	5·80	12·8	0·716
75		oil (0·88 kgs. per litre ; 8·8 lbs. per gal.)	Kilograms "	Pounds.
76	Fundamental	length	Millimetres and	Inches.	5·07	0·199	10·25	0·404	8·22
77		surface	Millimetres ² "	Inches ² .	25·7	0·0397	105	0·163	67·5
78		volume	10 ³ mms. ³ "	Inches ³ .	0·130	0·0079	1·08	0·0657	0·555
79	Flux density	(maximum)	10 ⁻⁶ V.S. per mm. ² & 10 ⁻³ V.S. per in. ²		0·332	0·214	0·562	0·362	0·950
80	Current density	(R.M.S.)	Amps. per mm. ² & Amps. per in. ²		1·74	1·120	0·529	341	1·20
81	Losses—	Specific iron loss	Watts per litre.	Watts per inch. ³	9·5	...	22
82		Iron loss	Watts and	Per cent.	0·807	4·3%	40·7	33·4%	12·6
83		Copper loss	"	"	3·11	16·4%	1·14	0·9%	12·4
84		Total loss	"	"	3·92	20·7%	41·8	34·3%	25·0
85		Geometrical mean	"	"	1·58	8·8%	6·81	5·6%	12·5
86	Dissipation intensity for conventional iron space		Watts per metre ² .	Watts per inch ² .	36·7	0·0237	348	0·224	209
87		iron, actual	"	"	40·3	0·0258	348	0·224	209
88		conventional copper space	"	"	90·2	0·0582	10	0·006	124
89		coils, actual	"	"	156	0·101	35	0·022	137
90		case	"	"
91		water pipes	"	"
92	Temperature rise for	iron, above oil and above room	° C.		...	6	...	52	...
93		copper, " "	Resistance ° C.		...	24	...	6	...
94		oil, above room	° C.	
95	Excitation—	Magnetising field (max.) for iron	Amp.-tns. per mm.	Amp.-tns. per inch.	0·10	...	0·15
96		for iron (R.M.S.)	Ampere-turns.		16	...	29·2	...	62
97		for joints	"		13	...	63·0	...	1778
98		Total effective	"		29	...	92·2	...	1840
99	Exciting current—	Idle component	Amperes and	Per cent.	1·93	9·65%	0·0566	75·5%	...
100		Working component	"	"	1·08	5·04%	0·0163	21·8%	...
101		Total	"	"	2·21	11·05%	0·0590	78·7%	...
102	Magnetic leakage—	Space between coils	Millimetres.	Inches.	8·0
103		$\frac{1}{3}$ of winding depths, or thicknesses	"	"	9·7
104		Total equivalent thickness	"	"	17·7 \div 2 ²
105	Total equivalent	resistance, referred to secondary	Ohms.		3·11	...	2·14
106		leakage reactance, "	"		0·376
107		internal impedance, "	"		2·17
108		characteristic lag, cos ϕ_T and ϕ_T	"		0·99	9°	...
109	Voltage drop at	unity power-factor	Volts and	Per cent.	3·11	4·83%	1·56	1·42%	...
110		zero (lagging) power-factor	"	"	0·274	0·25%	...
111		cos ϕ_T " "	"	"	1·58	1·44%	...
112	Efficiency at	$\frac{1}{4}$ load	Per cent.		79·0	...	32·5
113		$\frac{1}{2}$ "	"		82·5	...	49·1
114		$\frac{3}{4}$ "	"		81·5	...	59·0
115		full "	"		79·3	...	65·3	...	97·5
116		$\frac{5}{4}$ "	"		76·8	...	70·1

NOTE.—The figures marked m v

...	2 × 72.6	2 × 9.30	2 × 9.30	2 × 148	4 × 116	6 × 4.4	6 × 5.2	10 × 131	2 × 5.86	4 × 14.9	13.0 + 9.0	13.3 + 10.7	8 × 8.52	2 × 27.8
...	140	287	204	0.214	0.336	396	504	0.83	0.552	3160	1040 + 680	1080 + 840	3240	1930
...	22.8	29.1	21.5	1.70	1.71	65.8	49.7	2.39	1.47	428	1720	1920	820	600
...	7.34	9.36	6.94	15.1	15.2	21.2	16.0	21.3	13.1	138	745	714	264	193
...	0.124	0.235	0.174	12.4	0.155	9.74	0.0755	43.1	0.0534	0.651	0.00643	0.375	1.84	0.185
...	12.4	22.5	14.8	30.3	28.7	65.3	47.2	114	101	430	400	0.0169	534	383
...	0.088	0.079	0.073	142	86	0.114	0.094	138	183	0.263	0.204	0.195	0.165	0.198
...	21	19	17	18	11	23	19	17	23	53	41	39	33	40
x = 3	1	...	1 $\frac{3}{8}$	M 70	2.00	M 100	M 5	4 $\frac{1}{2}$
y = 1	3	x = 3.18	4 $\frac{3}{8}$	M 135	x = 1.93	M 310	x = 3.10	M 5 $\frac{1}{8}$	x = 3.33	15
z = 2.35	1	y = 0.86	1 $\frac{3}{16}$	M 64	y = 0.92	...	2.44	M 57.5	y = 0.58	...	M 3 $\frac{3}{4}$...	y = 0.75	3 $\frac{3}{8}$
...	2.35	z = 2.31	3.18	M 200	z = 2.86	...	4.75	M 100	z = 1.00	y = 0.75	M 16	y = 1.28	z = 2.22	10.0
...	2.00	...	2.70	150	4.00	M 89	...	z = 3.20	13.6	z = 4.09	...	8.0
...	14.0	...	19.0	528	23.8	1250	50	61.5
...	9.63	...	13.15	730	19.0	560	49.5	41.1
...	13.0	...	17.6	500	23.8	1150	45	57.0
...	2.35	...	4.37	14.0	9.50	10.00	80	45.0
...	2.00	...	3.71	10.5	8.00	8.90	68	36.0
...	6.00	...	10.38	8.64	20.60	35.7	56.2	101.3
...	2.37	...	3.85	4.695	6.09	6.90	15.77	34.6
...	0.852	...	0.849	0.750	0.843	0.890	0.850	0.800
...	0.395	...	0.371	0.544	0.295	0.193	0.281	0.341
...	0.580	...	0.560	0.639	0.500	0.415	0.489	0.523
...	93.8	...	174	0.358	322	0.500	2100	1780
...	93.8	...	174	0.394	405	0.500	2100	3320
...	155	...	282	0.291	935	0.824	3520	2950
...	140	...	491	0.235	1080	1.38	3160	5170
...	1070	M 1.18	3900	5000
0.459	28.0	1.16	70.5	5.55	338	3.12	190	11.25	686	55.8	3400	62.6	36.3	2214
0.374	22.8	0.83	50.6	3.41	208	1.90	115.5	3.86	236	13.7	836	23.9	23.3	1420
...	21	4.7	44	9.7	150	33	132	202	44.5
3.58	7.90	9.01	19.9	43.3	95.4	24.4	53.7	87.8	194	434	956	489	284	625
3.32	7.34	7.39	16.3	30.3	66.7	16.9	37.2	34.4	76	122	269	213	207	457
6.90	15.24	16.40	36.2	73.6	162.1	41.3	90.9	122	270	556	1225	702	491	1082
16.2	35.8	38.0	83.7	134	295	93.4	206	257	567	800	1763	1128	1120	2463
0.716	1.58	0.798	1.76	0.892	1.97	0.780	1.72	1.36	2.98	2.31	5.10	1.18	0.785	1.73
...	19	41	39	85	132	290	116	177	392
8.22	0.324	10.8	0.425	17.7	0.697	16.7	0.659	22.6	0.890	34.2	1.35	39.4	34.0	1.34
37.5	0.105	117	0.181	315	0.485	280	0.434	510	0.792	1170	1.82	1555	1160	1.80
0.555	0.0338	1.26	0.077	5.57	0.338	4.68	0.285	11.6	0.705	40.1	2.46	61.1	39.5	2.42
0.950	0.613	1.110	0.716	0.650	0.420	0.890	0.575	0.956	0.617	0.635	0.410	0.532	1.350	0.870
1.20	778	1.40	900	0.87	561	1.61	1040	1.585	1022	...	1050	1.152	1.315	847
...	0.45	...	0.29	10.1	0.31	13	0.185	0.395
2.6	1.26%	20.4	0.99%	M 56	1.80%	59	1.14%	146	1.41%	630	1.22%	0.138	...	0.86%
2.4	1.24%	37.3	1.82%	59	1.89%	112.5	2.18%	215	2.07%	830	1.62%	527	875	0.90%
5.0	2.50%	57.7	2.81%	115	3.69%	171.5	3.32%	361	3.48%	1460	2.84%	725	917	1.76%
2.5	1.25%	27.6	1.34%	57.5	1.85%	81.5	1.57%	177	1.71%	723	1.40%	1252	1792	0.88%
209	0.135	181	0.117	156	0.101	284	0.183	292	0.188	465	0.300	617	895	...
209	0.135	181	0.117	142	0.092	225	0.145	292	0.188	465	0.300	0.491
124	0.080	205	0.132	202	0.130	186	0.120	261	0.168	366	0.236	234	408	0.263
137	0.088	118	0.076	251	0.162	161	0.104	156	0.101	407	0.263	600	482	0.311
...	248	0.160	306	0.197	580	0.375	309	274	0.177
...	309	555	0.358
...	32	...	28	8	...	13	33	18	42	28	74
...	21	...	18	16	...	13	33	13	37	33	79	14	24	68
...	?	...	20	...	24	...	46	25	22	66
...	6.7	...	9.0	0.20	6	0.27	4.3	44
62	...	112	...	71	...	101	...	220	...	137	...	3.5	...	14
778	Gaps, 154 mils.	43	...	13	...	52	...	76	...	49	565	...
840	...	155	...	84	...	153	...	296	...	186	105	...
...	...	0.426	4.35%	0.0636	4.07%	0.125	4.83%	0.088	5.40%	0.575	2.24%
...	...	0.097	0.99%	0.0280	1.80%	0.030	1.16%	0.023	1.41%	0.315	1.23%	0.411	0.775	4.56%
...	...	0.436	4.45%	0.0695	4.45%	0.128	4.95%	0.091	5.58%	0.655	2.55%	0.264	0.146	0.85%
...	0.172	20	0.18	11	0.489	0.787	4.64%
...	0.260	36	0.52	10	0.60
...	0.432	56 ÷ 4 ²	0.70	21	0.69
...	0.438	0.319	...	0.180	...	0.114	...	0.0133	1.29
...	0.225	0.449	...	0.142	...	0.117	0.0327	0.444	...
...	0.493	0.550	...	0.229	...	0.163	0.0319	1.81	...
...	...	0.89	27°	0.58	55°	0.79	38°	0.70	46°	0.0456	1.87	...
...	0.72	0.24	76°
...	...	2.02	1.92%	4.34	1.97%	4.50	2.25%	4.95	2.15%	3.32	1.66%	4.86	20.2	0.92%
...	...	1.04	0.99%	6.10	2.78%	3.55	1.78%	5.10	2.22%	4.75	82.4	3.74%
...	...	2.28	2.18%	7.48	3.40%	5.73	2.86%	7.10	3.09%	6.80	85.1	3.87%
...	...	95.42	...	92.60	...	94.99	...	94.01	...	94.82	...	96.32	96.40	...
...	...	97.00	...	95.49	...	96.63	...	96.15	...	96.76	...	97.70	97.84	...
...	...	97.28	...	96.19	...	96.82	...	96.56	...	97.16	...	97.96	98.18	...
5	p.f. 0.025	97.13	...	96.31	...	96.68	...	96.52	...	97.16	...	97.96	98.24	...
...	...	96.86	...	96.20	...	96.36	...	96.28	...	97.00	...	97.83	98.19	...

ed m were supplied by the makers.

TABLE B.—PARTICULARS OF THREE-PHASE TRANSFORMERS

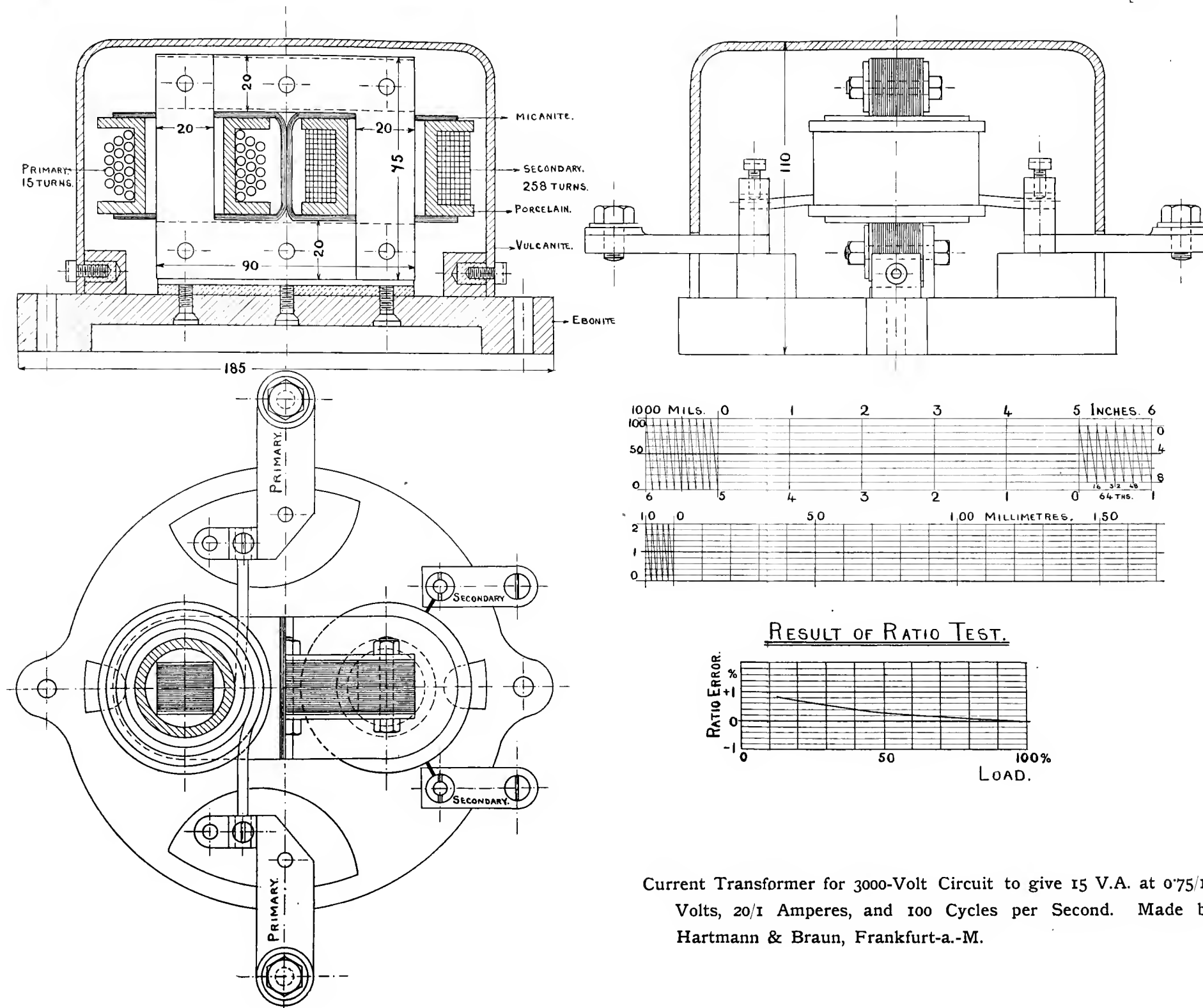
1	Plate No.			11.	12.	13.
2	Maker.		UNITS.	Elektricitäts Aktien Gesell- schaft (Kolben & Co.), Prague.	Ateliers de Construc- tions Electriques de Charleroi.	Brown, Boveri & Co., Baden.
3	Type—	Iron, magnetic circuit	...	Three-limb.	Three-limb.	Symmetrical
4		„ shape of section of core	...	Stepped.	Square.	Stepped.
5		„ number of ducts	Three.	One.
6		„ kind	...	Ordinary.	Ordinary.	Ordinary.
7		Coils, shape	...	Circular.	Circular.	Circular.
8		„ arrangement	...	Concentric.	Sandwiched.	Concentric.
9		Filling	...	Oil.	Oil.	Oil.
10		Cooling	...	Natural.	Natural.	Natural.
11		Remarks				
12	Units.		Mm. System. Inch System.	Millimetre.	Millimetre.	Millimetre.
13	Rating—	Output	Kilovolt-amperes.	20	75	110
14		Frequency	Cycles per second.	40	50	50
15		Connection	...	P. Star.	P. Star.	P. Star.
16		Voltage, line	Volts.	M 10,000	M 6300	M 5500
17		„ phase	„	5,780	3640	3180
18		Current, line	Amperes.	1.20	7.10	11.9
19		„ phase	„	1.20	7.10	11.9
20	Winding—	Number of turns per phase	...	M 3,454	M 1000	M 1120
21		Number of coils „	...	M 22	10	M 120
22		Turns per coil	...	M 157	125 & 62½	M 280
23		Layers „	...	M 11	21	M 11
24		Turns per layer	...	15 & 7	6 & 3	M 26 & 20
25	Conductor—	Gauge, if round	...	M 0.9φ	M 2.8φ	M 2.5φ
26		Diameter (φ) or dimensions, bare	Millimetres.	M 1.4φ	M 3.3φ	M 3.0φ
27		Ditto, covered	„	0.636	6.15	2 × 4.90
28		Section	10 ⁻³ inches ² .	33.2	4 × 84.6	10.98
29		Total section of copper per phase	Inches ² .	2.20	6.77	1.21
30		Current density	Amps. per mm. ²	1.89	1.07	0
31	Former—	Diameter (φ) or dimensions	Millimetres.	220φ	250φ	M 281φ
32		Perimeter of section	Inches.	691	785	883
33		Allowance for corners	„	„	„	„
34		Mean length of one turn	„	740	1004	992
35		Length of wire per phase	Metres.	2,555	1004	2 × 1110
36	Coils—	Depth of winding	Millimetres.	15.4	7.1	M 35
37		Axial length of each	Inches.	22.4	560	M 81
38		Exposed perimeter of section, per phase	„	22 × 44.8	1134	8 × 232
39		Cooling surface per phase	Metres ² .	0.730	0.613	1.84
40		Volume of copper „	Litres.	1.63	1.40	10.89
41		Mass of copper „	Kilograms.	14.5	12.5	96.9
42	Resistance—	Effective resistance, hot, per phase	Ohms.	92.3	0.0292	2.60
43		Copper loss per phase	Watts.	133	80.4	369
44		Dissipation intensity	Watts per metre ² .	183	131	200
45		Temperature rise	°C.	23	16	25
46	Dimensions—	Diameter or width of iron space	Millimetres.	140	...	180
47		Distance between yokes	Inches.	600	x = 4.29	M 800
48		Depth of copper space	„	75	y = 0.54	110
49		Depth of iron space	„
50		Net depth of iron	„
51	Length of	iron	Millimetres.	3,200	...	4140
52		copper	„	632	...	863
53		flux per phase (approx.)	„	960	...	1320
54	Section of	iron space	10 ³ mms. ²	15.4	...	25.5
55		iron, net	„	11.0	...	2 × 9.44
56		copper space	„	135	...	264
57		copper, net	„	14.37	...	76.4
58	Space-factor for	iron space	...	0.715	...	0.740
59		copper space	...	0.107	...	0.290
60		geometrical mean	...	0.277	...	0.463
61	Cooling surface of	iron space, conventional	Metres ² .	1.43	...	2.91
62		iron, actual	„	1.48	...	4.00
63		copper space, conventional	„	2.56	...	4.97
64		coils, actual	„	4.03	...	8.88
65		case	„	2.75	...	11.8
66		water pipes	„
	Volume of	iron	Litres and	35.2	2145	78.0
		copper	(10 ⁶ mms. ³)	9.09	554	65.9
		oil	„ Gallons.	220	48.5	680
	Mass of	iron (7.80 kgs. per litre ; 0.282 lbs. per inch ³)	Kilograms and	274	604	609
		copper (8.90 kgs. per litre ; 0.322 lbs. per inch ³)	Pounds.	81	178	587
		total active material	„	355	782	1196
		standard iron of same cost	„	517	1140	2370
		ditto × {√P _I P _C /½(I ₁ V ₁ + I ₂ V ₂)} ³ × {f ÷ 50 C.P.S.} ³	Grams and	7.52	16.6	5.55
		c (0.88 kg. per litre ; 8.8 lbs. per gal.)	Kilograms „	193	426	598
	fundamental	length	Millimetres and	32.1	1.26	43.4
		surface	„ Inches.	1,030	1.59	1880
		volume	10 ³ mms. ³ „	33.0	2.00	81.7
79	Flux density	(maximum)	10 ⁻⁶ V.S. per mm. ² & 10 ⁻³ V.S. per inch ² .	0.856	0.553	0.782
80	Current density	(R.M.S.)	Amperes per mm. ² & Amperes per inch ² .	1.73	1.115	1.08
81	Losses—	Specific iron loss	Watts per litre and Watts per inch ³ .	14	...	15.8
82		Iron loss	Watts „ Per cent.	492	2.33%	M 1230

TRANSFORMERS.											
13.		14.		15.		16.		17.		18.	
Crown, Boveri Co., Baden.		Ateliers de Constructions Electriques de Charleroi.		Gesellschaft für Elektrische Industrie, Karlsruhe.		Elektricitäts Aktien Gesellschaft (Kolben & Co.), Prague.		Maschinenfabrik Oerlikon. Oerlikon.		Brown, Boveri & Co., Baden.	
Symmetrical. Stepped. One. Ordinary. Circular. Concentric. Oil. Natural.		Three-limb. Square. Three. Ordinary. Circular. Sandwiched. Oil. Natural.		Symmetrical. Stepped. Three. Ordinary. Circular. Concentric. Oil. Natural.		Three-limb. Stepped. Three. Ordinary. Circular. Concentric. Oil. Water.		Three-limb. Rectangular. Two. Ordinary. Rectangular. Concentric. Air. Forced draught. See fig. 11-01.		Three-limb. Ordinary. Circular. Concentric. Oil. Water.	
Millimetre.		Millimetre.		Millimetre.		Millimetre.		Millimetre.		Millimetre.	
110 50		130 50		200 50		350 40		770 50		1400 50	
P. Star.	S. Star.	P. Star.	S. Star.	P. Star.	S. Star.	P. Star.	S. Star.	P. Star.	S. Star.	P. Mesh.	S. Star.
500	M 440	M 550	M 2000	M 5000	M 215	M 10,000	M 2,000	M 5000	M 20,000	M 3,000	M 26,000
180	254	318	1155	2886	124	5,780	1,154	2,886	11,540	M 3,000	M 15,000
9	145	M 140	M 37·5	23·9	538	20·7	101	91	22·2	274	31·1
9	145	140	37·5	23·9	538	20·7	101	91	22·2	158	31·1
120	M 94	M 81	M 304	M 736	M 33	768	156	M 158	M 630	M 252	M 1,344
1/2 x 8	M 1	M 7	M 12	M 16	M 1	24	6	1	15	M 6	M 16
280	M 94	13 & 8	25 & 26	M 46	M 33	32	26	158	1/2 x 84	M 42	M 84
111	M 2	13 & 8	9	M 5	M 1	4	3	2	14	3	6
3 & 20	M 47	1	3 & 2	M 10 & 6	M 33	8	9 & 8	79	6	14	14
5φ	M 11 x 14	M 29 x 4·5	M 6·7φ	M 4·3φ	M 4·23 x 3·5	4·0φ	14 x 4·5	M 8 x 6·5	M 3·6φ	M 10·5 x 10·5	M 4·6φ
0φ	M 12 x 15	30 x 5·5	M 7·2φ	M 4·8φ	bare	4·5φ	15 x 5·5	9 x 7·5	4·0	M 11·5 x 11·5	M 5·2φ
4·90	154	130·5	35·2	14·5	322	12·58	63	52	2 x 10·18	110·2	16·6
98	14·50	10·57	10·69	10·68	10·62	9·65	9·83	8·22	12·80	27·80	22·35
21	0·94	1·073	1·062	1·65	1·67	1·65	1·61	1·75	1·09	1·43	1·87
281φ	M 214φ	M 296φ	M 296φ	M 310φ	M 240φ	370φ	450φ	M 294 x 574	M 350 x 630	335φ	440φ
883	672	930	930	974	754	1,162	1,415	1,736	1,960	1,050	1,380
992	751	1130	1130	M 1060	M 800	1,220	1,465	1,790	2,130	M 1,160	M 1,485
1110	M 72	91·5	344	780	26·4	938	229	283	1,340	292	1,998
M 35	M 25	71·5 & 44	64·8	24·0	15·0	18·0	16·5	18	56	34·5	31·2
M 81	M 720	30	21·6	52·8	980	40·5	150	600	28	173	78
x 232	1490	590	580	16 x 154	33 x 77	2,000	6 x 333	1,236	940	6 x 416	2,560
84	1·12	0·666	0·655	2·61	2·03	2·44	2·93	2·21	2·00	2·90	3·80
89	11·09	11·95	12·10	11·31	8·50	11·80	14·40	14·70	27·3	32·2	33·2
9	98·6	106·2	107·7	100·7	75·7	105·0	128·2	131	243	287	296
60	0·0108	0·0161	0·225	1·24	0·00189	1·72	0·0835	0·125	1·515	0·0610	2·77
369	226	317	316	707	546	735	850	1,035	745	1,520	2,680
200	202	475	483	271	269	300	290	469	373	525	705
25	26	60	60	34	34	38	36	52	42	66	88
180	...	270	...	205	...	330	...	230	...	270	...
800	x = 4·45	600	x = 2·22	1000	x = 4·88	1,200	x = 3·64	700	x = 3·04	1,300	x = 4·81
110	y = 0·61	105	y = 0·39	95	y = 0·46	100	y = 0·30	140	y = 0·61	145	y = 0·54
...	480	z = 2·09
...	385
140	...	4120	...	5840	...	6,320	...	5,090	...	6,680	...
863	...	1130	...	930	...	1,350	...	2,000	...	1,305	...
320	...	1180	...	1630	...	1,900	...	1,380	...	2,000	...
5	...	57·2	...	33·0	...	85·7	...	112·0	...	57·2	...
9·44	...	M 29·5	...	2 x 11·24	...	57·0	...	88·5	...	M 42·8	...
264	...	189	...	285	...	360	...	294	...	566	...
4	...	63·8	...	63·9	...	58·4	...	63·1	...	150·5	...
740	...	0·516	...	0·680	...	0·665	...	0·790	...	0·748	...
290	...	0·338	...	0·224	...	0·162	...	0·215	...	0·266	...
463	...	0·418	...	0·390	...	0·328	...	0·412	...	0·446	...
91	...	3·50	...	3·22	...	6·61	...	6·31	...	5·67	...
00	...	7·93	...	17·5	...	15·8	...	10·96	...	15·5	...
97	...	5·00	...	6·20	...	10·52	...	10·08	...	11·3	...
88	...	3·96	...	13·9	...	16·1	...	12·63	...	20·1	...
8	...	5·10	...	M 17·1	...	35	...	8·3	...	11·3	...
...	3·14	11·02	...
0	4760	121·5	7410	130·9	7980	360	21,900	450	27,450	286	17,450
9	4015	72·2	4400	59·4	3620	78·6	4,800	126	7,690	196	11,950
380	150	550	121	830	183	2,770	610	1,960	431
609	1340	948	2090	1021	2250	2,810	6,200	3,510	7,740	2,230	4,910
587	1292	642	1420	530	1170	700	1,540	1,122	2,475	M 1,750	3,860
196	2640	1590	3510	1551	3420	3,510	7,740	4,632	10,200	3,980	8,760
370	5220	2874	6350	2611	5760	4,910	10,820	6,876	15,150	7,480	16,500
55	12·2	4·44	9·80	13·0	28·7	6·42	14·2	5·70	12·6	4·59	10·15
998	1320	484	1065	730	1610	2,435	5,360	1,725	3,800
4	1·70	48·3	1·90	41·6	1·64	60·1	2·37	56·9	2·24	61·0	2·40
880	2·91	2330	3·61	1730	2·69	3,610	5·60	3,230	5·01	3,720	5·76
7	4·98	112·5	6·87	72·0	4·39	218	13·25	184	11·22	227	13·80
782	0·504	0·599	0·386	0·908	0·585	0·743	0·479	0·930	0·600	1·250	0·806
08	696	1·068	688	1·66	1070	1·63	1,050	1·42	915	1·65	1,063
8	...	10·0	...	24·5	...	11·0	...	22·2	...	40	...
230	1·09%	1215	0·91%	M 3200	1·55%	3,960	1·10%	M 10,000	1·27%	M 11,400	0·80%
85	1·58%	1900	1·43%	M 3760	1·82%	4,755	1·33%	5,340	0·68%	12,600	0·88%

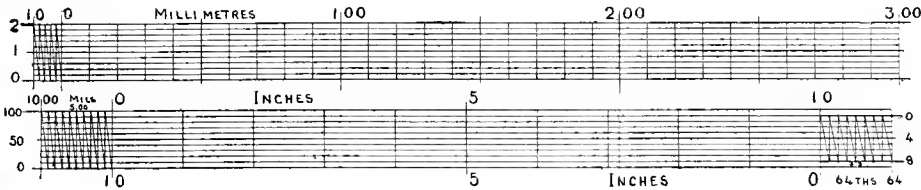
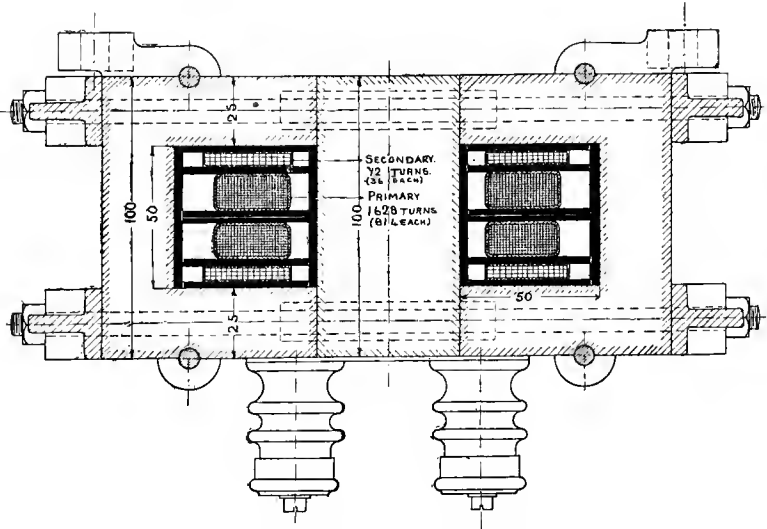
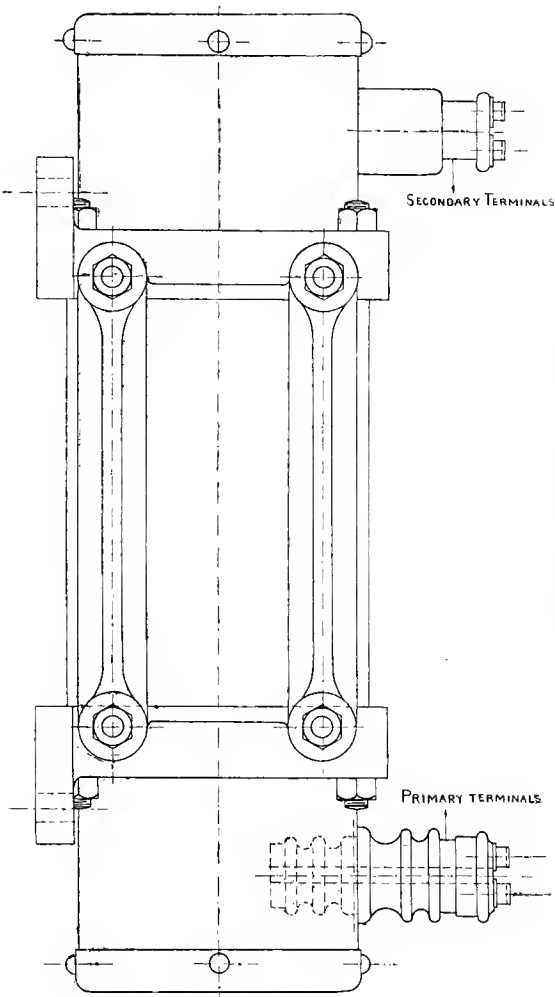
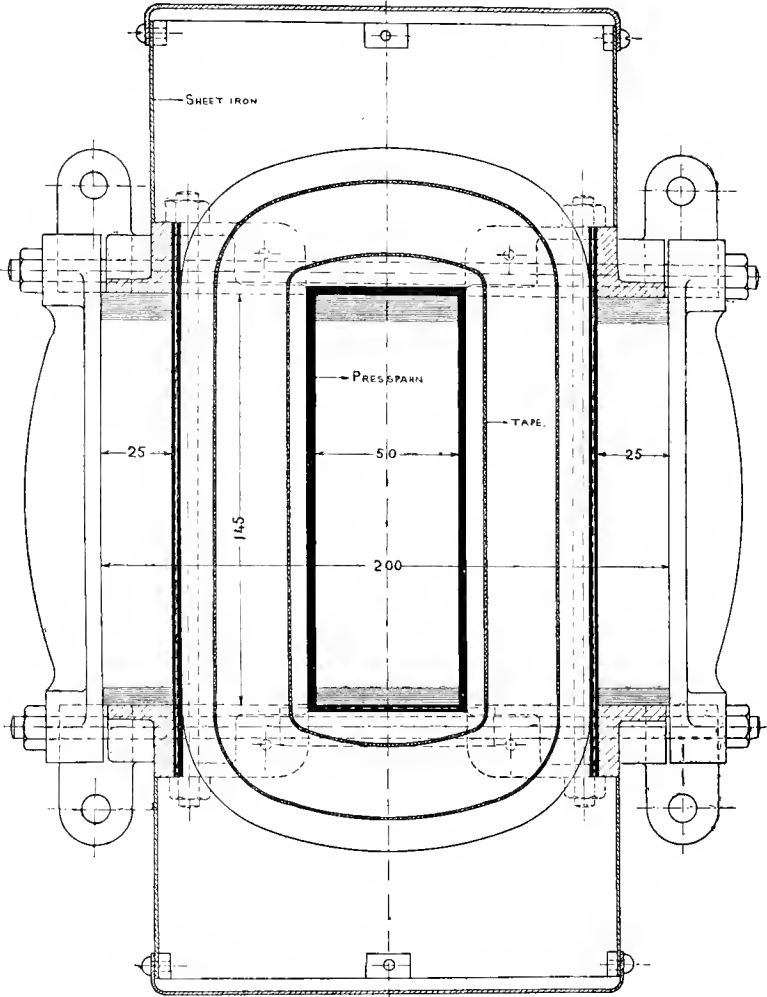
46	Dimensions—	Diameter or width of iron space	l_{IS}	Millimetres.	Inches.	140	...	220	...	180
47		Distance between yokes	$L_{CS} = x l_{IS}$	"	"	600	$x = 4.29$	480	$x = 2.18$	M 800
48		Depth of copper space	$l_{CS} = y l_{IS}$	"	"	75	$y = 0.54$	95	$y = 0.43$	110
49		Depth of iron space	$L_{IS} = z l_{IS}$	"	"
50		Net depth of iron		"	"
51	Length of	iron	$L_I = V_I / S_I$	Millimetres.	Inches.	3,200	...	3370	...	4140
52		copper	$L_C = V_C / S_C$	"	"	632	...	975	...	863
53		flux per phase (approx.)		"	"	960	...	1000	...	1320
54	Section of	iron space	S_I	10^3 mms.^2	Inches ² .	15.4	...	38.0	...	25.5
55		iron, net		"	"	11.0	...	M 19.3	...	2 x 9.44
56		copper space	S_C	"	"	135	...	137	...	264
57		copper, net		"	"	14.37	...	38.8	...	76.4
58	Space-factor for	iron space		0.715	...	0.509	...	0.740
59		copper space		0.107	...	0.283	...	0.290
60		geometrical mean		0.277	...	0.380	...	0.463
61	Cooling surface of	iron space, conventional		Metres ² .	Inches ² .	1.43	...	2.43	...	2.91
62		iron, actual		"	"	1.48	...	5.29	...	4.00
63		copper space, conventional		"	"	2.56	...	3.42	...	4.97
64		coils, actual		"	"	4.03	...	2.67	...	8.88
65		case		"	"	2.75	...	15.5	...	11.8
66		water pipes		"	"
67	Volume of	iron		Litres and	Inches ³ .	35.2	2145	65.0	3960	78.0
68		copper		(10^6 mms.^3)	"	9.09	554	37.7	2300	65.9
69		oil		"	Gallons.	220	48.5	600	132	680
70	Mass of	iron (7.80 kgs. per litre ; 0.282 lbs. per inch ³)		Kilograms and	Pounds.	274	604	507	1118	609
71		copper (8.90 kgs. per litre ; 0.322 lbs. per inch ³)		"	"	81	178	336	741	587
72		total active material		"	"	355	782	843	1860	1196
73		standard iron of same cost		"	"	517	1140	1515	3340	2370
74		ditto $\times \{ \sqrt{P_I P_C} / \frac{1}{2} (l_1 V_1 + l_2 V_2) \}^3 \times \{ f \div 50 \text{ C.P.S.} \}^3$		Grams and	10^{-3} lbs.	7.52	16.6	4.80	10.6	5.55
75		c (0.88 kg. per litre ; 8.8 lbs. per gal.)		Kilograms "	Pounds.	193	426	528	1160	598
76	Fundamental	length		Millimetres and	Inches.	32.1	1.26	39.8	1.57	43.4
77		surface		Millimetres ² "	Inches ² .	1,030	1.59	1590	2.46	1880
78		volume		10^3 mms.^3 "	Inches ³ .	33.0	2.00	63.0	3.85	81.7
79	Flux density	(maximum)		10^{-6} V.S. per mm. ² & 10^{-3} V.S. per inch ² .		0.856	0.553	0.850	0.548	0.782
80	Current density	(R.M.S.)		Amperes per mm. ² & Amperes per inch ² .		1.73	1115	1.11	716	1.08
81	Losses—	Specific iron loss	P_I	Watts per litre and Watts per inch ³ .		14	...	18	...	15.8
82		Iron loss		Watts "	Per cent.	492	2.33%	1170	1.52%	M 1230
83		Copper loss	P_C	" "	"	640	3.03%	1068	1.38%	1785
84		Total loss		" "	"	1,132	5.36%	2238	2.90%	3015
85		Geometrical mean	$\sqrt{P_I P_C}$	" "	"	561	2.66%	1118	1.45%	1480
86	Dissipation intensity for	conventional iron space		Watts per metre ² and Watts per inch ² .		344	0.222	482	0.311	423
87		iron, actual		" "	"	332	0.214	221	0.143	308
88		conventional copper space		" "	"	250	0.161	312	0.201	360
89		coils, actual		" "	"	159	0.103	400	0.258	201
90		case		" "	"	412	0.266	144	0.093	255
91		water-pipes		" "	"
92	Temperature rise for	iron, above oil and above room		°C.		20°	53°	13°	25°	18°
93		copper, " "		Resistance °C.		13°	46°	32°	44°	16°
94		oil, above room		°C.		...	33°	...	12°	...
95	Excitation—	Magnetising field (max.) for iron		Amp.-tns. per mm. & Amp.-tns. per inch.		0.25	...	0.25	...	0.23
96		for iron (R.M.S.) per phase (approx.)		Ampere-turns.		170	...	177	...	215
97		for joints " "		" "		35	...	35	...	32
98		Total, effective " "		" "		205	...	212	...	247
99	Exciting current—	Idle component, per phase		Amperes and Per cent.		0.0594	4.94%	0.212	2.99%	0.221
100		Working " "		" "		0.0284	2.36%	0.107	1.51%	0.129
101		Total " "		" "		0.0658	5.48%	0.238	3.35%	0.256
102	Magnetic leakage—	Space between coils		Millimetres and Inches.		20.4	...	24	...	10
103		$\frac{1}{2}$ of winding depths, or thicknesses		" "		7.5	...	133	...	20
104		Total equivalent thickness		" "		27.9	...	157	...	30
105	Total equivalent	resistance per phase, referred to secondary		Ohms.		0.0775	...	0.00273	...	0.0283
106		leakage reactance, " "		"		0.0572	...	0.0256	...	0.115
107		internal impedance, " "		"		0.0962	...	0.0251	...	0.118
108		characteristic lag, $\cos \phi_T$ and ϕ_T		"		0.81	36°	0.106	84°	0.24
109	Voltage drop at	unity power-factor	$= R_{T2} I_2$	Volts and Per cent.		4.06	3.20%	0.986	1.42%	4.10
110		zero (lagging) " "	$= \mathcal{L}_{TL2} \omega I_2$	" "		3.00	2.36%	9.24	13.3%	16.7
111		$\cos \phi_T$ " "	$= Z_{T2} I_2$	" "		5.05	3.98%	9.26	13.4%	17.1
112	Efficiency at	$\frac{1}{4}$ load		Per cent.		90.38	...	93.81	...	95.35
113		$\frac{1}{2}$ " "		"		93.88	...	96.31	...	97.04
114		$\frac{3}{4}$ " "		"		94.63	...	96.95	...	97.36
115		full " "		"		94.64	...	97.10	...	97.33
116		$\frac{5}{4}$ " "		"		94.36	...	97.06	...	97.16

NOTE.—The figures marked M were supplied by the manufacturer.

800	...	270	...	205	...	330	...	230	...	270	...
110	$x=4.45$	600	$x=2.22$	1000	$x=4.88$	1,200	$x=3.64$	700	$x=3.04$	1,300	$x=4.81$
...	$y=0.61$	105	$y=0.39$	95	$y=0.46$	100	$y=0.30$	140	$y=0.61$	145	$y=0.54$
...	480	$z=2.09$
...	385
4140	...	4120	...	5840	...	6,320	...	5,090	...	6,680	...
863	...	1130	...	930	...	1,350	...	2,000	...	1,305	...
1320	...	1180	...	1630	...	1,900	...	1,380	...	2,000	...
25.5	...	57.2	...	33.0	...	85.7	...	112.0	...	57.2	...
$\times 9.44$...	$\times 29.5$...	2×11.24	...	57.0	...	88.5	...	$\times 42.8$...
264	...	189	...	285	...	360	...	294	...	566	...
76.4	...	63.8	...	63.9	...	58.4	...	63.1	...	150.5	...
0.740	...	0.516	...	0.680	...	0.665	...	0.790	...	0.748	...
0.290	...	0.338	...	0.224	...	0.162	...	0.215	...	0.266	...
0.463	...	0.418	...	0.390	...	0.328	...	0.412	...	0.446	...
2.91	...	3.50	...	3.22	...	6.61	...	6.31	...	5.67	...
4.00	...	7.93	...	17.5	...	15.8	...	10.96	...	15.5	...
4.97	...	5.00	...	6.20	...	10.52	...	10.08	...	11.3	...
8.88	...	3.96	...	13.9	...	16.1	...	12.63	...	20.1	...
11.8	...	5.10	...	$\times 17.1$...	35	...	8.3	...	11.3	...
...	3.14	11.02	...
78.0	4760	121.5	7410	130.9	7980	360	21,900	450	27,450	286	17,450
65.9	4015	72.2	4400	59.4	3620	78.6	4,800	126	7,690	196	11,950
680	150	550	121	830	183	2,770	610	1,960	431
609	1340	948	2090	1021	2250	2,810	6,200	3,510	7,740	2,230	4,910
587	1292	642	1420	530	1170	700	1,540	1,122	2,475	$\times 1,750$	3,860
1196	2640	1590	3510	1551	3420	3,510	7,740	4,632	10,200	3,980	8,760
2370	5220	2874	6350	2611	5760	4,910	10,820	6,876	15,150	7,480	16,500
5.55	12.2	4.44	9.80	13.0	28.7	6.42	14.2	5.70	12.6	4.59	10.15
598	1320	484	1065	730	1610	2,435	5,360	1,725	3,800
43.4	1.70	48.3	1.90	41.6	1.64	60.1	2.37	56.9	2.24	61.0	2.40
1880	2.91	2330	3.61	1730	2.69	3,610	5.60	3,230	5.01	3,720	5.76
81.7	4.98	112.5	6.87	72.0	4.39	218	13.25	184	11.22	227	13.80
0.782	0.504	0.599	0.386	0.908	0.585	0.743	0.479	0.930	0.600	1.250	0.806
1.08	696	1.068	688	1.66	1070	1.63	1,050	1.42	915	1.65	1,063
15.8	...	10.0	...	24.5	...	11.0	...	22.2	...	40	...
$\times 1230$	1.09%	1215	0.91%	$\times 3200$	1.55%	3,960	1.10%	$\times 10,000$	1.27%	$\times 11,400$	0.80%
1785	1.58%	1900	1.43%	$\times 3760$	1.82%	4,755	1.33%	5,340	0.68%	12,600	0.88%
3015	2.67%	3115	2.34%	6960	3.37%	8,715	2.43%	15,340	1.95%	24,000	1.68%
1480	1.31%	1520	1.14%	3470	1.68%	4,340	1.21%	7,320	0.93%	12,000	0.84%
423	0.273	348	0.224	994	0.640	600	0.387	1,585	1.023	2,010	1.297
308	0.199	153	0.099	183	0.118	250	0.161	914	0.589	735	0.474
360	0.232	380	0.245	607	0.392	452	0.292	530	0.342	1,115	0.720
201	0.130	480	0.310	271	0.175	295	0.190	422	0.272	628	0.405
255	0.165	611	0.394	407	0.262	250	0.161	1,850	1.19	2,120	1.37
...	2,780	1.79	2,180	1.41
18°	38°	10°	59°	11°	44°	15°	$\times 50^\circ$	44°	...
16°	36°	38°	87°	22°	55°	24°	$\times 50^\circ$	50°	...
...	20°	...	49°	...	33°
0.23	...	0.16	...	0.28	...	0.20	...	0.30	...	0.58	...
215	...	134	...	323	...	269	...	293	...	820	...
32	...	71	...	37	...	30	...	37	...	50	...
247	...	205	...	360	...	299	...	330	...	870	...
0.221	1.86%	2.53	1.81%	0.489	2.05%	0.390	1.88%	2.09	2.30%	3.45	2.18
0.129	1.09%	1.27	0.91%	0.370	1.55%	0.228	1.10%	1.15	1.26%	1.27	0.80
0.256	2.15%	2.84	2.03%	0.614	2.57%	0.451	2.18%	2.38	2.62%	3.67	2.32
10	...	75	...	20	...	23	...	10	...	18	...
20	...	162	...	13	...	11.5	...	25	...	22	...
30	...	237	...	33	...	34.5	...	35	...	40	...
0.0283	...	0.451	...	0.00433	...	0.155	...	3.61	...	4.35	...
0.115	...	0.92	...	0.0134	...	0.302	...	15.9	...	28.9	...
0.118	...	1.02	...	0.0140	...	0.340	...	16.3	...	29.2	...
0.24	76°	0.44	64°	0.31	72°	0.46	63°	0.22	77°	0.15	81°
4.10	1.61%	16.9	1.46%	2.33	1.88%	15.7	1.36%	80.2	0.69%	135	0.90%
16.7	6.57%	34.5	2.99%	7.21	5.81%	30.5	2.65%	353	3.06%	900	6.00%
17.1	6.74%	38.3	3.32%	7.55	6.09%	34.4	2.98%	362	3.14%	908	6.05%
95.35	...	96.05	...	93.56	...	95.35	...	94.90	...	96.64	...
97.04	...	97.77	...	96.04	...	97.14	...	96.10	...	97.96	...
97.36	...	97.71	...	96.57	...	97.54	...	97.80	...	98.27	...
97.33	$\times 97$	97.66	...	96.63	...	97.57	$\times 97.6$	98.05	$\times 98$	98.32	$\times 98.4$
97.16	...	97.49	...	96.49	...	97.46	...	98.13	...	98.25	...

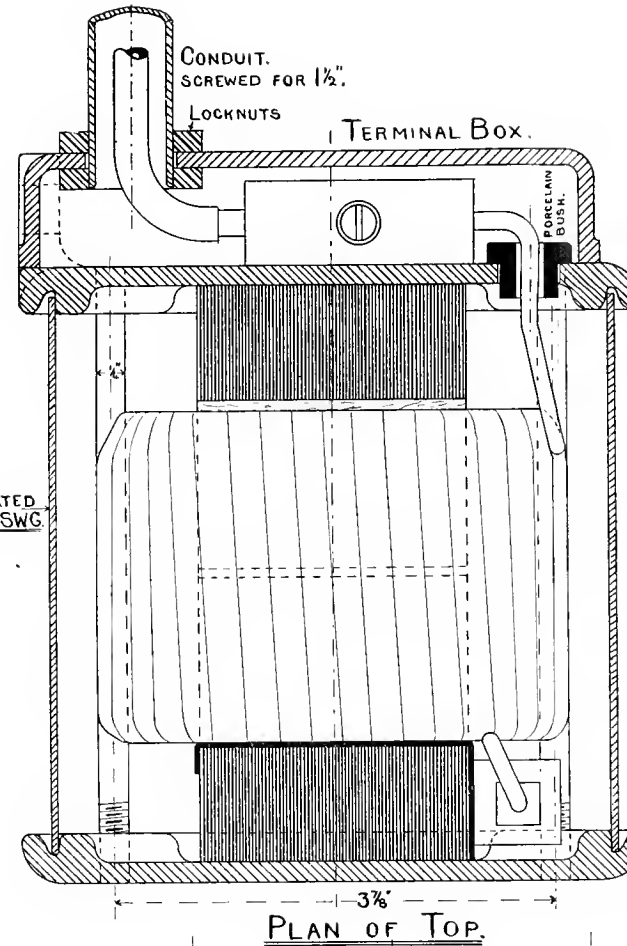
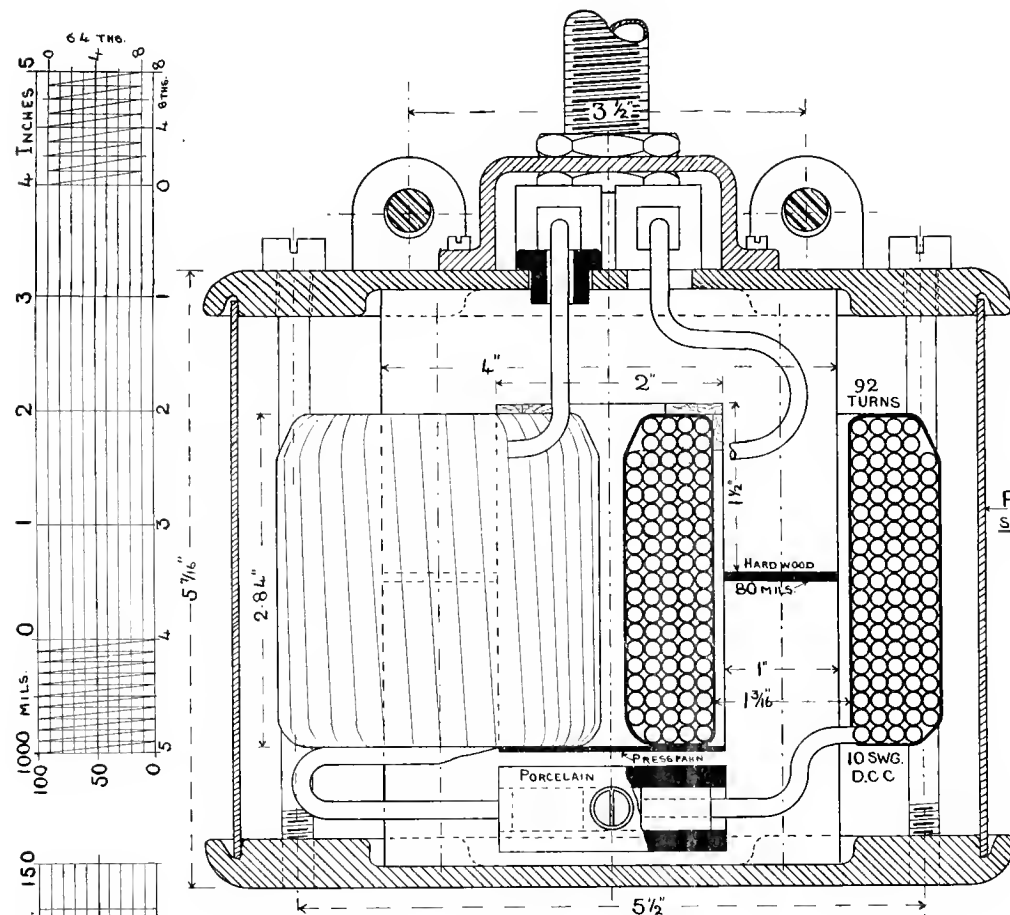


Current Transformer for 3000-Volt Circuit to give 15 V.A. at 0.75/15 Volts, 20/1 Amperes, and 100 Cycles per Second. Made by Hartmann & Braun, Frankfurt-a.-M.



Voltmeter Transformer of the Rectangular Coil Shell Type to give 0.08 K.V.A. at 2500/110 Volts, 0.06, 0.7 Amperes, and 100 Cycles per Second. Made by Hartmann & Braun, Frankfurt-a.-M.

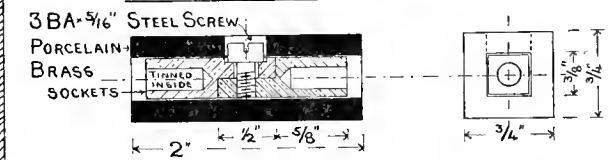
Iron loss	41	watts = 33 per cent.
Copper loss	1.1	„ = 0.9 „
Total „	42	„ = 34 „
Iron mass	14.4	kgs. = 31.8 lbs.
Copper mass	1.45	„ = 3.2 „
Standard „	18.8	„ = 41.5 „
Flux density (max.)	5620	lines/cm. ² = 0.362 M.V.S./in. ²
Current density (R.M.S.)	0.53	amps./mm. ² = 340 amps./in. ²
Resistance (hot):	$R_1 = 221$ ohms, $R_2 = 0.55$ ohms, $R_{T_2} = 2.14$ ohms.		
Turns:	$N_1 = 1628 = 2 \times 814$, $N_2 = 72 = 2 \times 36$.		



Design for a Choking Coil to absorb 1 K.V.A.
at 100 Volts, 10 Amperes, and 100 Cycles
per Second. (Transformer Rating, 1/2
K.V.A.) (See pages 243 and 245.)

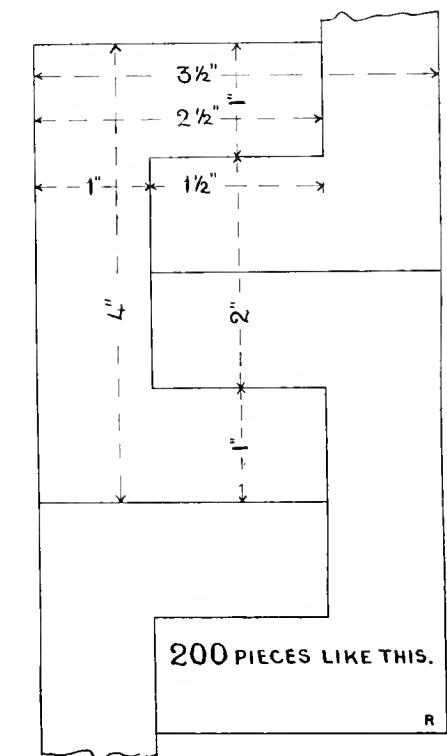
Iron loss	12.6 watts = 1.26 %
Copper loss	12.4 " = 1.24 %
Total	25 " = 2.5 %
Resistance (hot)	0.124 ohms.
Total turns	184.
Iron mass	7.90 lbs.
Copper mass	7.34 "
Standard	35.8 "
$B = 0.613$ M.V.S./in. ²	= 9500 lines/cm. ²
$I = 778$ amps./in. ²	= 1.20 amps. mm. ²

DETAIL OF TERMINAL BLOCKS.



METHOD OF CUTTING SHEETS.

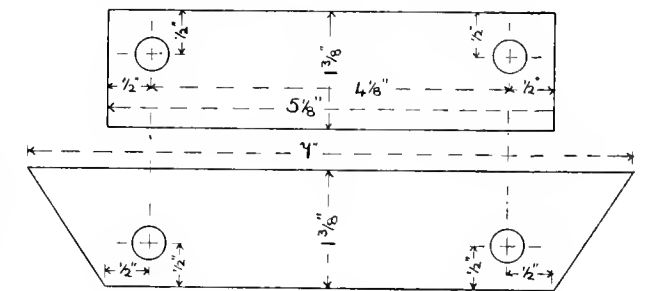
STALLOY IRON. 20 MILS THICK.



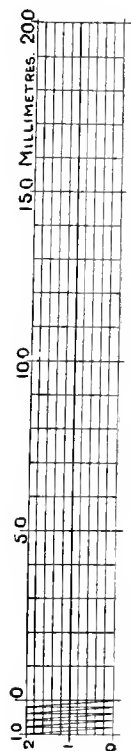
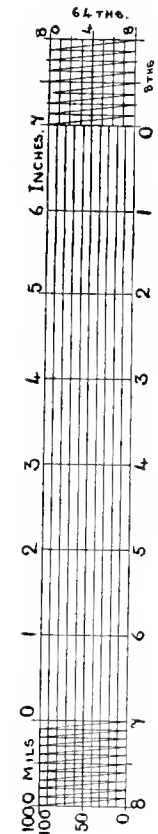
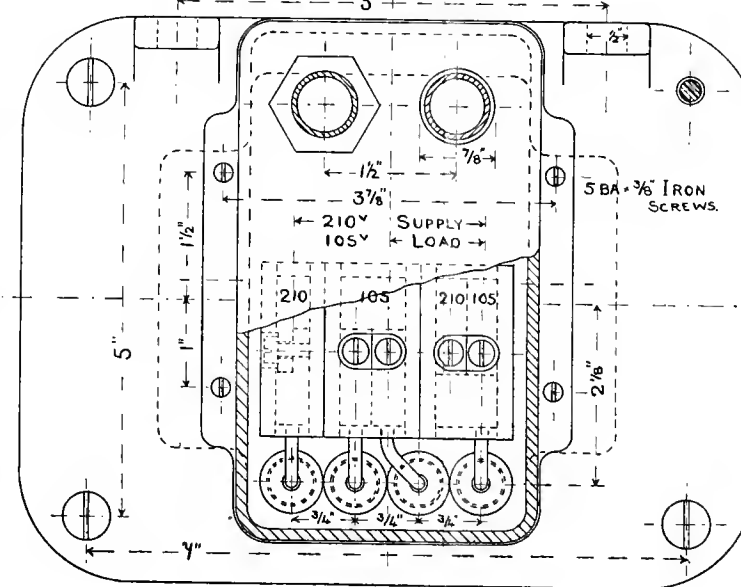
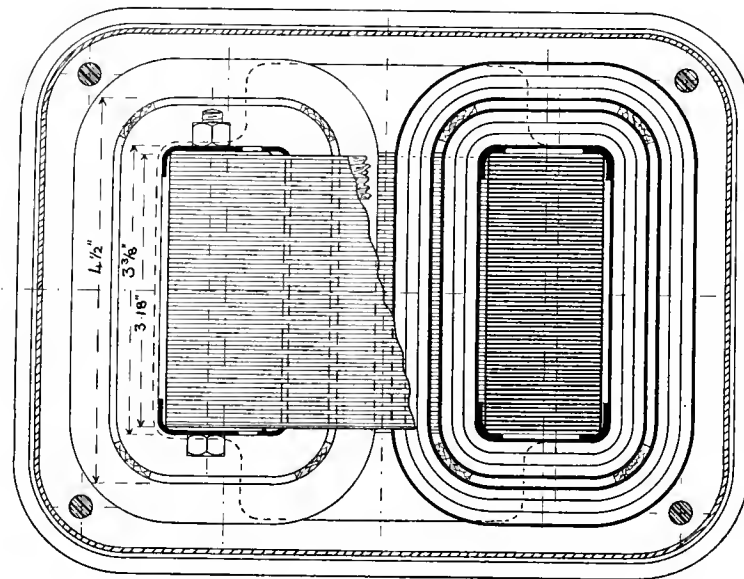
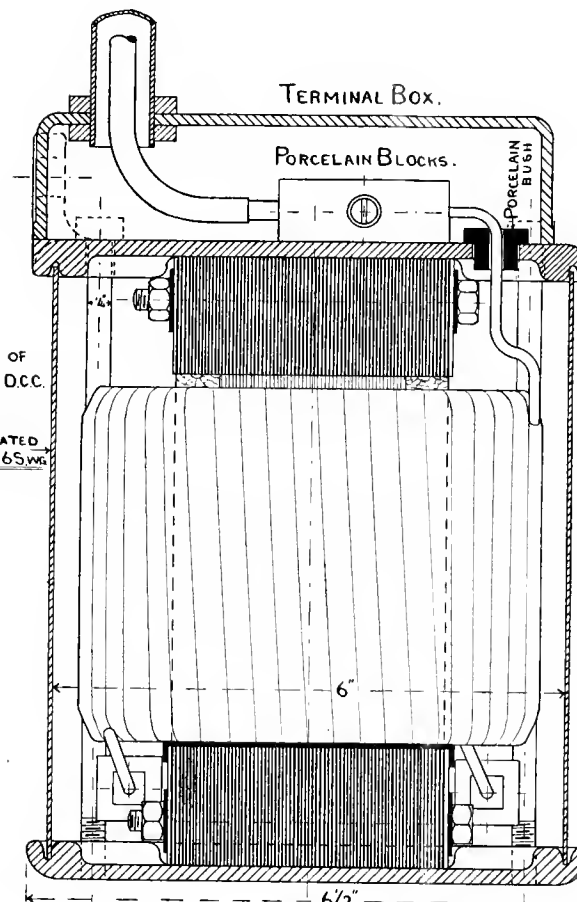
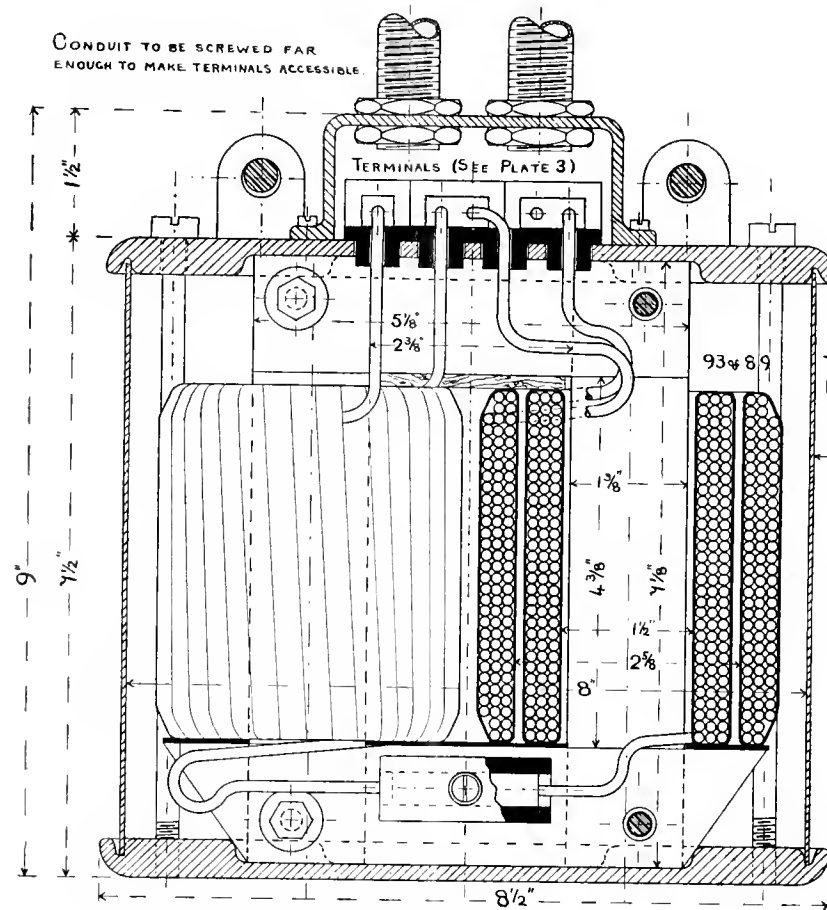
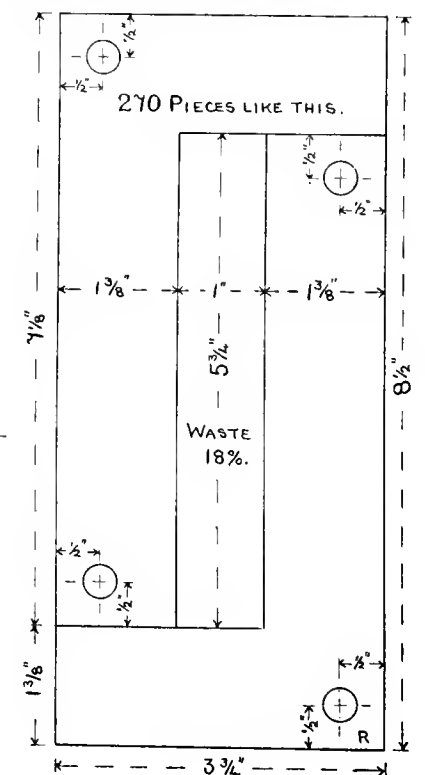
Design for an Auto-Transformer to give 2 K.V.A. at 210/105 Volts, 10/19 Amperes, and 50 Cycles per Second. (Transformer Rating, 1 K.V.A.) (See pages 247 and 250.)

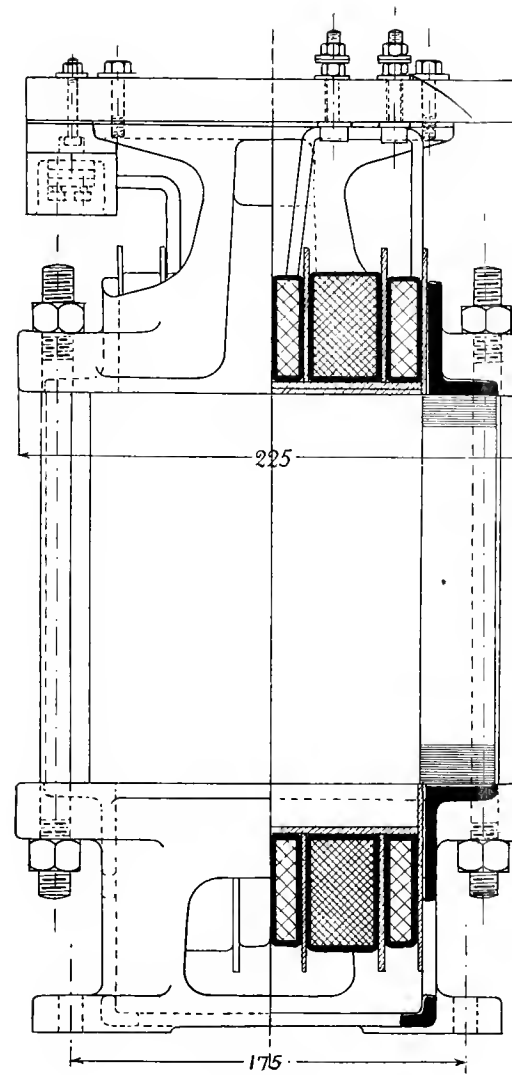
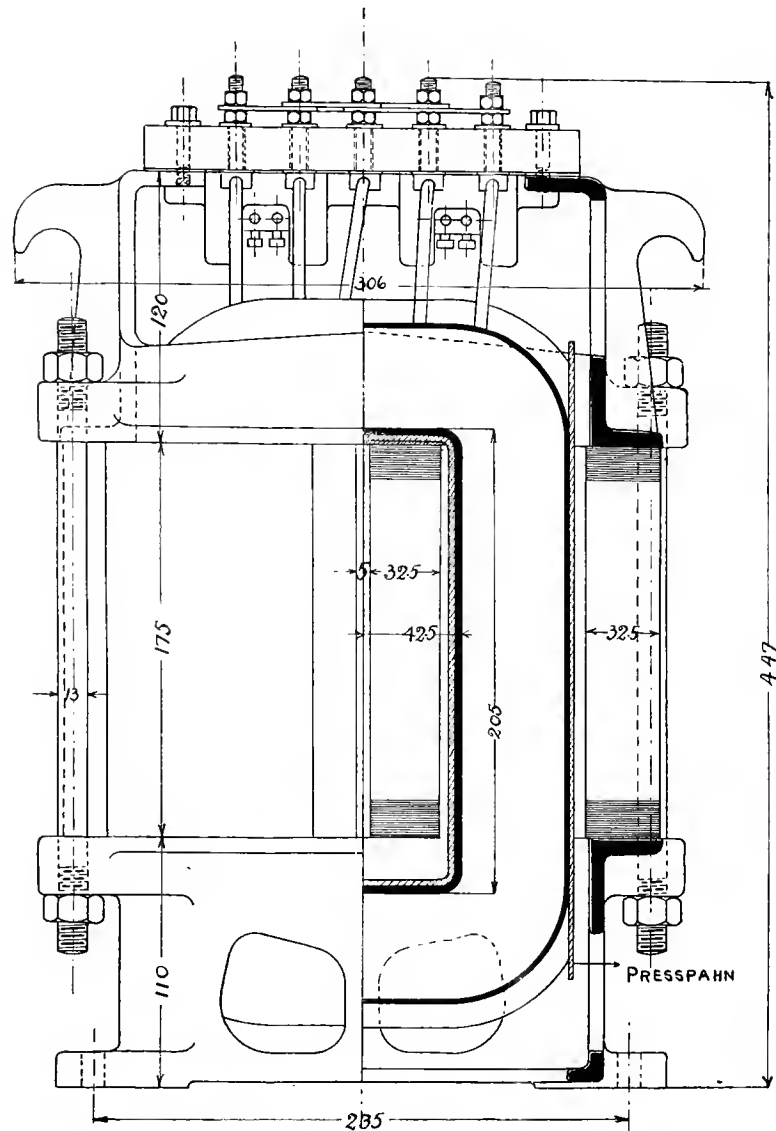
Iron loss	20 watts = 1 %
Copper loss	40 " = 2 %
Total	60 " = 3 %
$R_1 = 0.235\omega$ $R_2 = 0.174\omega$ $R_{T_2} = 0.440\omega$ (hot).	
$N_1 = 178 = 2 \times 89$ $N_2 = 186 = 2 \times 93$	
Iron mass	20 lbs.
Copper mass	16 "
Standard "	84 "
$B = 0.72$ M.V.S./in. ² = 11,200 lines/cm. ²	
$f = 928$ amps./in. ² = 1.44 amps./mm. ²	

SIDE PLATES FOR TOP & BOTTOM.
TWO EACH, 16 SWG.



METHOD OF CUTTING SHEETS.
STALLOY IRON 20 MILS THICK.



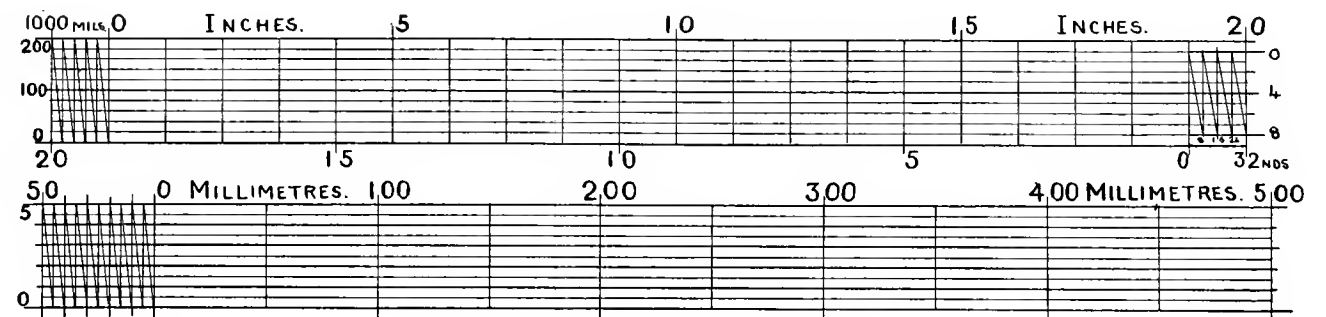
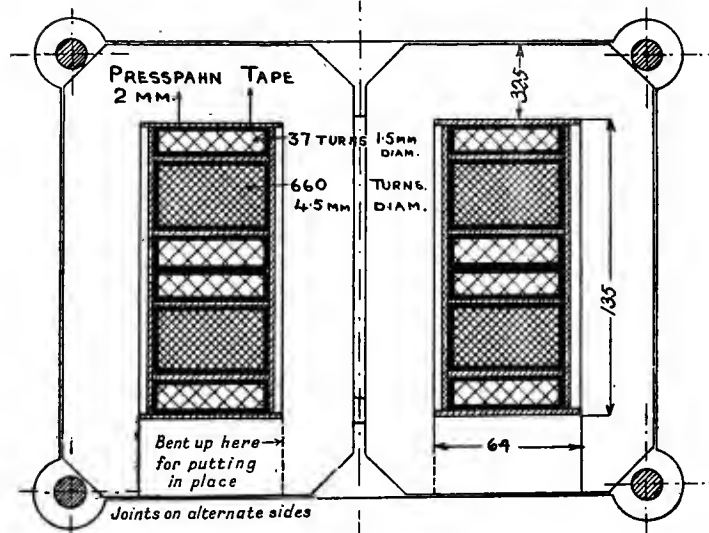


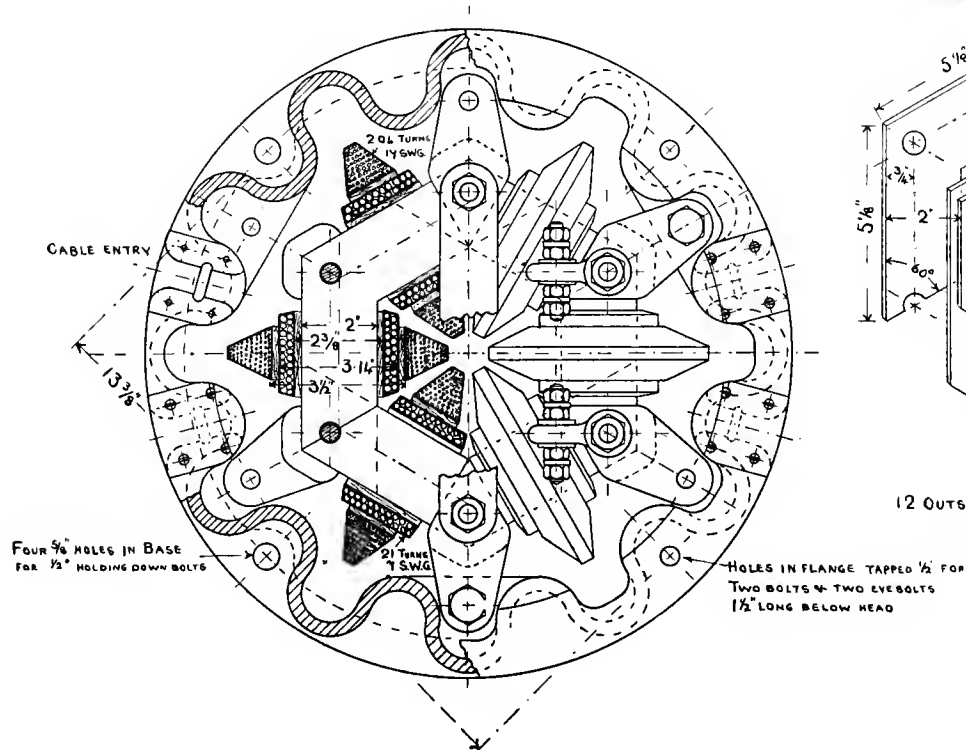
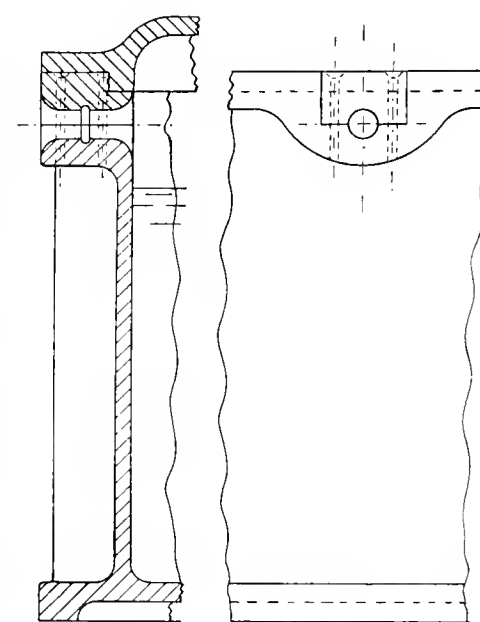
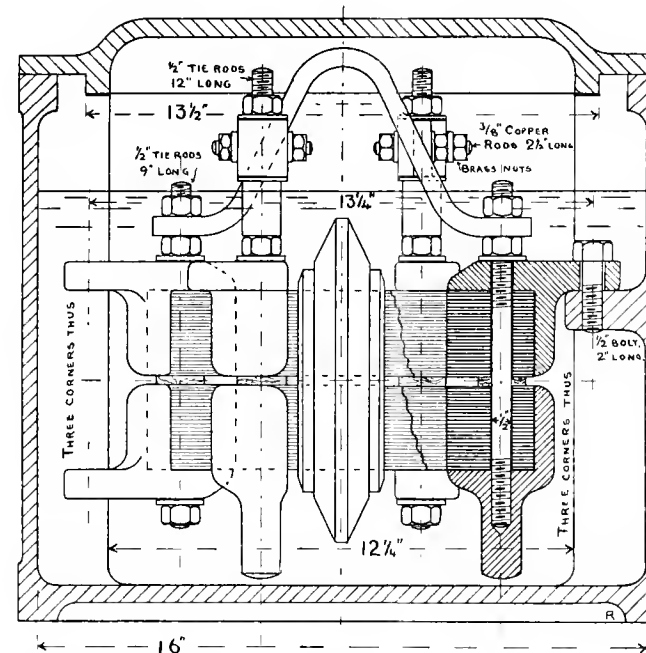
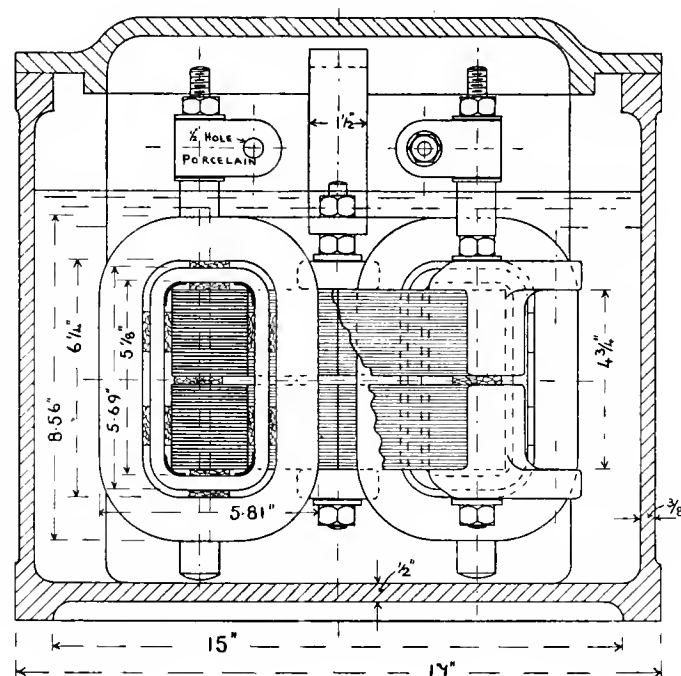
Oil-Insulated Rectangular Coil Shell Transformer
to give 3 K.V.A. at 2000/220 Volts, 1.6/14
Amperes, and 50 Cycles per Second. Made by
Elektricitäts Allgemeine Gesellschaft, Cölne.
(See also fig. 9.10.)

Iron loss	56 watts = 1.80 per cent.
Copper loss	59 „ = 1.90 „
Total „	115 „ = 3.70 „

Iron mass	43 kgs.
Copper mass	30 „
Standard mass of same cost (296 lbs.)	134 „

Resistance (hot): $R_1 = 12.4\omega$	$R_2 = 0.155\omega$	$R_{T2} = 0.32\omega$
Turns:	$N_1 = 1320 = 2 \times 660$	$N_2 = 148 = 4 \times 37$
Flux density (max.)	$B = 6500$ lines per cm. ²	
Current density	$j = 0.87$ amperes per mm. ²	





STALLOY IRON
20 MILS THICK

10 SHEETS TO SET
+ 10 SETS TO PACKET

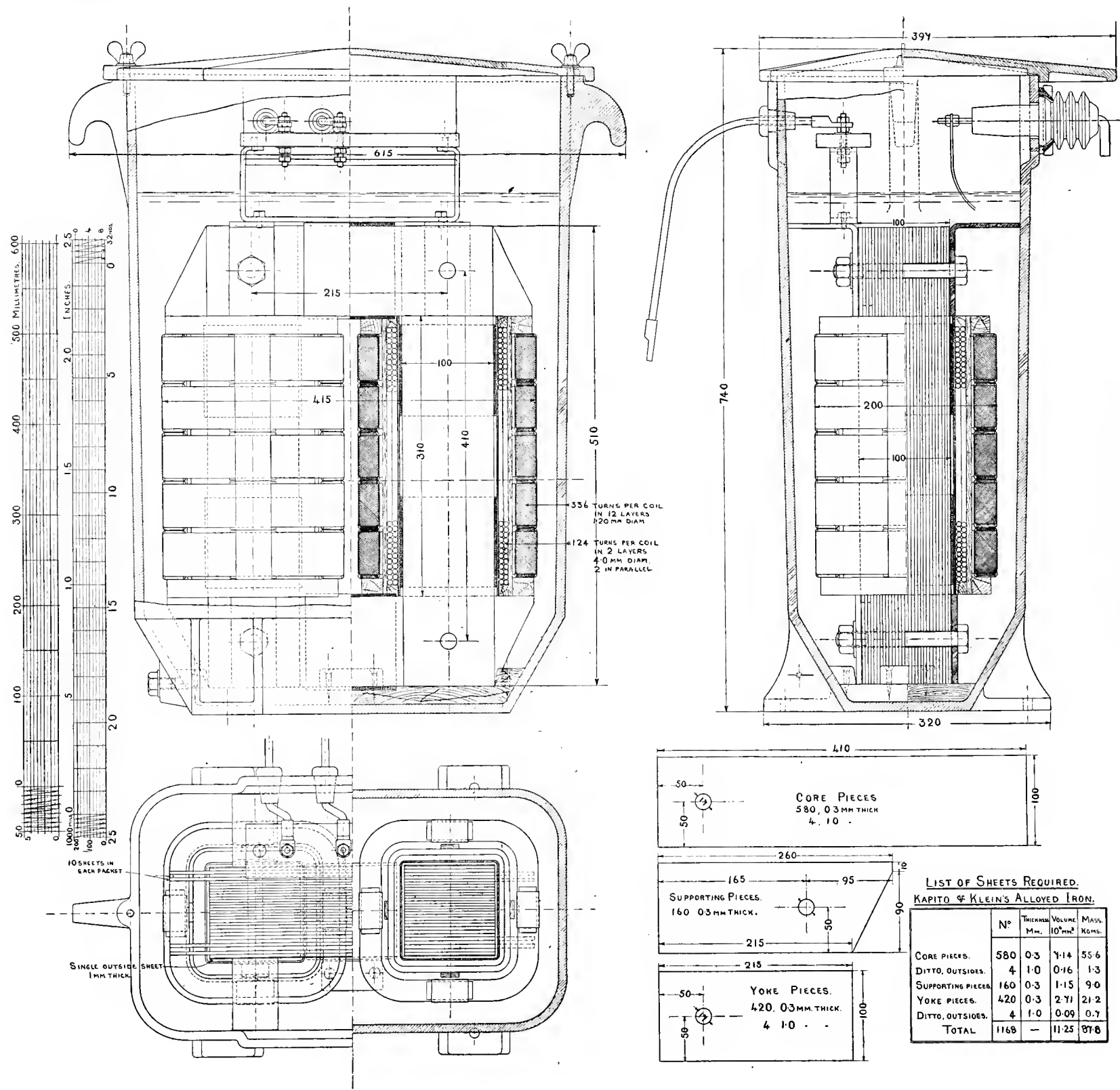
5 1/8" 2" 60° 2" 2" 60° 2" 120° 120° 120°

12 OUTSIDE PIECES
LIKE THIS →

Diagram illustrating a zigzag pattern for a quilt, showing dimensions and piece counts:

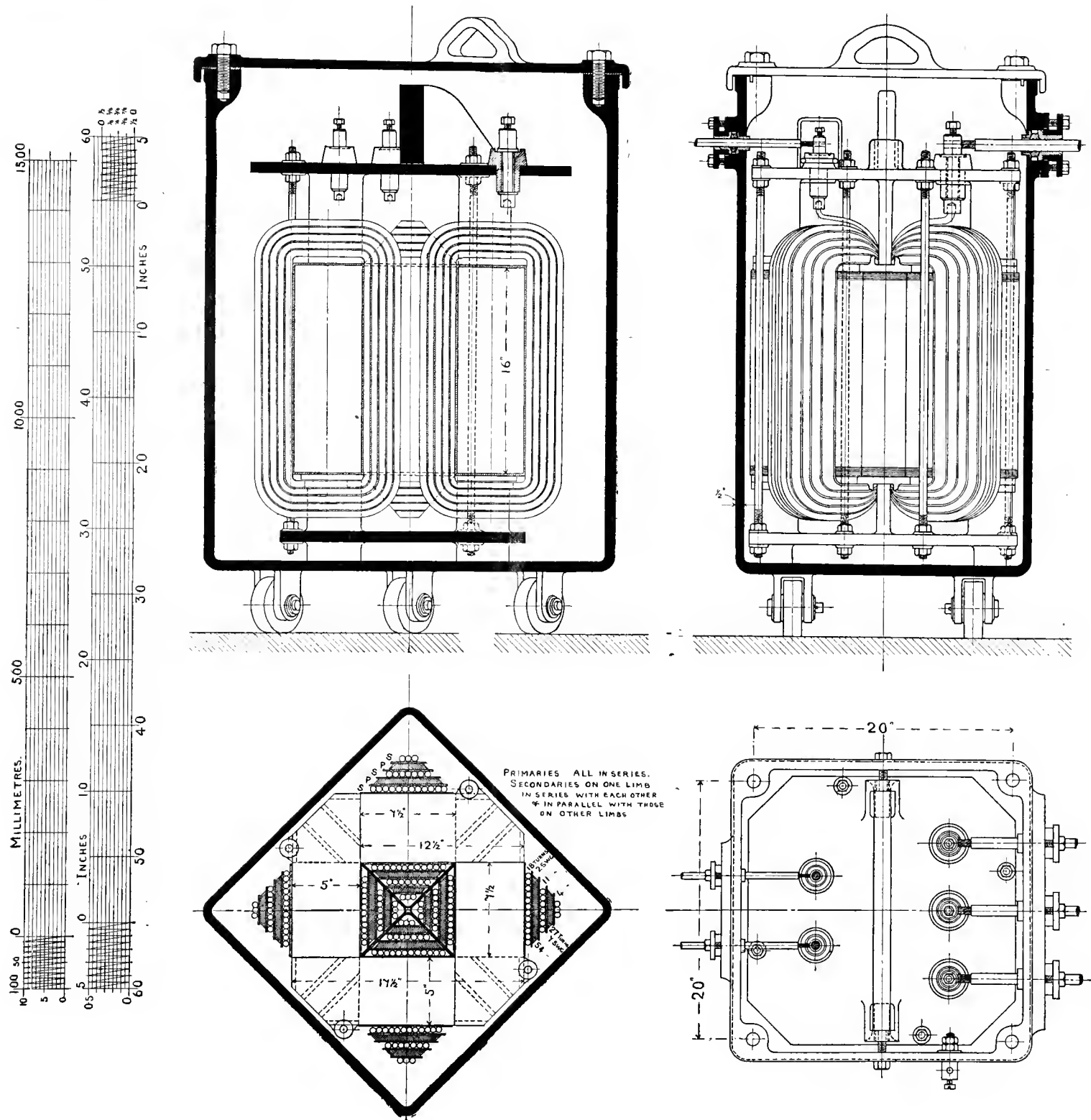
- Overall width: $28\frac{1}{2}$
- Overall height: $23\frac{1}{2}$
- Segment width: $5\frac{1}{8}$
- Segment height: $2\frac{1}{2}$
- Angle: 23°
- Segment length: $4\cdot26$
- Text: 1500 PIECES LIKE THIS
- Small dimension: $9\frac{1}{16}$ round

Iron loss	.	.	60 watts = 1.16 per cent.
Copper loss	.	113	„ = 2.18 „
Total	„	173	„ = 3.34 „
$R_1 = 9.7 \omega$ (hot). $R_2 = 0.076 \omega$. $R_{T_2} = 0.180 \omega$.			
$N_1 = 1224 = 6 \times 204$. $N_2 = 126 = 6 \times 21$.			
Iron mass	.	.	54 lbs.
Copper mass	.	.	37 „
Standard	„	.	206 „
$\beta = 0.58$ M.V.S./in. ² = 9000 lines/cm. ²			
$f = 1040$ amps./in. ² = 1.62 amps./mm. ²			



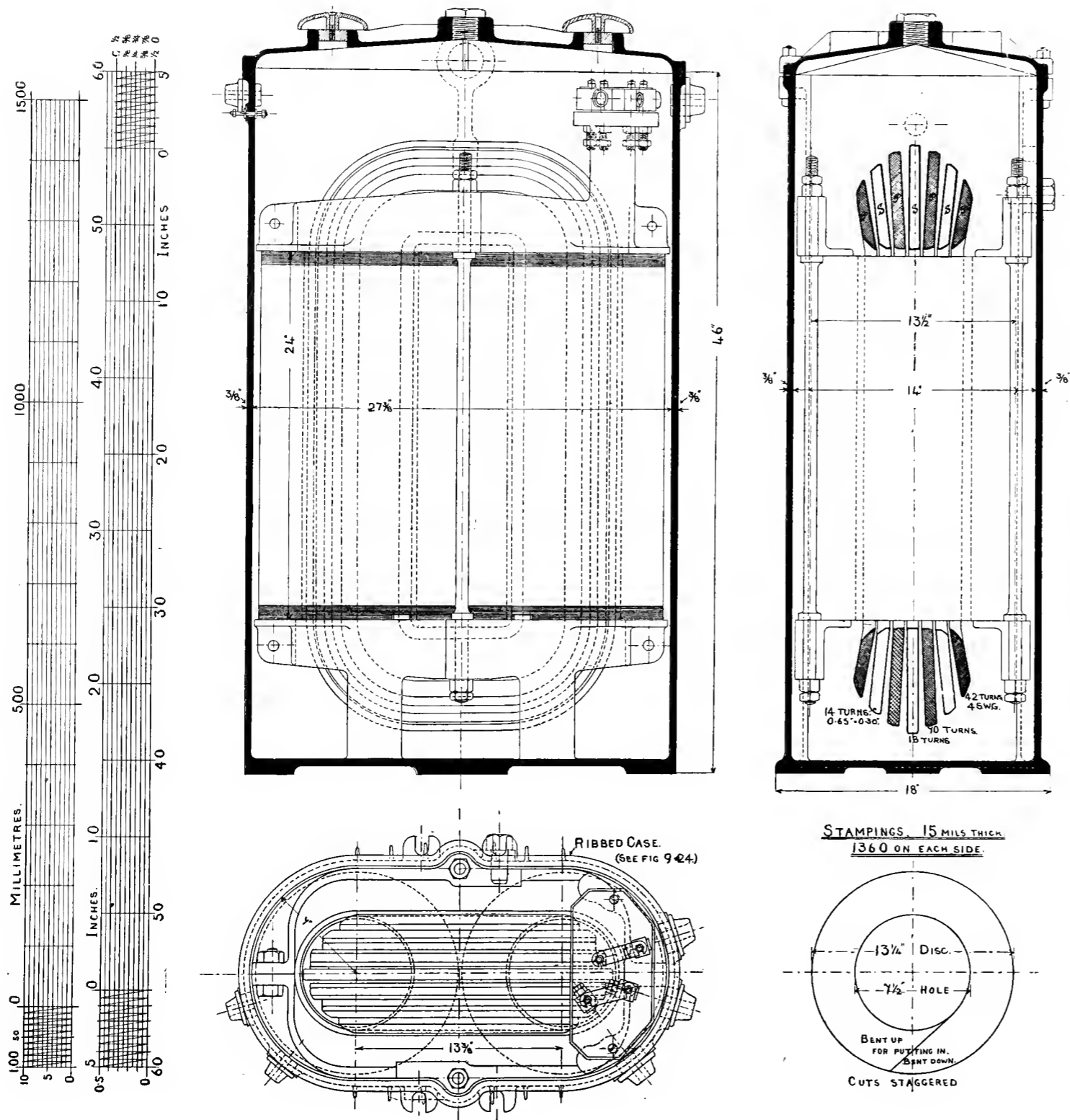
Oil-Insulated Transformer of the Square Coil Core Type to give 10 K.V.A. at 6350/230 Volts, 1.6/43 Amperes, and 50 Cycles per Second. Made by Gesellschaft für Elektrische Industrie, Karlsruhe.

Iron loss	146 watts	= 1.41 per cent.
Copper loss	215 „	= 2.07 „
Total „	361 „	= 3.48 „
Iron mass	87.8 kgs.	= 194 lbs.
Copper mass	34.4 „	= 76 „
Standard „	257 „	= 567 „
Flux density (max.)	9560 lines/cm. ²	= 0.617 M.V.S./in. ²
Current density (R.M.S.)	1.59 amps./mm. ²	= 1025 amps./in. ²
Resistance (hot): R ₁ = 43.1 ohms. . R ₂ = 0.0534 ohms. . R _{T2} = 0.114 ohms.		
Turns: N ₁ = 3360 = 10 × 336. . N ₂ = 124 = 1/2 × 2 × 124.		



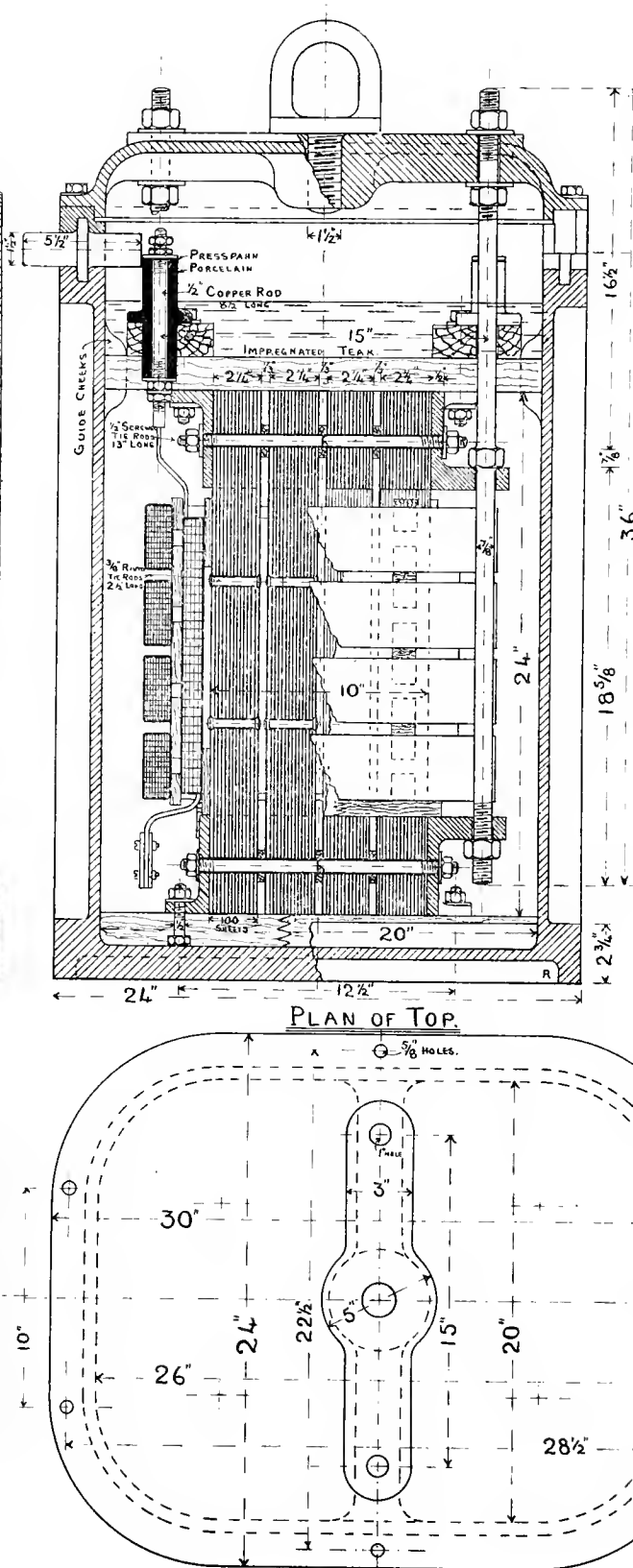
Oil-Insulated Transformer of the Rectangular Coil Square Ring Type, to give 50 K.V.A. at 2000 200 Volts, 25'7 250 Amperes, and 50 Cycles per Second. Made by W. E. Burnand & Co., Sheffield.

Iron loss	630 watts	= 1.22 per cent.
Copper loss	830 „	= 1.62 „
Total „	1460 „	= 2.84 „
Iron mass	434 kgs.	= 956 lbs.
Copper mass	122 „	= 269 „
Standard „	800 „	= 1763 „
Flux density (max.)	6350 lines/cm. ²	= 0.41 M.V.S./in. ²
Current density (R.M.S.)	1.63 amps./mm. ²	= 1050 amps./in. ²
Resistance (hot): R ₁ = 0.65 ohms.	R ₂ = 0.0064 ohms.		R _{T2} = 0.0133 ohms.				
Turns:	N₁ = 324 = 4(54 + 27). N₂ = 33 = $\frac{1}{4} \times 4(18 + 11 + 4)$.						



Oil-Insulated Transformer of the Rectangular Coil, Circular Iron, Shell Type to give 60 K.V.A. at 2000/404 Volts, 30·6/149 Amperes, and 50 Cycles per Second. Made by The Electric Construction Company, Limited, Wolverhampton. (See also fig. 9·23.)

Iron loss	527 watts	= 0·86 per cent.
Copper loss	725 „	= 1·18 „
Total „	1252 „	= 2·04 „
Iron mass	489 kgs.	= 1076 lbs.
Copper mass	213 „	= 470 „
Standard „	1128 „	= 2486 „
Flux density (max.)	5320 lines/cm. ²	= 0·343 M.V.S./in. ²
Current density (R.M.S.)	1·15 amps./mm. ²	= 7·45 amps./in. ²
Resistance (hot):	$R_1 = 0·375$ ohms. $R_2 = 0·0169$ ohms. $R_{T2} = 0·0327$ ohms.		
Turns:	$N_1 = 224 = 2(70 + 42)$. $N_2 = 46 = (14 + 18 + 14)$.		



Iron loss . . . 880 watts = 0.87 per cent.	Iron mass 625 lbs.
Copper loss . . . 920 „ = 0.90 „	Copper mass 457 „
Total „ . . . 1800 „ = 1.77 „	Standard „ 2463 „
$R_1 = 1.84\omega$. $R_2 = 0.185\omega$. $R_{T_2} = 0.44\omega$.	$\mathcal{B} = 0.87$ M.V.S./in. ² = 13,500 lines/cm. ²
$N_1 = 864 = 8 \times 108$. $N_2 = 320 = 2 \times 160$.	$\mathcal{I} = 850$ amps./in. ² = 1.32 amps./mm. ²

10 SHEETS.
10 SETS TO
PACKET.

19 1/2

11 1/2

4 1/2

3 3/8

5 5/8

2 1/4

6 1/2

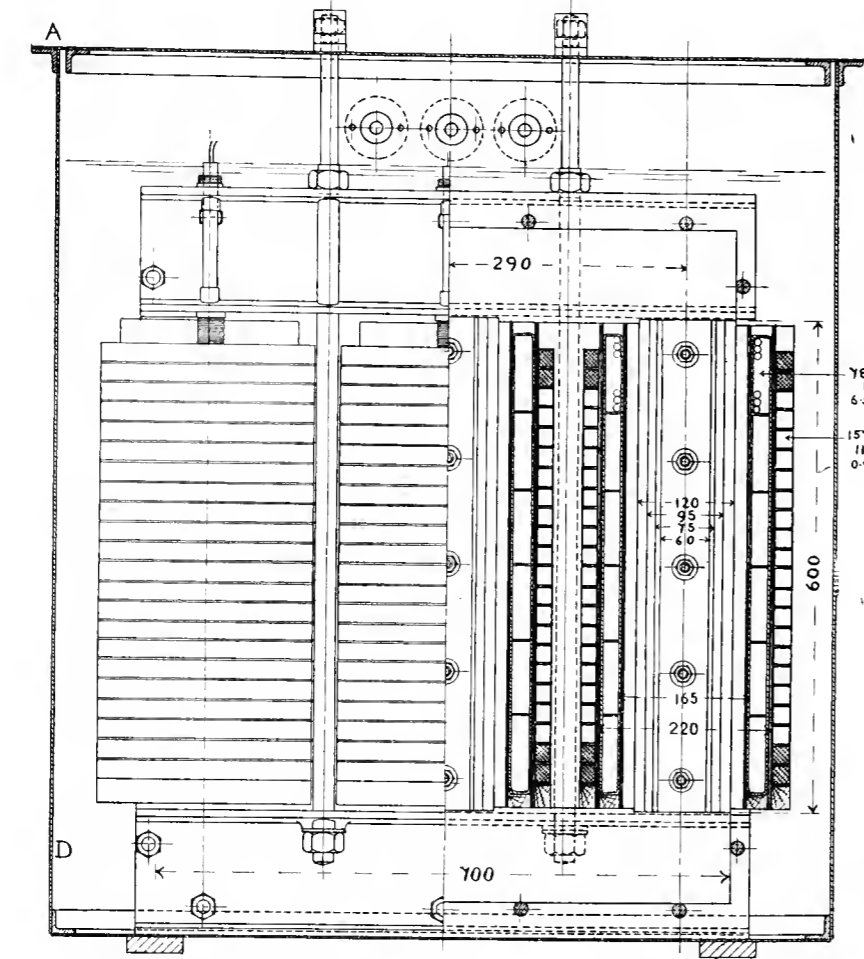
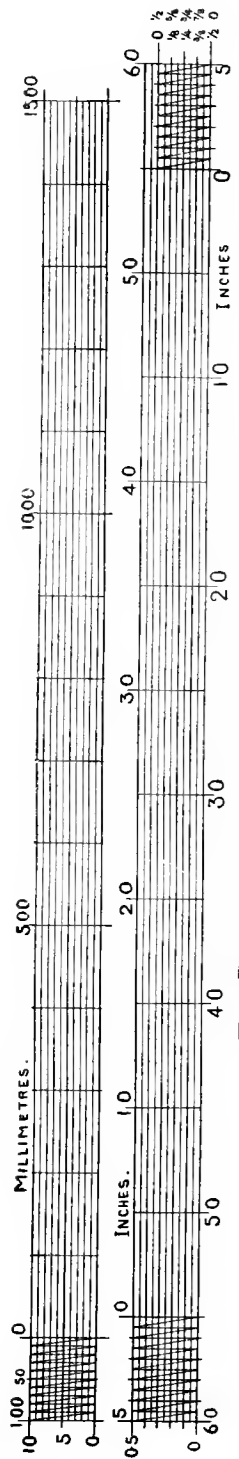
6 1/2

4 1/2

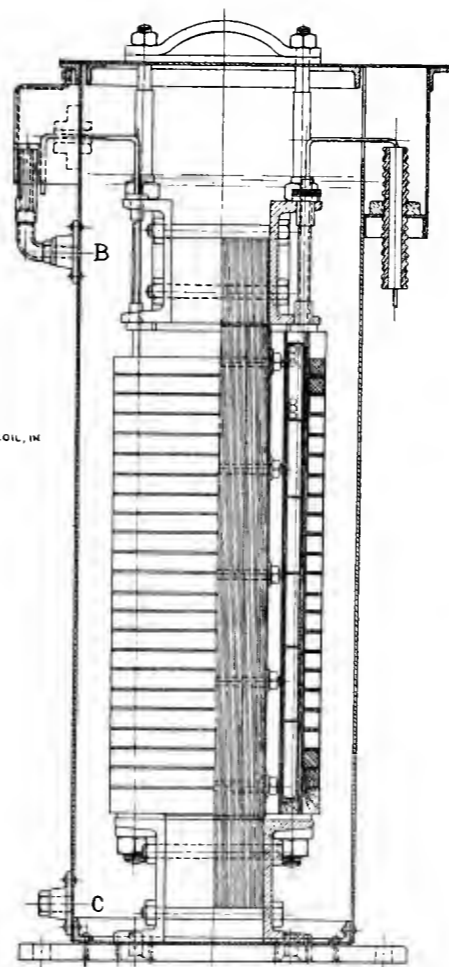
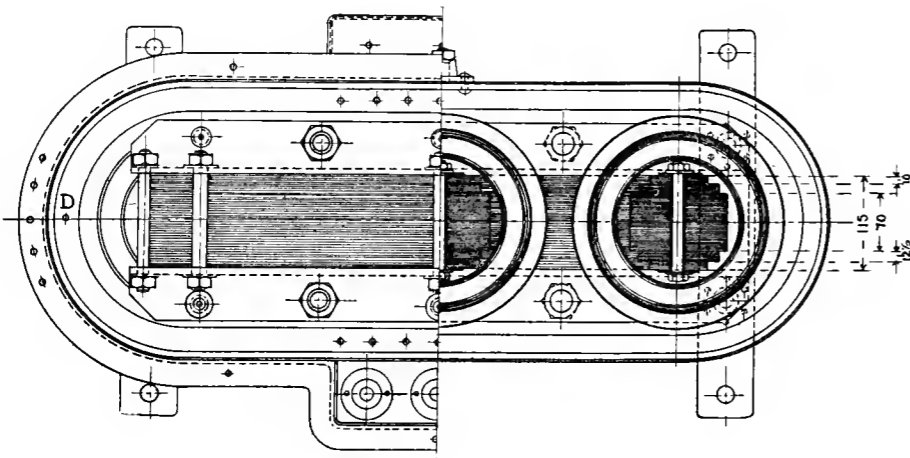
1 1/2

1/2

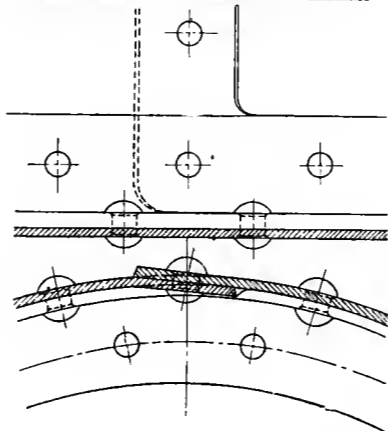
3/8



76 TURNS IN
1 LAYER
6.5 MM DIAM
157 TURNS PER COIL, IN
11 LAYERS
0.9 MM DIAM

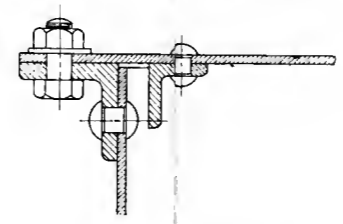


DETAIL OF JOINT IN CASE AT D.

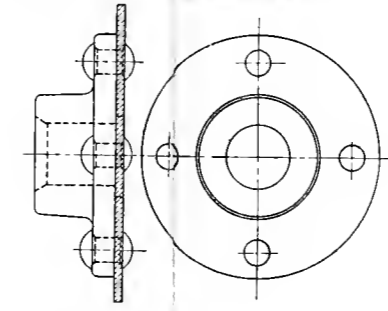


Oil-Insulated Three-Phase Transformer of the Circular Coil Three-Limb Type to give 20 K.V.A. at 10,000/220 Volts, 1.20/52.5 Amperes, and 40 Cycles per Second. Made by Elektricitäts Aktien Gesellschaft, Prague (formerly Kolben & Co.).

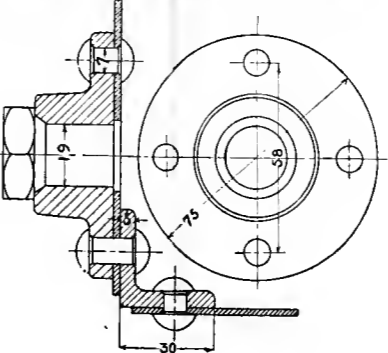
DETAIL OF EDGE OF LID AT A.



DETAIL OF NIPPLE AT B.



DETAIL OF LOWER CORNER C.

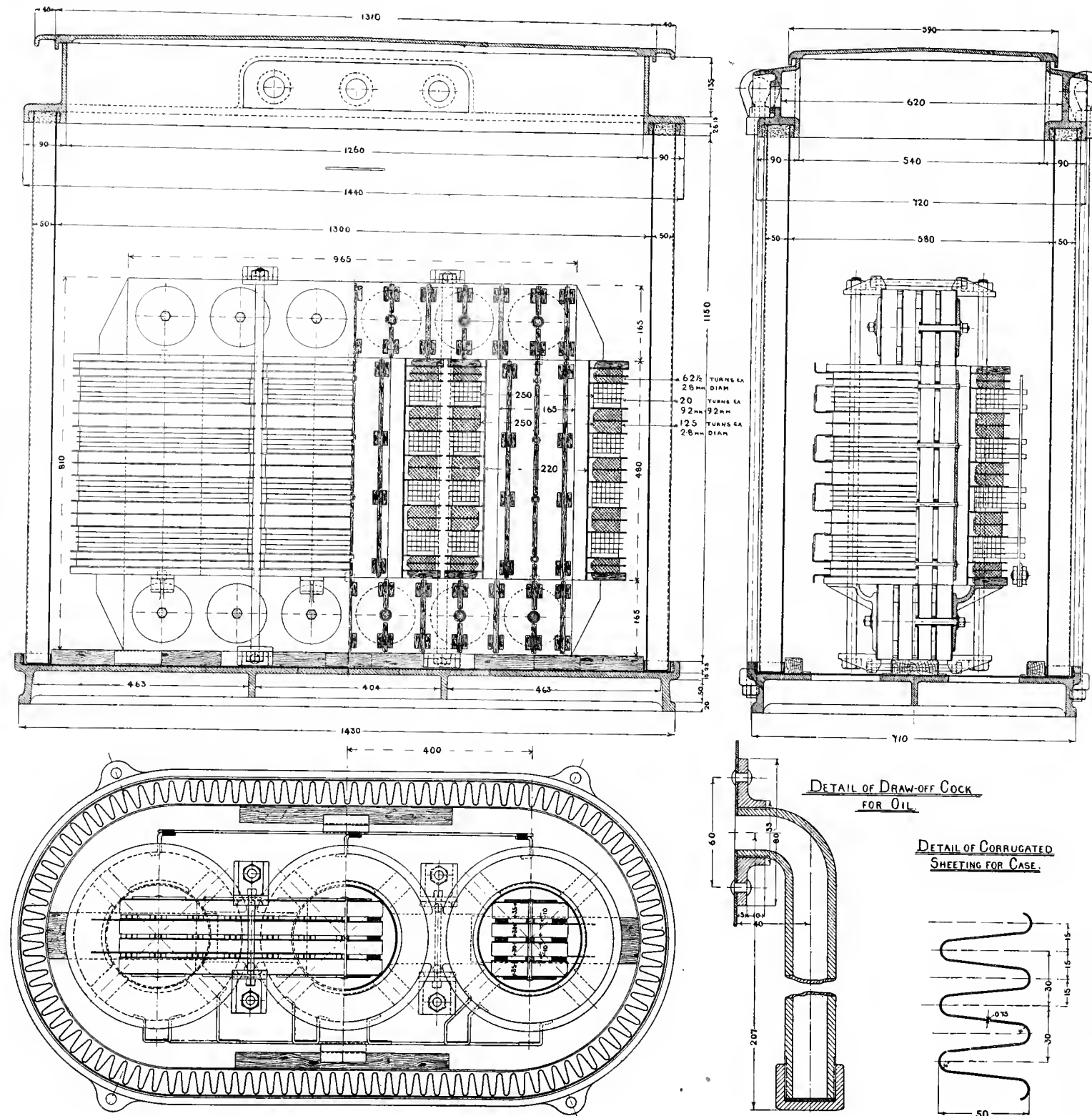
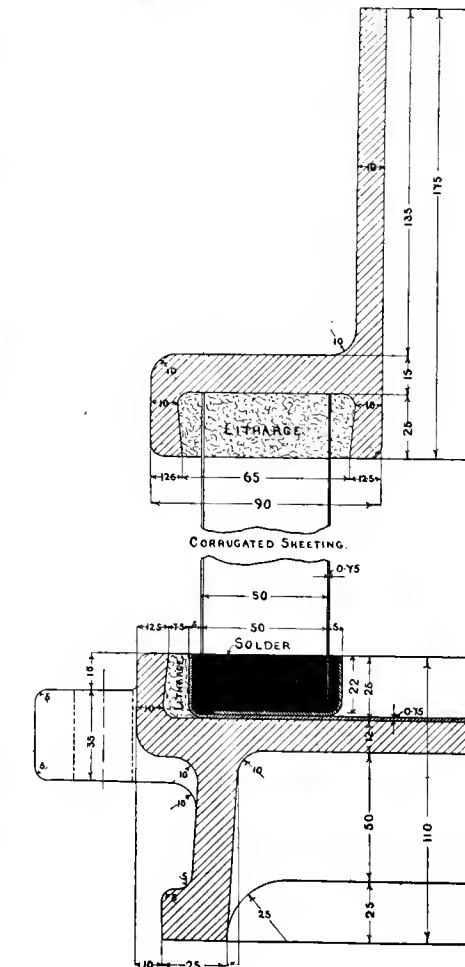


Iron loss	492 watts.	= 2.33 per cent.
Copper loss	640 „	= 3.03 „
Total „	1132 „	= 5.36 „
Iron mass	274 kgs.	= 604 lbs.
Copper mass	81 „	= 178 „
Standard mass of same cost	517 „	= 1140 „
Flux density (max.)	8560 lines/cm. ²	= 0.553 M.V.S./in. ²
Current density (R.M.S.) . .	1.73 amps./mm. ²	= 1115 amps./in. ²
Resistance (hot): R ₁ = 92.3 ohms.		
	R ₂ = 0.0292 „	
	R _{T2} = 0.0775 „	
Turns per phase: N ₁ = 3454 = 22 × 157.		
	N ₂ = 78.	

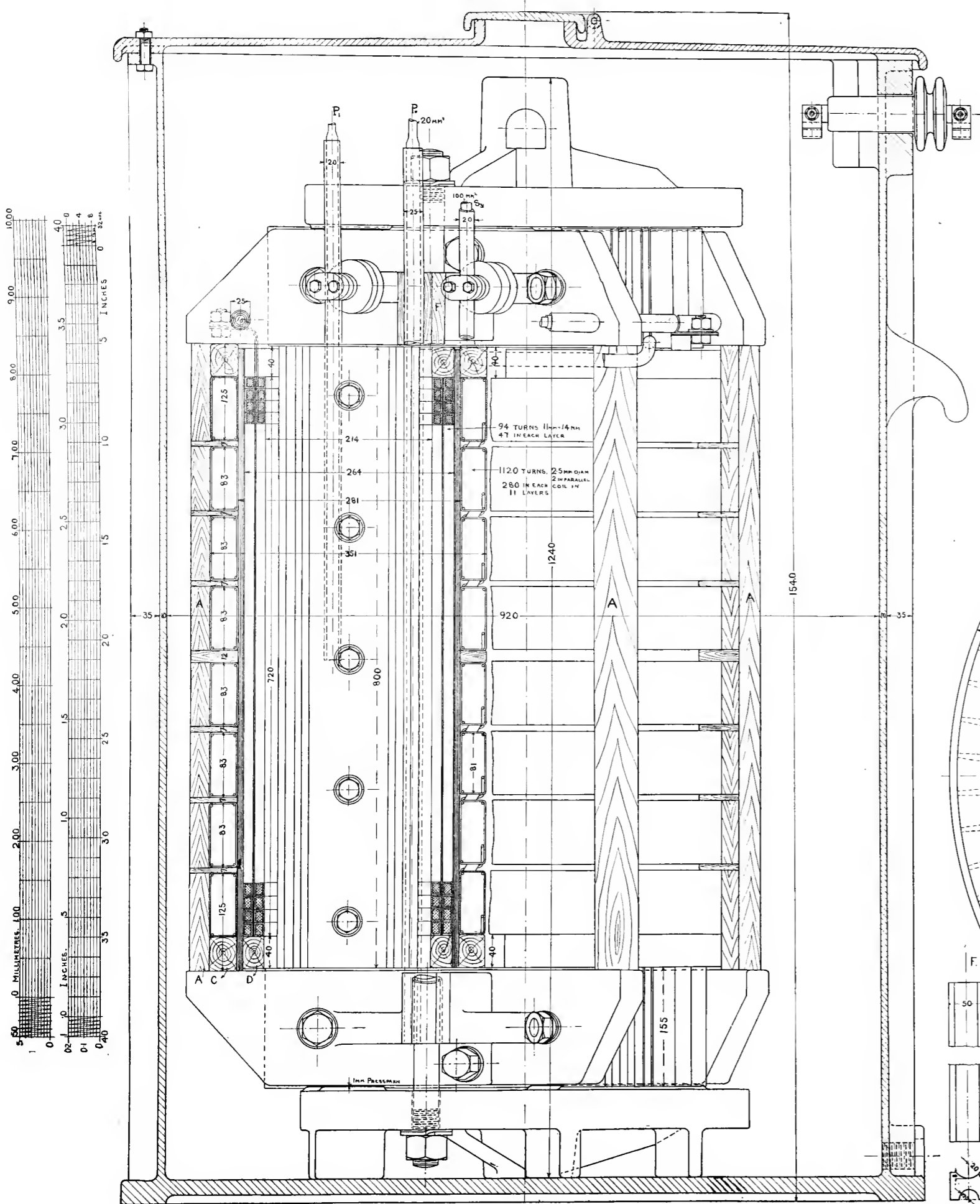
Oil-Insulated Three-Phase Transformer of the Circular Coil, Three-Limb Type to give 75 K.V.A. at 6300/120 Volts, 710/361 Amperes, and 50 Cycles per Second. Made by Ateliers de Constructions Electriques de Charleroi.

Iron loss	1170 watts	= 1.52 per cent.
Copper loss	1068 "	= 1.38 "
Total	„	2238 "	= 2.90 "
Iron mass	507 kgs.	= 1118 lbs.
Copper mass	336 "	= 741 "
Standard mass of same cost	1515 "	= 3340 "
Flux density (max.)	8500 lines/cm. ²	= 0.548 M.V.S./in. ²
Current density (R.M.S.)	1.11 amps./mm. ²	= 716 amps./in. ²
Resistance (hot):	R ₁ = 3.76 ohms.	R ₂ = 0.00128 ohms.	R _{T2} = 0.00273 ohms.				
Turns per phase:	N ₁ = 1000 = (6 × 125 + 4 × 62½).	N ₂ = 20 = ¼ × 4 × 20.					

DETAIL OF JOINTS IN CASE.

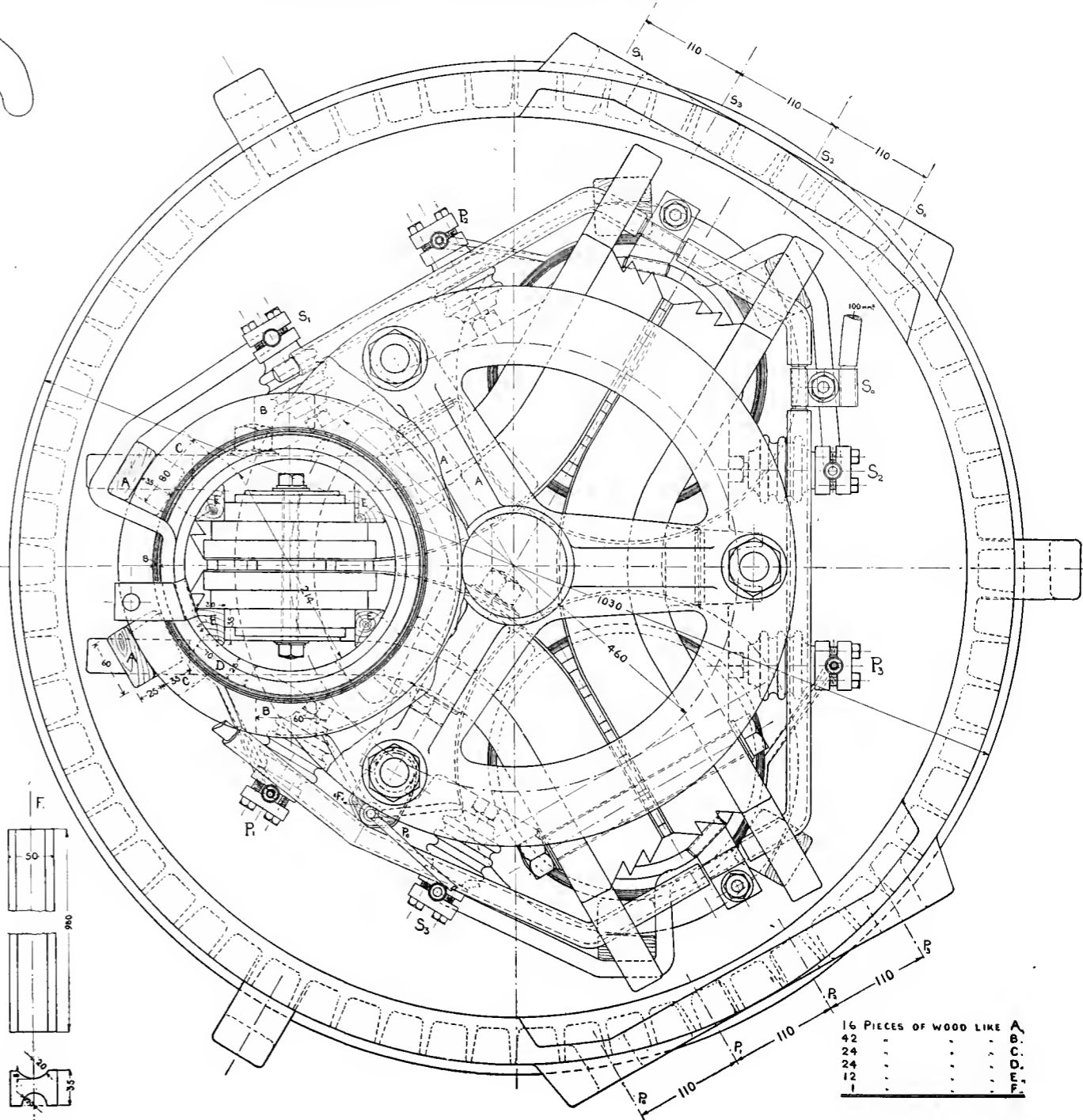


THE DETAILS ARE DRAWN TO FOUR TIMES THIS SCALE.

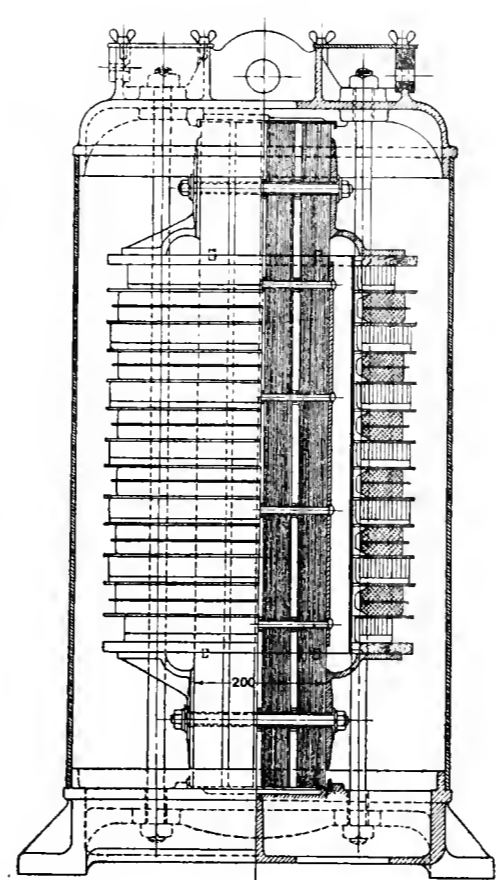
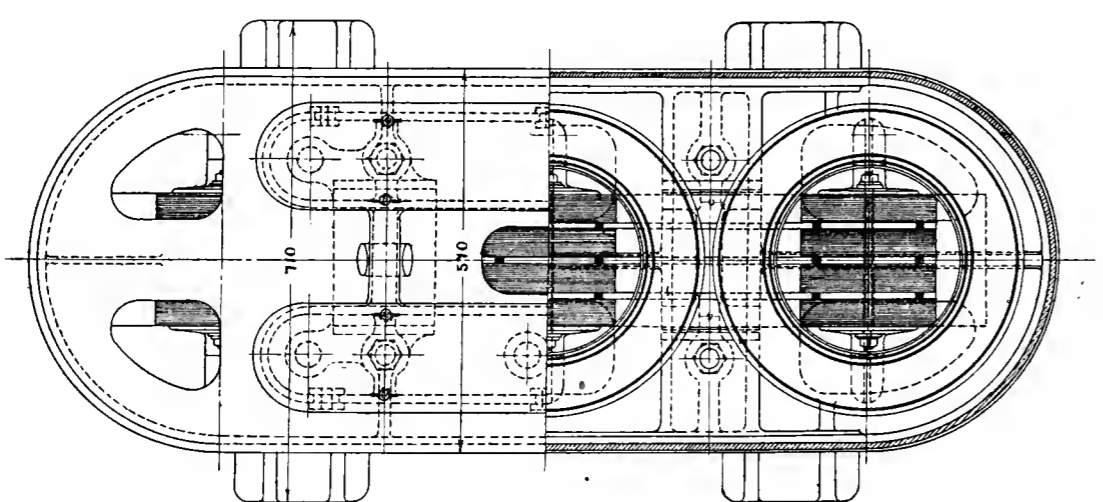
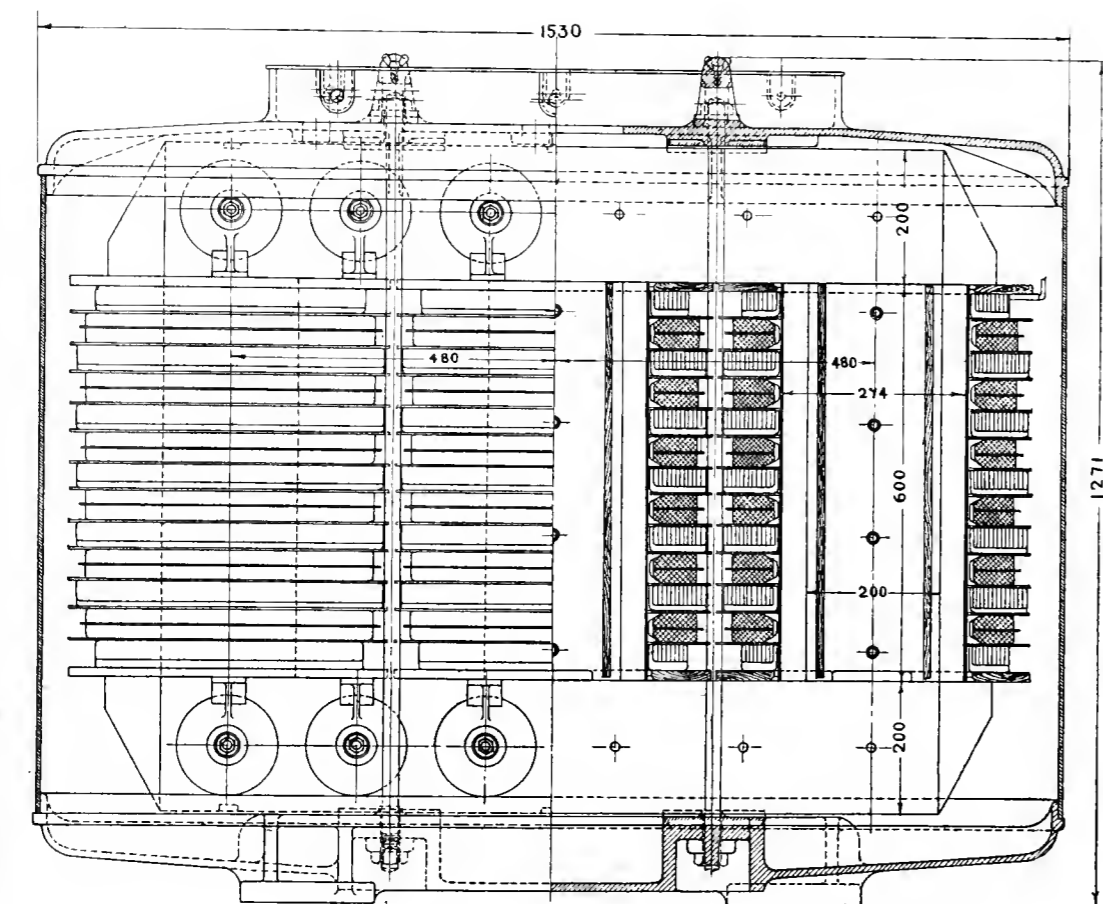
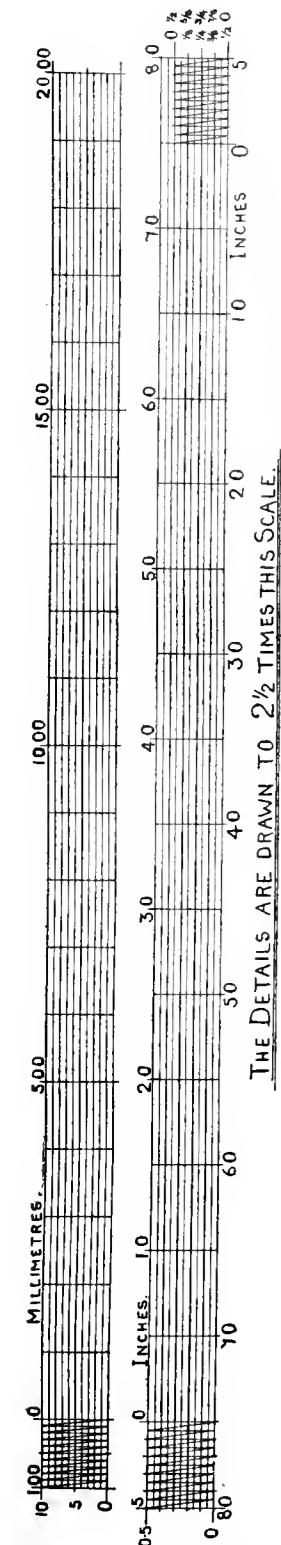


Oil-Insulated Three-Phase Transformer of the Circular Coil Symmetrical Type to give 110 K.V.A. at 5500/440 Volts, 11.9/145 Amperes, and 50 Cycles per Second. Made by Brown, Boveri & Co., Baden.

Iron loss	1230 watts	= 1.09 per cent.
Copper loss	1785 "	= 1.58 "
Total "	3015 "	= 2.67 "
Iron mass	609 kgs.	= 1340 lbs.
Copper mass	587 "	= 1292 "
Standard mass of same cost	2370 "	= 5220 "
Flux density (max.)	7820 lines/cm. ²	= 0.504 M.V.S./in. ²
Current density	1.08 amps./mm. ²	= 696 amps./in. ²
Resistance (hot):	$R_1 = 2.60$ ohms. $R_2 = 0.0108$ ohms. $R_T = 0.0283$ ohms.	
Turns per phase:	$N_1 = 1120 = \frac{1}{2} \times 8 \times 280$. $N_2 = 94$.	

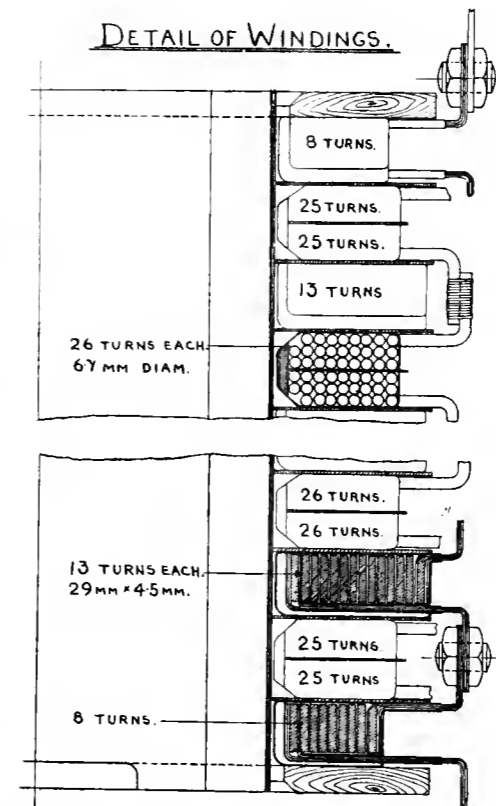


16 PIECES OF WOOD LINE A
42
24
24
12
1



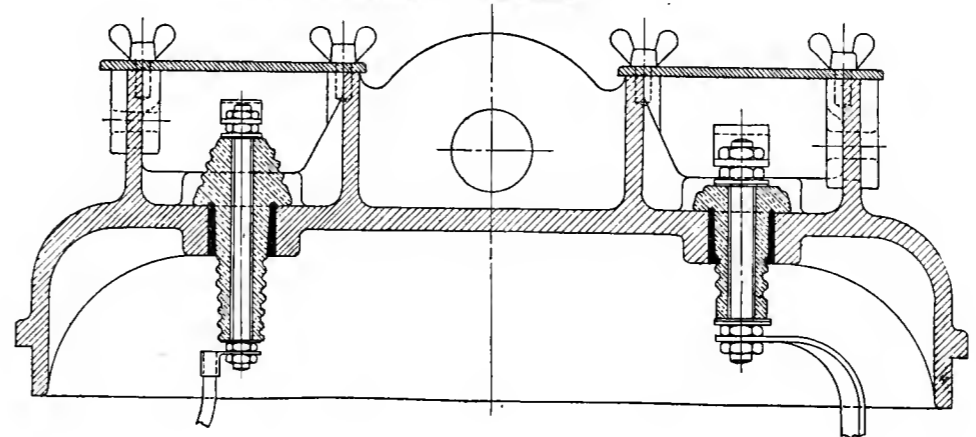
Oil-Insulated Three-Phase Transformer of the Circular Coil Three-Limb Type to give 130 K.V.A. at 550/2000 Volts, 140/37.5 Amperes, and 50 Cycles per Second. Made by Ateliers de Constructions Electriques de Charleroi.

DETAIL OF WINDINGS.



Iron loss	1215 watts	= 0.91 per cent.
Copper loss	1900 „	= 1.43 „
Total „	3115 „	= 2.34 „
Iron mass	948 kgs.	= 2090 lbs.
Copper mass	642 „	= 1420 „
Standard mass of same cost .	2874 „	= 6350 „
Flux density (max.)	5990 lines/cm. ²	= 0.386 M.V.S./in. ²
Current density (R.M.S.) . .	1.068 amps./mm. ²	= 688 amps./in. ²
Resistance (hot): R ₁ = 0.0161 ohms.		
R ₂ = 0.225 „		
R _{T2} = 0.451 „		
Turns per phase: N ₁ = 81 = (5 × 13 + 2 × 8).		
N ₂ = 304 = (8 × 25 + 4 × 26).		

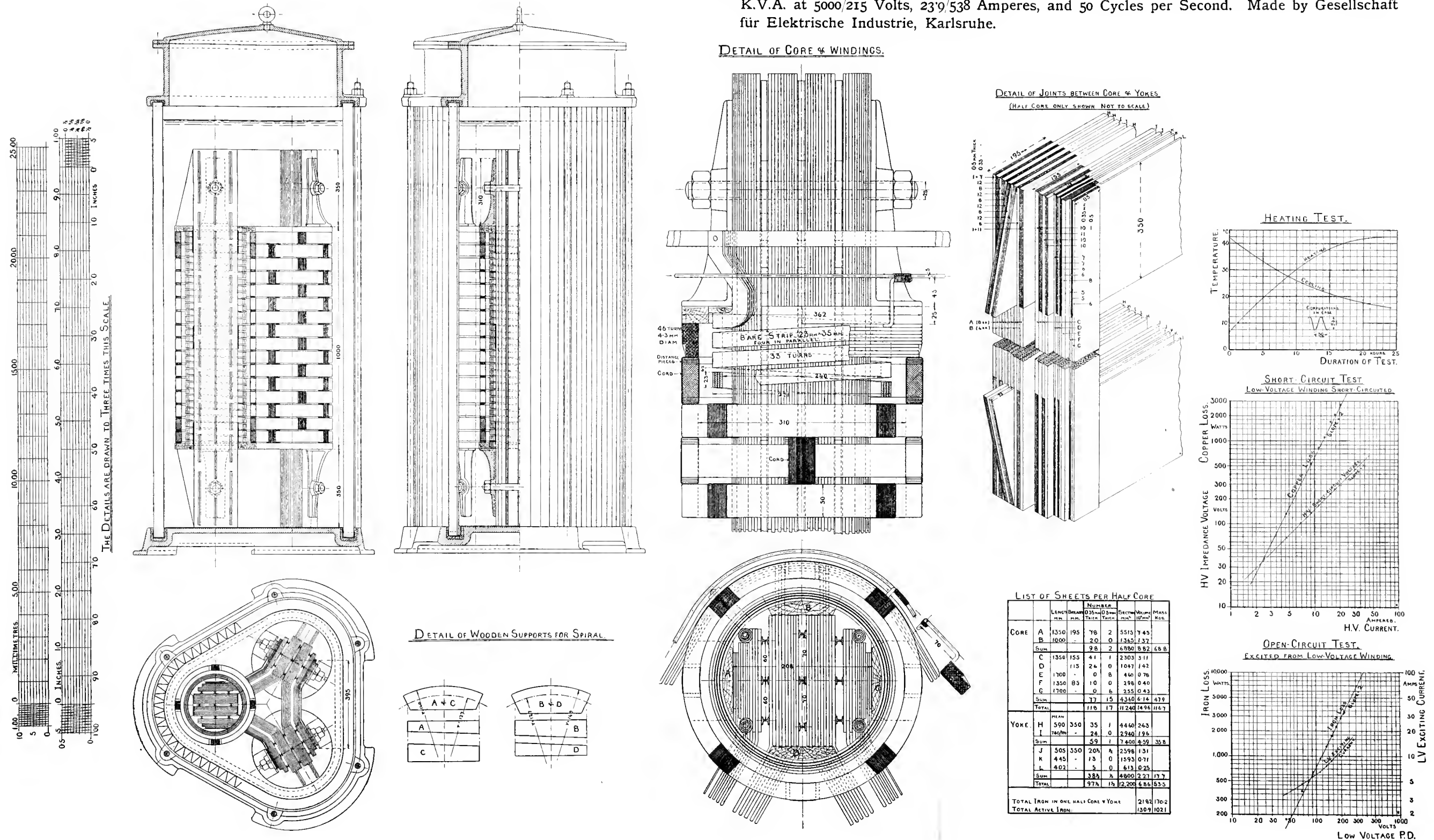
DETAIL OF TERMINALS & COVER.



Oil-Insulated Three-Phase Transformer of the Circular Coil Symmetrical Type to give 200 K.V.A. at 5000/215 Volts, 239/538 Amperes, and 50 Cycles per Second. Made by Gesellschaft für Elektrische Industrie, Karlsruhe.

DETAIL OF CORE & WINDINGS.

DETAIL OF JOINTS BETWEEN CORE & YOKES
(HALF CORE ONLY SHOWN NOT TO SCALE)



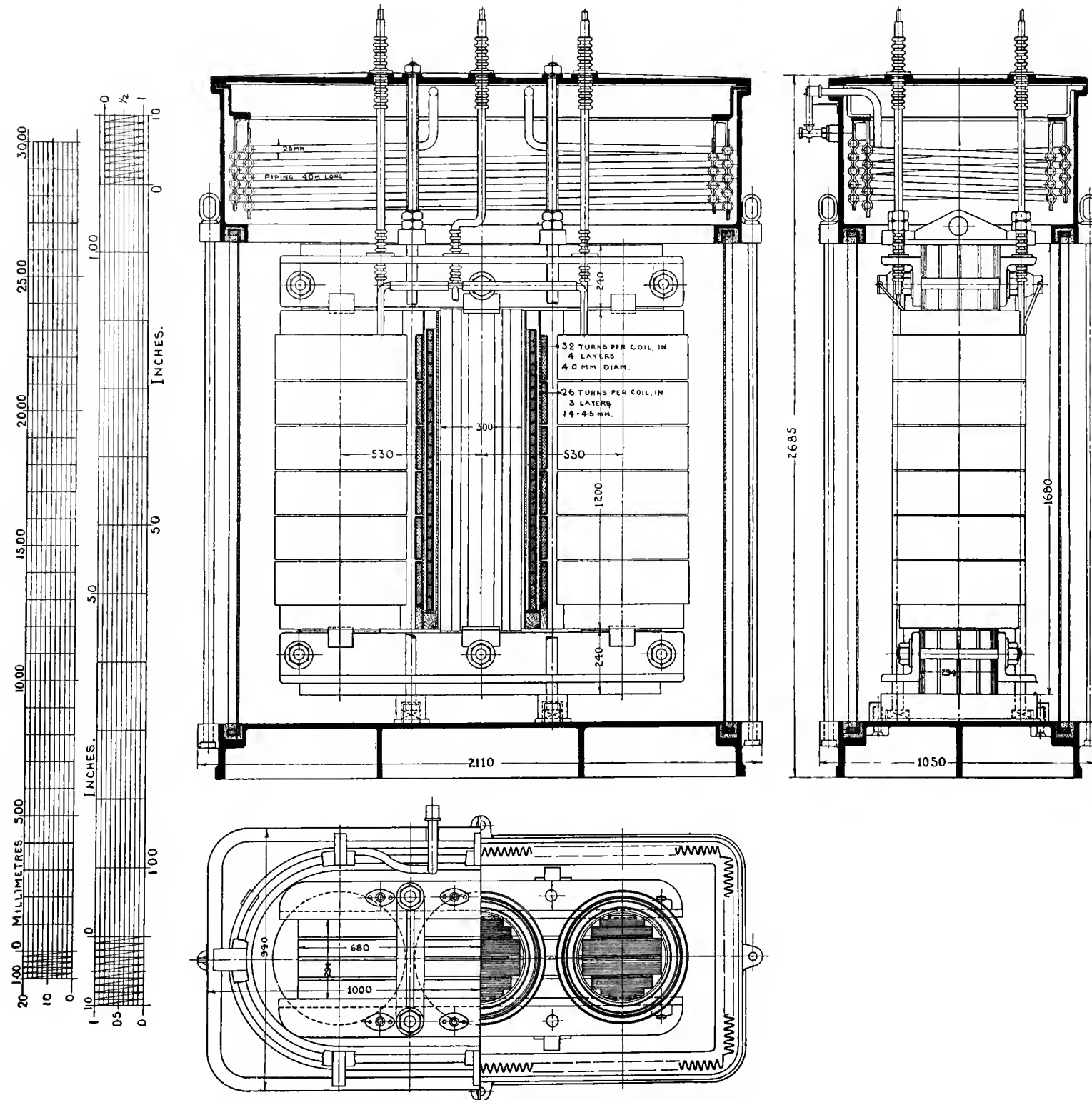
Leading Particulars.

Iron loss	.	.	.	3200 watts = 1.55 per cent.
Copper loss	.	.	.	3760 „ = 1.82 „
Total	„	.	.	6960 „ = 3.37 „

Iron mass	1021 kgs. = 2250 lbs.
Copper mass	530 „ = 1170 „
Standard mass of same cost .	2611 „ = 5750 „

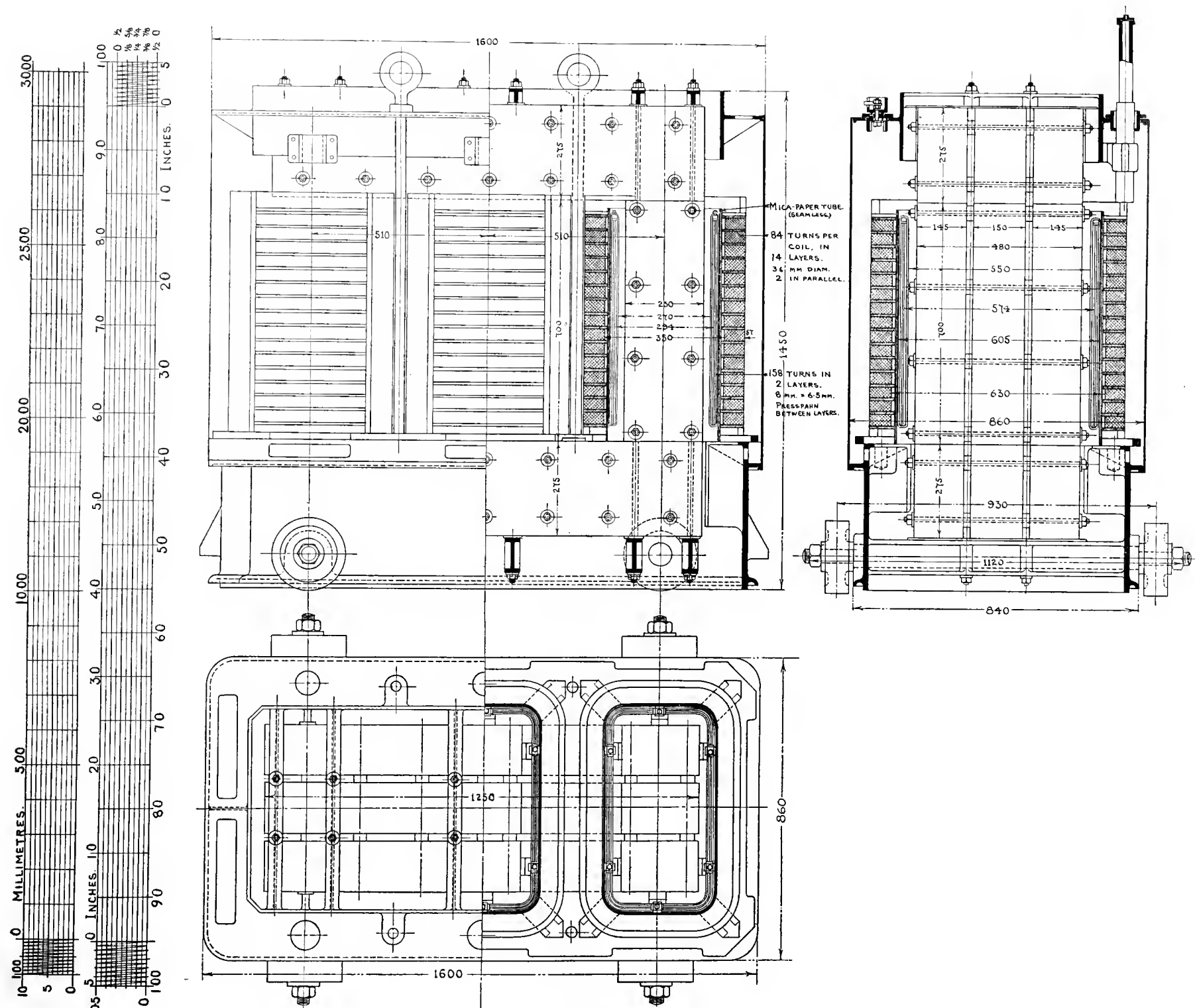
Flux density (max.) . . . 9080 lines/cm.² = 0.585 M.V.S./in.²
 Current density (R.M.S.) . . 1.66 amps./mm.² = 1070 amps./in.²
 Turns per phase: $N_1 = 736 = 16 \times 46$. $N_2 = 33$.

Resistance (hot): $R_1 = 1.24$ ohms.
 $R_2 = 0.00189$ ohms.
 $R_{T2} = 0.00433$ „



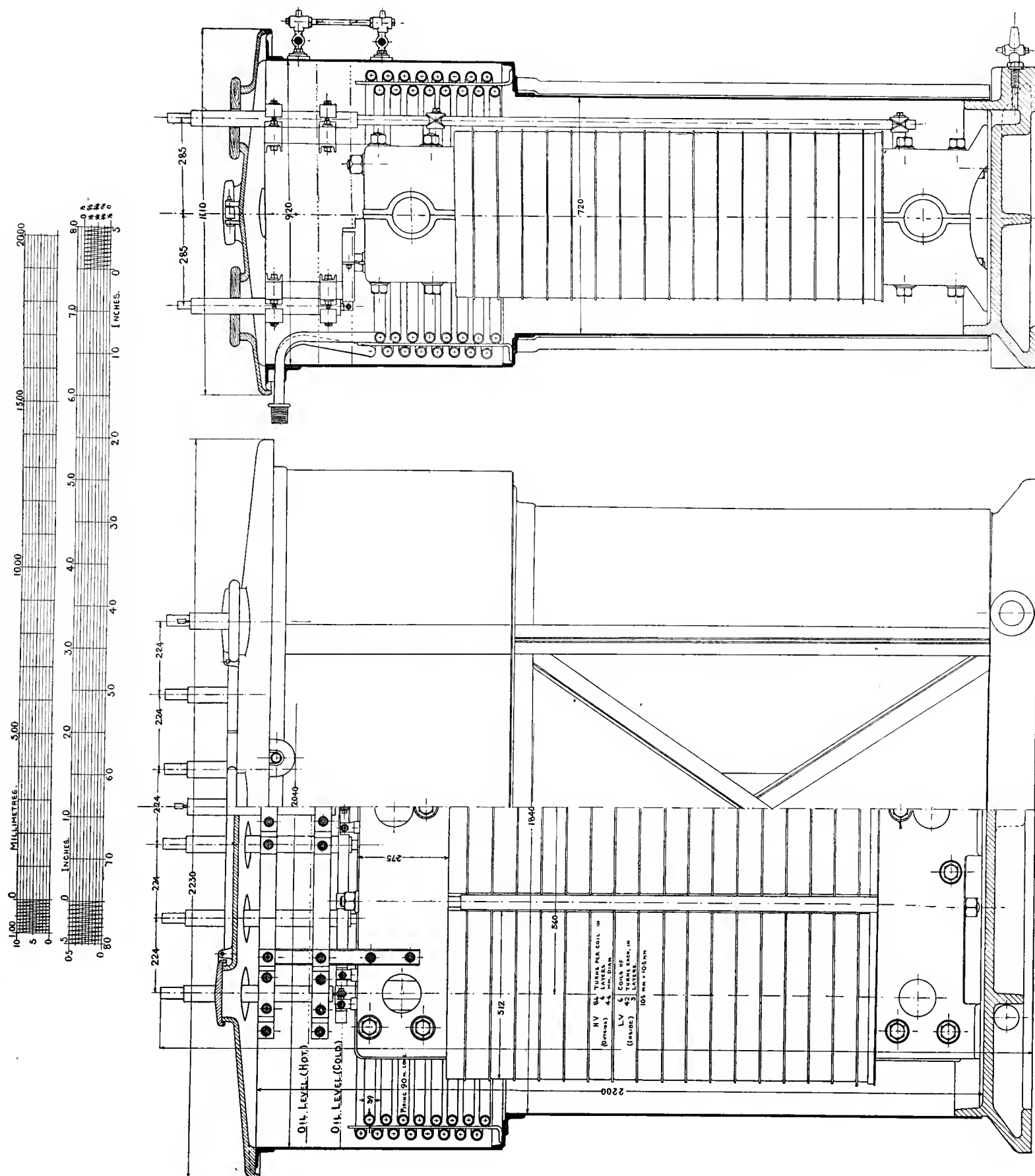
Oil-Insulated,* Water-Cooled, Three-Phase Transformer of the Circular Coil Three-Limb Type to give 350 K.V.A. at 10,000/2000 Volts, 20.7/101 Amperes, and 40 Cycles per Second. Made by Elektricitäts Aktien Gesellschaft, Prague (late Kolben & Co.).

Iron loss	3960 watts	= 1.10 per cent.
Copper loss	4755 „	= 1.33 „
Total „	8715 „	= 2.43 „
Iron mass	2810 kgs.	= 6,200 lbs.
Copper mass	700 „	= 1,540 „
Standard mass of same cost	4910 „	= 10,820 „
Flux density (max.)	7430 lines/cm. ²	= 0.479 M.V.S./in. ²
Current density (R.M.S.)	1.63 amps./mm. ²	= 1050 amps./in. ²
Resistance (hot): $R_1 = 1.72$ ohms. $R_2 = 0.0835$ ohms. $R_{T2} = 0.155$ ohms.		
Turns per phase: $N_1 = 768 = 24 \times 32$. $N_2 = 156 = 6 \times 26$.		



Three-Phase Transformer of the Rectangular Coil Three-Limb Type, with Forced Draught, to give 770 K.V.A. at 5000/20,000 Volts, 91/22·2 Amperes, and 50 Cycles per Second. Made by Maschinenfabrik Oerlikon, Oerlikon, Switzerland, for Drammen, Scandinavia. (See also figs. 9·30 and 11·01.)

Iron loss	10,000 watts	= 1.27 per cent.
Copper loss	5,340 "	= 0.68 "
Total „	15,340 "	= 1.95 "
Iron mass	3,510 kgs.	= 7,740 lbs.
Copper mass	1,122 "	= 2,475 "
Standard mass of same cost	6,876 "	= 15,150 "
Flux density (max.)	9,300 lines/cm. ²	= 0.600 M.V.S./in. ²
Current density (R.M.S.)	1.42 amps./mm. ²	= 915 amps./in. ²
Resistance (hot): R ₁ = 0.125 ohms. R ₂ = 1.515 ohms. R _{T2} = 3.61 ohms.		
Turns per phase: N ₁ = 158. N ₂ = 630 = $\frac{1}{3} \times 15 \times 84$.		



Oil-Insulated, Water-Cooled, Three-Phase Transformer of the Circular Coil Three-Limb Type to give 1400 K.V.A. at 3000/26,000 Volts, 274/311 Amperes, and 50 Cycles per Second. Made by Brown, Boveri & Co., Baden.

Iron loss	11,400 watts	= 0.80 per cent.
Copper loss	12,600 „	= 0.88 „
Total „	24,000 „	= 1.68 „
Iron mass	2,230 kgs.	= 4,910 lbs.
Copper mass	1,750 „	= 3,860 „
Standard mass of same cost	7,480 „	= 16,500 „
Flux density (max.)	12,500 lines/cm. ²	= 0.806 M.V.S./in. ²
Current density (R.M.S.)	1.65 amps/mm. ²	= 1063 amps./in. ²
Resistance (hot): R ₁ = 0.0610 ohms.	R ₂ = 2.77 ohms.	R _{T2} = 4.35 ohms.
Turns per phase : N ₁ = 252 = 6 × 42.	N ₂ = 1344 = 16 × 84.	

